UNIT IV

MULTIPATH MITIGATION TECHNIQUES

Equalisation – Adaptive equalization, Linear and Non-Linear equalization, Zero forcing and LMS Algorithms. Diversity – Micro and Macrodiversity, Diversity combining techniques, Error probability in fading channels with diversity reception, Rake receiver,

INTRODUCTION

Equalization, diversity, and channel coding are three techniques which can be used independently or in tandem to improve received signal quality.

Equalization compensates for intersymbol interference (ISI) created by multipath within time dispersive channels. If the modulation bandwidth exceeds the coherence bandwidth of the radio channel, ISI occurs and modulation pulses are spread in time. An equalizer within a receiver compensates for the average range of expected channel amplitude and delay characteristics. Equalizers must be adaptive since the channel is generally unknown and time varying.

Diversity is another technique used to compensate for fading channel impairments, and is usually implemented by using two or more receiving antennas. As with an equalizer, the quality of a mobile communications link is improved without increasing the transmitted power or bandwidth. However, while equalization is used to counter the effects of time dispersion (ISI), diversity is usually employed to reduce the depth and duration of the fades experienced by a receiver in a flat fading (narrowband) channel.

Diversity techniques can be employed at both base station and mobile receivers. The most common diversity technique is called spatial diversity, whereby multiple antennas are strategically spaced and connected to a common receiving system. While one antenna sees a signal null, one of the other antennas may see a signal peak, and the receiver is able to select the antenna with the best signal at any time. Other diversity techniques include antenna polarization diversity, frequency diversity, and time diversity. CDMA systems often use a RAKE receiver, which provides link improvement through time diversity.

Channel coding improves mobile communication link performance by adding redundant data bits in the transmitted message. At the baseband portion of the transmitter, a channel coder maps a digital message sequence into another specific code sequence containing a greater number of bits than originally contained in the message. The coded message is then modulated for transmission in the wireless channel.

Channel coding is used by the receiver to detect or correct some (or all) of the errors introduced by the channel in a particular sequence of message bits. Because decoding is performed after the demodulation portion of the receiver, coding can be considered to be a post detection technique. The added coding bits lowers the raw data transmission rate through the channel (expands the occupied bandwidth for a particular message data rate). There are two general types of channel codes: block codes and convolutional codes. Channel coding is generally treated independently from the type of modulation used, although this has changed recently with the use of trellis coded modulation schemes that combine coding and modulation to achieve large coding gains without any bandwidth expansion.

The three techniques of equalization, diversity, and channel coding are used to improve radio link performance (i.e. to minimize the instantaneous bit error rate), but the approach, cost, complexity, and effectiveness of each technique varies widely in practical wireless communication systems.

PRINCIPLE OF DIVERSITY

Diversity is a technique used to compensate for fading channel impairments, and is usually implemented by using two or more receiving antennas.

Diversity is usually employed to reduce the depth and duration of the fades experienced by a receiver in a flat fading (narrowband) channel. Diversity techniques can be employed at both base station and mobile receivers.

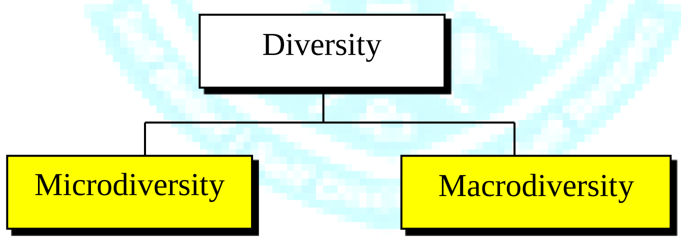
The principle of diversity is to ensure that the same information reaches the receiver (RX) on statistically independent channels.

The basic principle of diversity is that the RX has multiple copies of the transmit signal, where each of the copies goes through a statistically independent channel.

In Rayleigh fading, the BER decreases only linearly with the SNR. The reason for this different performance is the fading of the channel. A way to improve the BER is thus to change the effective channel statistics. Diversity is a way to achieve this.

Consider the simple case of an RX with two antennas. The antennas are assumed to be far enough from each other that small-scale fading is independent at the two antennas. As the signals are statistically independent, the probability that both antennas are in a fading dip simultaneously is low. The diversity thus changes the SNR statistics at the detector input.

Diversity is a powerful communication receiver technique that provides wireless link improvement at relatively low cost. Diversity exploits the random nature of radio propagation by finding independent (or at least highly uncorrelated) signal paths for communication.



To combat small-scale fading To combat large-scale fading

There are two types of fading

1. Small-Scale fading
2. Large-Scale fading.

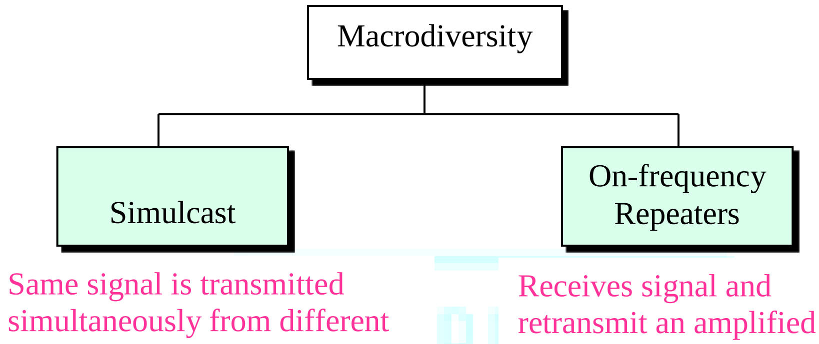
Small-scale fades are characterized by deep and rapid amplitude fluctuations. In order to prevent deep fades from occurring, **microscopic diversity techniques** are used.

Large-scale fading is caused by shadowing due to variations in both the terrain profile and the nature of the surroundings. In order to prevent large-scale fades from occurring, **microscopic diversity techniques** are used.

MACRODIVERSITY

Macrodiversity is used to combat large-scale fading due to shadowing. Macrodiversity can be achieved by

1. Simulcast
2. On-frequency repeaters



SIMULCAST

In **simulcast**, the same signal is transmitted simultaneously from different BSs. In cellular applications the two BSs should be synchronized, and transmit the signals intended for a specific user in such a way that the two waves arrive at the RX almost simultaneously.

Simulcast is also widely used for broadcast applications, especially digital TV. In this case, the exact synchronization of all possible RXs is not possible. Each RX would require a different timing advance from the TXs.

Disadvantage:
A large amount of signaling information has to be carried on landlines. Synchronization information as well as transmit data has to be transported on landlines (or microwave links) to the BSs.

ON-FREQUENCY REPEATERS

The simplest method for macrodiversity is the use of **on-frequency repeaters** that receive the signal and retransmit an amplified version of it.

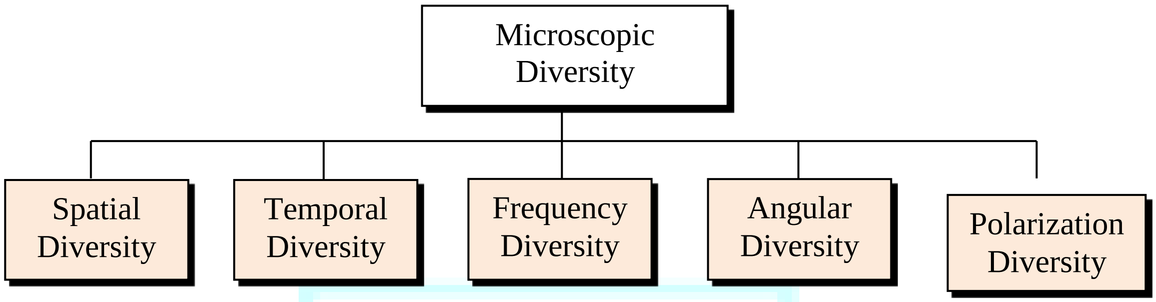
Advantage:
The use of on-frequency repeaters is simpler than that of simulcast, as no synchronization is required.

Disadvantage:
Delay dispersion is larger, because

1. The runtime from BS to repeater, and repeater to MS is larger and
2. The repeater introduces additional delays due to the group delays of electronic components, filters, etc.

MICROSCOPIC DIVERSITY

Small-scale fades are characterized by deep and rapid amplitude fluctuations which occur as the mobile moves over small distances. In order to prevent deep fades from occurring, **microscopic diversity** techniques are used.



Microscopic Diversity Techniques

Microscopic diversity is used to combat small-scale fading. The five most common methods are as follows:

Spatial diversity: several antenna elements separated in space.

Temporal diversity: transmission of the transmit signal at different times.

Frequency diversity: transmission of the signal on different frequencies.

Angular diversity: multiple antennas (with or without spatial separation) with different antenna patterns.

Polarization diversity: multiple antennas with different polarizations (e.g. vertical and horizontal).

The Space/ Spatial Diversity Techniques

Space diversity also known as antenna diversity, is one of the most popular forms of diversity used in wireless systems. Conventional wireless systems consist of an elevated base station antenna and a mobile antenna close to the ground. The existence of a direct path between the transmitter and the receiver is not guaranteed and the possibility of a number of scatterers in the vicinity of the mobile suggests a Rayleigh fading signal.

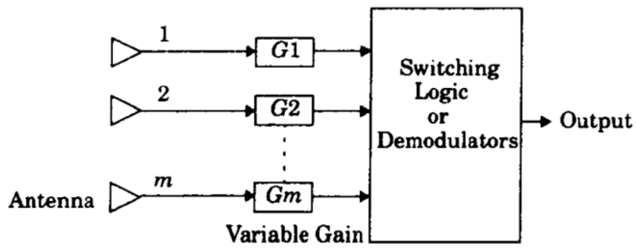
From this model, Jakes deduced that the signals received from spatially separated antennas on the mobile would have essentially uncorrelated envelops for antenna separations of one-half wavelength or more.

The concept of antenna space diversity is also used in base station design. At each cell site, multiple base station receiving antennas are used to provide diversity reception. However, since the important scatterers are generally on the ground in the vicinity of the mobile, the base station antennas must be spaced considerably far apart to achieve decorrelation. Separations on the order of several tens of wavelengths are required at the base station. Space diversity can thus be used at either the mobile or base station, or both.

Space diversity reception method can be classified into four categories.

- Selection diversity
- Feedback Diversity
- Maximal Ratio Combining
- Equal Gain Diversity

Selection diversity:

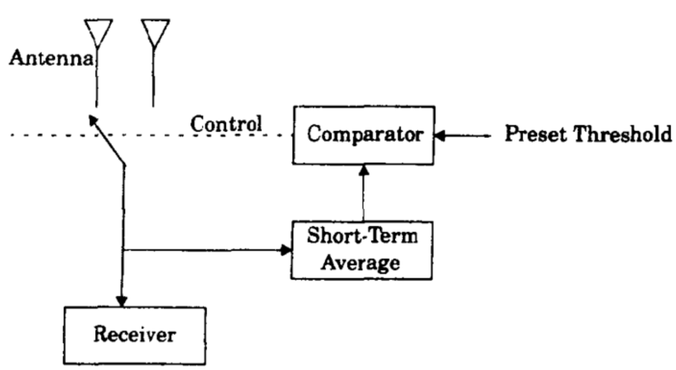


Selection diversity is the simplest diversity techniques similar to that shown in figure. Where m demodulators are used to provide m diversity branches whose gains are adjusted to provide the same average SNR for each branch. The receiver branch having the highest instantaneous SNR is connected to the demodulator.

The antenna signals themselves could be sampled and the best one sends to a signal demodulator. In practice, the branch with the largest (S+N)/N is used, since it is difficult to measure SNR alone. A practical selection diversity system cannot function on a truly instantaneous basis. But must be designed so that the internal time constants of the selection circuitry are shorter than the reciprocal of the signal fading rate.

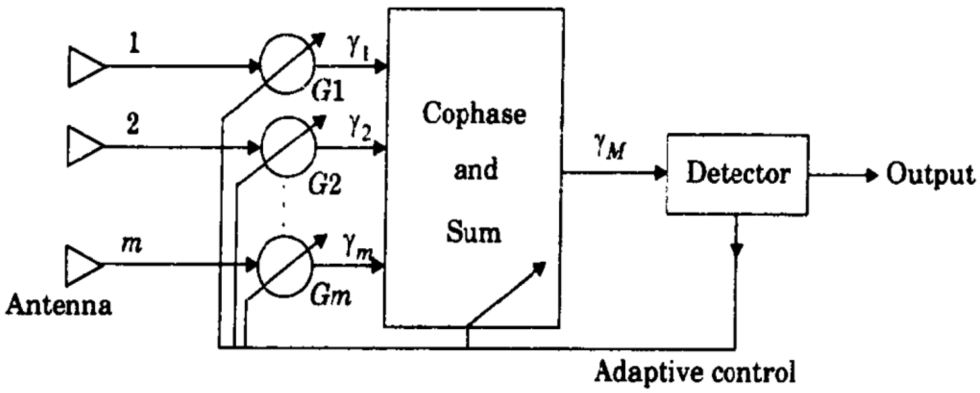
Feedback Diversity or scanning diversity

Scanning diversity is very similar to selection diversity except that instead of always using the best of M signals, the M signals are scanned in a fixed sequence until one is found to be a above predetermined threshold. This signal is then received until it falls below threshold and the scanning process is again initiated. The resulting fading statistics are somewhat inferior to those obtained by the other methods, but the advantage with this method is that it is very simple to implement – only one receiver is required. A block diagram of this method is shown in figure



Maximal Ratio Combining

In this method first proposed by Kahn, the signals from all of the M branches are weighted according to their individual signal voltage to noise power ratios and summed. Figure shows a block diagram of the technique.

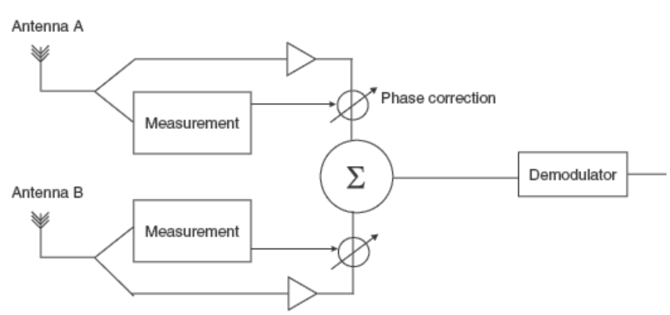


Here, the individual signals must be co phased before being summed which generally requires an receiver and phasing circuit for each antenna element. Maximal Ratio Combining produces an output SNR equal to the sum of the individual SNRs. Thus, it has the advantage of producing an output with an acceptable SNR even when none of the individual signals are themselves acceptable. This technique gives the best statistical reduction of fading of any known linear diversity combiner. Modern DSP techniques and digital receiver are now making this optimal form of diversity practical.

Equal Gain combining:

In certain cases it is not convenient to provide for the variable weighting capability required for true maximal ratio combining.

In such cases, the branch weights are all set to unity, but the signals from each branch are co phased to provide the equal gain combining diversity. This allows the receiver to exploit signals that are simultaneously received on each branch. The possibility of producing an acceptable signal from a number of unacceptable inputs is still retained, and performance is only marginally inferior to maximal ratio combining and superior to selection diversity.



The Polarization, Time and Frequency Diversity Techniques

POLARIZATION DIVERSITY

At the base station, space diversity is considerably less practical than at the mobile because the narrow angle of incident fields requires large antenna spacing. The comparatively high cost of using space diversity at the base station prompts the consideration of the using orthogonal polarization to exploit polarization diversity. While this only provides two diversity branches, it does allow the antenna elements to the co-located.

In the early days of cellular radio, all subscriber units were mounted in vehicles and used vertical whip antennas. Today, however, over half the subscriber units are portable. This means that most of subscribers are no longer using vertical polarization due to hand-tilting when the portable cellular phone is used. This recent phenomenon has sparked interest in polarization diversity at the base station.

Measured horizontal and vertical polarization paths between a mobile and a base station are reported to be uncorrelated by Lee and Yeh. The decorrelation for the signals in each polarization is caused by multiple reflections in the channel between the base station and antennas. The reflection coefficient for each polarization is different, which results in different amplitudes and phases for each, or at least some, of the reflections. After sufficient random reflections, the polarization state of the signal will be independent of the transmitted polarization. In practice, however, there is some dependence of the received polarization on the transmitted polarization.

Circular and linear polarization antennas have been used to characterize multipath inside buildings. When the path was obstructed, polarization diversity was found to dramatically reduce the multipath delay spread without significantly decreasing the received power. While polarization diversity has been studied in the past, it has primarily been used for fixed radio links which vary slowly in time. Line-of-sight microwave links, for example, typically use polarization diversity to support two simultaneous users on the same radio channel. Since the channel does not change much in such a link, there is little likelihood of cross polarization interference. As portable users proliferate, polarization diversity is likely to become more important for improving link margin and capacity. An outline of theoretical model for the base station polarization diversity reception as suggested by Kozono is given below.

Theoretical model for Polarization Diversity

It is assumed that the signal is transmitted from a mobile with vertical (or horizontal) polarization. It is received at the base station by a polarization diversity antenna with two branches.

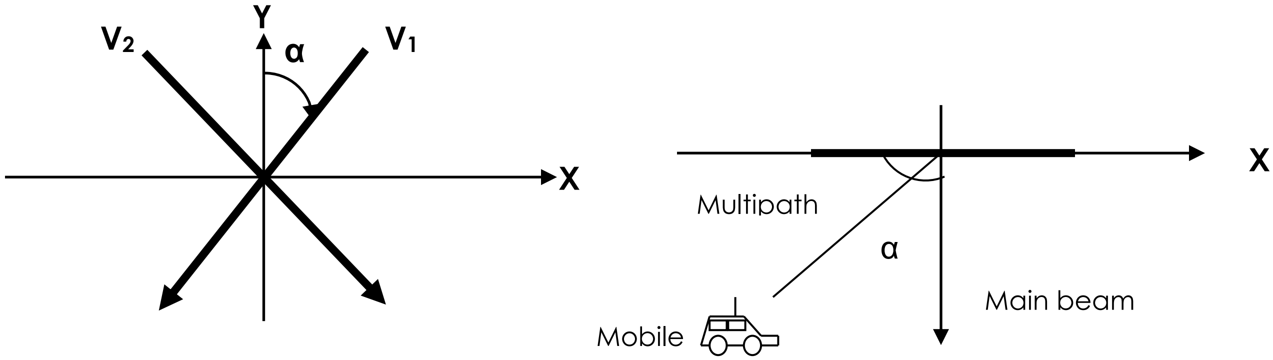


Figure shows the theoretical model and the system coordinates. As seen in the figure, polarization diversity antenna is composed of two antenna elements V_1 and V_2, which make $\pm\alpha$ angle (polarization angle) with the Y axis. A mobile station is located in the direction of offset angleβ from the main beam direction of the diversity antenna as seen in figure above.

Some of the vertically polarized signals transmitted are converted to the horizontal polarizatied signal because of multipath propagation. The signal arriving at the base station can be expressed as

$$x = r_1 \cos(\omega t + \phi_1)$$
$$y = r_2 \cos(\omega t + \phi_2)$$

where x and y are signal levels which are received when $\beta=0$. it is assumed that r_1 and r_2 have independent Rayleigh distributions, and ϕ_1 and ϕ_2 have independent uniform distributions.

The received signal values at elements V_1 and V_2 can be written as:

$$V_1 = (a\, r_1 \cos\phi_1 + r_2\, b \cos\phi_2) \cos\omega t - (a\, r_1 \sin\phi_1 + r_2\, b \sin\phi_2) \sin\omega t$$
$$V_2 = (-\, a\, r_1 \cos\phi_1 + r_2\, b \cos\phi_2) \cos\omega t - (-\, a\, r_1 \sin\phi_1 + r_2\, b \sin\phi_2) \sin\omega t$$

$a=\sin\alpha \cos\beta$ and $b=\cos\alpha$

The correlation coefficient ρ can be written as

$$\rho = \left((\tan^2(\alpha) \cos^2(\beta) - \Gamma) / (\tan^2(\alpha) \cos^2(\beta) + \Gamma) \right)^2$$

where,

$X = \langle R_2^{\,2} \rangle \;/\; \langle R_1^{\,2} \rangle$ And

$$R_1 = (r_1^2\ a^2 + r_2^2\ b^2 + 2\, r_1 r_2\, ab \cos(\phi_1 + \phi_2))^{(1/2)}$$

$$R_2 = (r_1^2\ a^2 + r_2^2\ b^2 - 2\, r_1 r_2\, ab \cos(\phi_1 + \phi_2))^{(1/2)}$$

Here, X is the cross polarization discrimination of the propagation path between a mobile and a base station.

The correlation coefficient is determined by three factors: Polarization angle, Offset angle from the main beam direction of the diversity antenna, and the cross polarization discrimination. The correlation coefficient generally becomes higher as offset angle offset angle β becomes larger. Also, ρ generally becomes lower as polarization angle α increases. This is because the horizontal polarization component becomes larger as α increases.

Because antenna elements V_1 and V_2 are polarized at $\pm$ to the vertical, the received signal is lower than that received by a vertically polarized antenna. The average value of signal loss L, relative to that received signal using vertical polarization is given by

$$L = a^2 / X + b^2$$

The results of practical experiments carried out using polarization diversity show that polarization diversity is viable diversity reception technique, and is exploited within wireless handsets as well as at base stations.

Frequency Diversity

Frequency diversity is implemented by transmitting information on more than one carrier frequency. The rationale behind this technique is that frequencies separated by more than the coherence bandwidth of the channel will be uncorrelated and will thus not experience the same fades. Theoretically, if the channels are uncorrelated, the probability of simultaneous fading will be the product of the individual fading probabilities.

Frequency diversity is often employed in microwave line-of-sight links which carry several channels in a frequency division multiplex mode (FDM).

Due to tropospheric propagation and resulting refraction, deep fading sometimes occurs. In practice, *1:N* protection switching is provided by a radio licensee, wherein one frequency is nominally idle but is available on a stand-by basis to provide frequency diversity switching for any one of the N other carriers (frequencies) being used on the same link, each carrying independent traffic. When diversity is needed, the appropriate traffic is simply switched to the backup frequency.

This technique has the disadvantages that it is not only requires spare bandwidth but also requires that there be as many receivers as there are channels used for the frequency diversity. However, for critical traffic, the expense may be justified.

Repeating the same information at two different frequencies will decrease spectral efficiency. So information is spread over a large bandwidth, so that small parts of the information are conveyed by different frequency components. The RX can then sum over the different frequencies to recover the original information.

This spreading can be done by different methods:

1. Compressing the information in time
2. Code Division Multiple Access (CDMA).
3. Multicarrier CDMA and coded orthogonal frequency division multiplexing
4. Frequency hopping in conjunction with coding

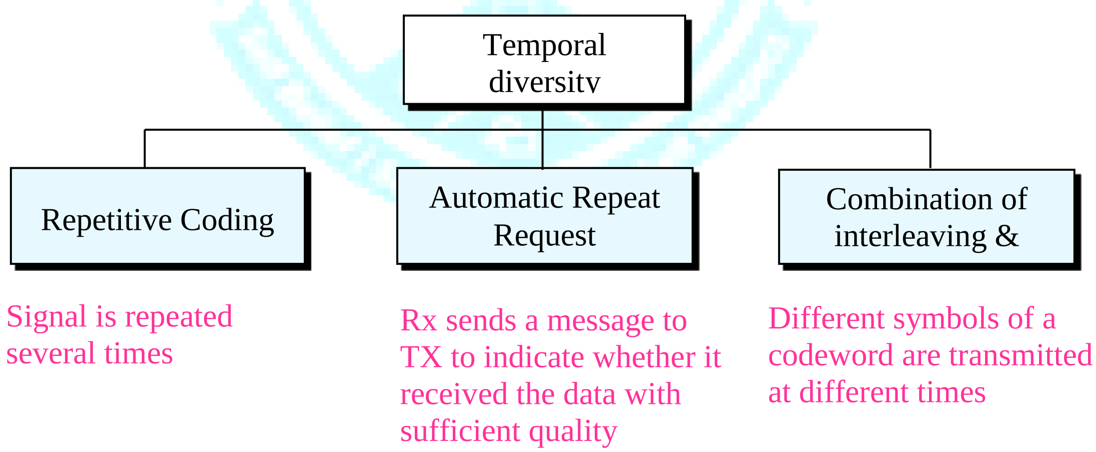
New OFDM modulation and access techniques exploit frequency diversity by providing simultaneous modulation signals with error control coding across a large bandwidth, so that if a particular frequency undergoes a fade, the composite signal will still be demodulated.

TIME DIVERSITY

Time diversity repeatedly transmits information at time spacing that exceed the coherence time of the channel, so that multiple repetitions of the signal will be received with independent fading conditions, thereby providing for diversity. One modern implementation of time diversity involves the use of the RAKE receiver for spread spectrum CDMA, where the multipath channels provide a redundancy in the transmitted message. By demodulating several replicas of the transmitted CDMA signal, where each replica experience a particular multipath delay, the RAKE receiver is able to align the replicas so that a better estimate of the original signal may be formed at the receiver.

The wireless propagation channel is time variant. So the signals that are received at different times are uncorrelated. For sufficient decorrelation, the temporal distance must be at least $1/(2v_{\max})$, where v_{max} is the maximum Doppler frequency.

Temporal diversity can be realized in different ways:



Repetition coding

This is the simplest form. The signal is repeated several times, where the repetition intervals are long enough to achieve decorrelation.

Disadvantages:

1. It is highly bandwidth inefficient.
2. Spectral efficiency decreases by a factor that is equal to the number of repetitions.

Automatic Repeat reQuest (ARQ)

The RX sends a message to the TX to indicate whether it received the data with sufficient quality. If this is not the case, then the transmission is repeated after a wait period.

Advantage: The spectral efficiency of ARQ is better than that of repetition coding, since it requires multiple transmissions only when the first transmission occurs in a bad fading state.

Disadvantage: ARQ requires a feedback channel.

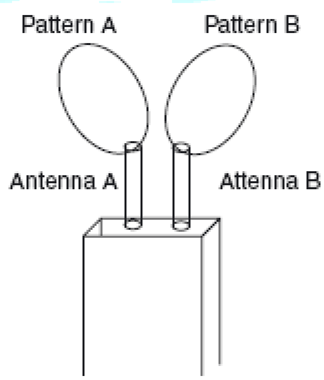
Combination of interleaving and coding

Advanced version of repetition coding is forward error correction coding with interleaving. The different symbols of a code word are transmitted at different times. This increases the probability that at least some of them arrive with a good SNR. The transmitted code word can then be reconstructed.

ANGLE DIVERSITY

A fading dip is created when MPCs interfere destructively. If some of these waves are attenuated or eliminated, then the location of fading dips changes In other words, two co-located antennas with different patterns see differently weighted MPCs, so that the MPCs interfere differently for the two antennas. This is the principle of **angle diversity** (also known as **pattern diversity**).

Angular diversity is usually used in conjunction with spatial diversity. Different antenna patterns can be achieved very easily. Different types of antennas have different patterns. But even identical antennas can have different patterns when mounted close to each other. This effect is due to **mutual coupling**.



Antenna B acts as a reflector for antenna A, whose pattern is therefore skewed to the left. Similarly, the pattern of antenna B is skewed to the right due to reflections from antenna A. Thus, the two patterns are different.

COMBINATION OF SIGNALS

Two ways of exploiting signals from the multiple diversity branches are

1. Selection diversity
2. Combining diversity

- Combination of Signals
  - Selection Diversity
    - RSSI-Driven
    - BER-Driven
  - Combining Diversity
    - Maximum Ratio Combining
    - Equal Gain Combining
    - Optimum
    - Hybrid-Selection

Combination of Signals

Selection diversity: In selection diversity the best signal copy is selected and processed (demodulated and decoded), while all other copies are discarded.

Advantage: Requires only one RF (down conversion) chain.

Disadvantage: Selection diversity wastes signal energy by discarding $(N_r - 1)$ copies of the received signal

Combining diversity: In combining diversity all copies of the signal are combined (before or after the demodulator), and the combined signal is decoded.

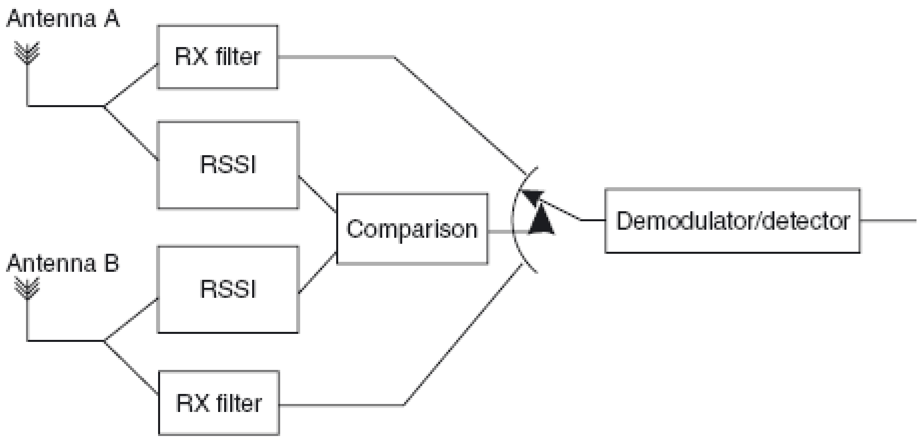
Advantage: Combining diversity leads to better performance, as all available information is exploited.

Disadvantage: It requires a more complex RX than selection diversity. A RX with combining diversity needs to downconvert all available signals, and combine them appropriately in the baseband. Thus, it requires N_r antenna elements as well as N_r complete Radio Frequency (RF) (downconversion) chains.

SELECTION DIVERSITY

Received-Signal-Strength-Indication-Driven Diversity

In this method, the RX selects the signal with the largest instantaneous power (or **Received Signal Strength Indication – RSSI**), and processes it further. This method requires N_r antenna elements, *N*r RSSI sensors, and a N_r-to-1 multiplexer (switch), but only one RF chain.



The method allows simple tracking of the selection criterion even in fast-fading channels. Thus, we can switch to a better antenna as soon as the RSSI becomes higher there.

1. If the BER is determined by noise, then RSSI-driven diversity is the best of all the selection diversity methods, as maximization of the RSSI also maximizes the SNR.
2. If the BER is determined by co-channel interference, then RSSI is not a good selection criterion.
3. Similarly, RSSI-driven diversity is suboptimum if the errors are caused by the frequency selectivity of the channel.

Assume that the instantaneous signal amplitude is Rayleigh distributed. SNR distribution of nth diversity branch is given by

$$pdf_{\gamma_n}(\gamma_n) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma_n}{\bar{\gamma}}\right)$$

where $\bar{\gamma}$ is the mean branch SNR. The cumulative distribution function (cdf) is then

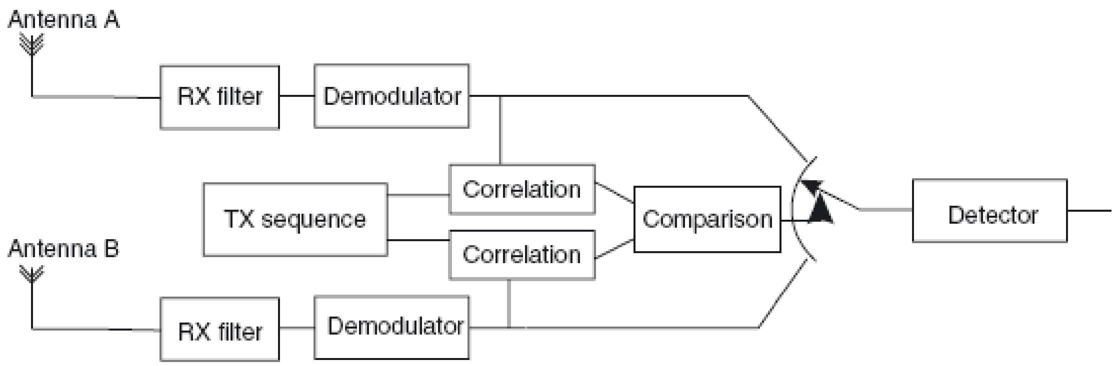
$$cdf_{\gamma_n}(\gamma_n) = 1 - \exp\left(-\frac{\gamma_n}{\bar{\gamma}}\right)$$

The cdf of the selected signal is the product of the cdfs of each branch

$$cdf_{\gamma}(\gamma) = \left[1 - \exp\left(-\frac{\gamma_n}{\bar{\gamma}}\right)\right]^{N_r}$$

Bit-Error-Rate-Driven Diversity

For BER-driven diversity, we first transmit a *training sequence* – i.e., a bit sequence that is known at the RX. The RX then demodulates the signal from each receive antenna element and compares it with the transmit signal. The antenna whose signal results in the smallest BER is judged to be the best, and used for the reception of data signals.



If the channel is time variant, the training sequence has to be repeated at regular intervals and selection of the best antenna has to be done anew. The necessary repetition rate depends on the coherence time of the channel.

Drawbacks:

BER-driven diversity has several drawbacks:

1. The RX needs either *N*r RF chains and demodulators (which makes the RX more complex), or the training sequence has to be repeated *N*r times (which decreases spectral efficiency), so that the signal at all antenna elements can be evaluated.
2. If the RX has only one demodulator, then it is not possible to continuously monitor the selection criterion (i.e. the BER) of all diversity branches. This is critical if the channel changes quickly.
3. Since the duration of the training sequence is finite, the selection criterion – i.e., bit error probability – cannot be determined exactly.

COMBINING DIVERSITY

Basic Principle

Selection diversity wastes signal energy by discarding $(N_r - 1)$ copies of the received signal. This drawback is avoided by combining diversity, which exploits all available signal copies. Each signal copy is multiplied by a (complex) weight and then added up. Each complex weight w_n^* can be thought of as consisting of a phase correction, plus a (real) weight for the amplitude:

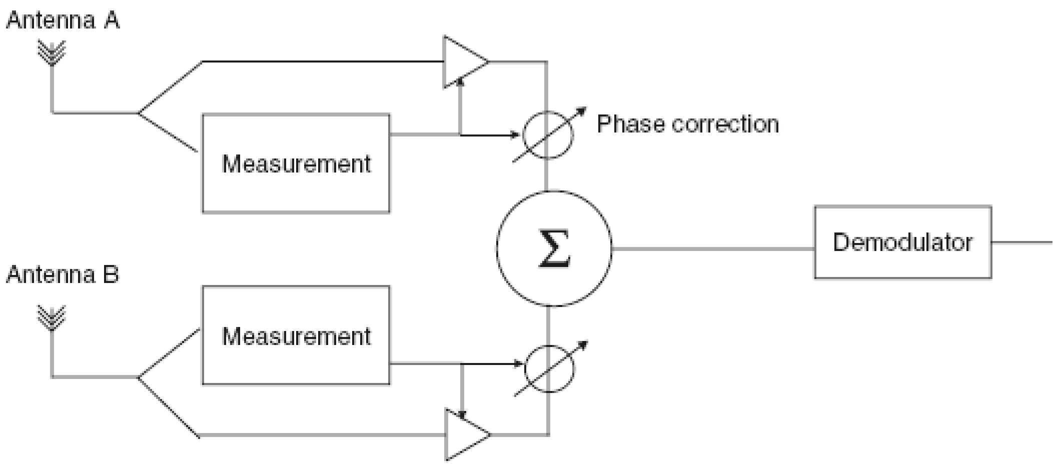
Phase correction causes the signal amplitudes to add up, and noise is added incoherently, so that noise powers add up.

For amplitude weighting, two methods are widely used:

1. ***Maximum ratio combining:*** Maximum Ratio combining weighs all signal copies by their amplitude. This is an optimum combination strategy.
2. ***Equal gain combining:*** In Equal Gain Combining, all amplitude weights are the same (in other words, there is no weighting, but just a phase correction).

Maximum ratio combining

MRC compensates for the phases, and weights the signals from the different antenna branches according to their SNR.



Let us assume a propagation channel that is slow fading and flat fading. The only disturbance is AWGN. Under these assumptions, each channel realization can be written as a time-invariant filter with impulse response:

$$h_n(\tau) = \alpha_n \delta(\tau)$$

where α_n is the gain of diversity branch n.

The signals at the different branches are multiplied with weights w_n^* and added up. The SNR is maximized by choosing the weights as

$$w_{MRC} = \alpha_n$$

The output SNR of the diversity combiner is the *sum* of the branch SNRs:

$$\gamma_{MRC} = \sum_{n=1}^{N_r} \gamma_n$$

If the SNR distribution in each branch is exponential (corresponding to Rayleigh fading), and all branches have the same mean SNR $\bar{\gamma}_n = \bar{\gamma}$

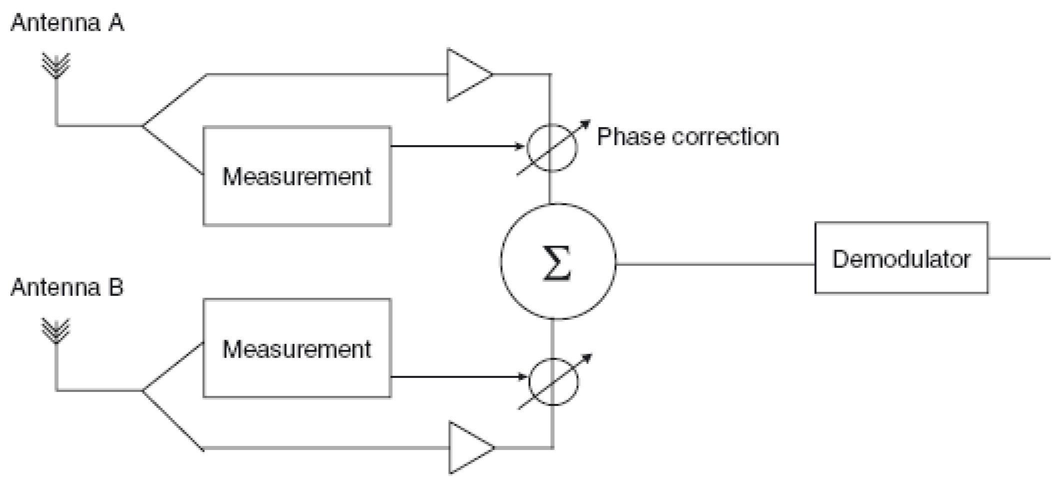
$$pdf_{\gamma_n}(\gamma_n) = \frac{1}{(N_r - 1)!} \frac{\gamma^{N_r - 1}}{\bar{\gamma}^{N_r}} \exp\left(-\frac{\gamma_n}{\bar{\gamma}}\right)$$

and the mean SNR of the combiner output is

$$\bar{\gamma}_{MRC} = N_r \bar{\gamma}$$

Equal Gain Combining

In Equal Gain Combining, all amplitude weights are the same (in other words, there is no weighting, but just a phase correction



For EGC, we find that the SNR of the combiner output is

$$\gamma_{ECG} = \frac{\left(\sum_{n=1}^{N_r} \sqrt{\gamma_n}\right)^2}{N_r}$$

The mean SNR of the combiner output can be found to be

$$\bar{\gamma}_{ECG} = \bar{\gamma}\left(1 + (N_r - 1)\frac{\pi}{4}\right)$$

Optimum Combining

One of the assumptions in the derivation of MRC was that only AWGN disturbs the signal. If it is interference that determines signal quality, then MRC is no longer the best solution. In order to maximize the Signal-to-Interference-and-Noise Ratio (SINR), the weights should then be determined according to a strategy called **optimum combining.**

The correlation matrix of noise and interference at the different antenna elements:

$$\mathbf{R} = \sigma_n^2 \mathbf{I} + \sum_{k=1}^{K} E[\mathbf{r}_k \mathbf{r}_k^*]$$

where **r***k* is the receive signal vector of the *k*th interferer. The complex transfer function of the *N*r diversity branches are written into the vector $\mathbf{h}_d$.

The vector containing the optimum receive weights is then

$$\mathbf{w}_{opt} = \mathbf{R}^{-1}\mathbf{h}_d$$

Hybrid Selection – Maximum Ratio Combining

In hybrid selection scheme, the best *L* out of *N*r antenna signals are chosen, downconverted, and processed. This reduces the number of required RF chains from N_r to *L*, and thus leads to significant savings. The approach is called **Hybrid Selection/Maximum Ratio Combining** (H-S/MRC), or **Generalized Selection Combining** (GSC).

The instantaneous output SNR of H-S/MRC is

$$\gamma_{H-S/MRC} = \sum_{n=1}^{N_r} \gamma_{(n)}$$

where $\gamma_{(n)}$ are ordered SNRs - $\gamma_{(1)} > \gamma_{(2)} > \cdots > \gamma_{(N_r)}$

H-S/MRC schemes provide good diversity gain, as they select the best antenna branches for combining. However, they do not provide full beamforming gain. the SNR gain of H-S/MRC is only L, compared with N_r for an MRC scheme.

The mean and variance of the output SNR for H-S/MRC is thus

$$\bar{\gamma}_{H-S/MRC} = L\left(1 + \sum_{n=L+1}^{N_r} \frac{1}{n}\right)\bar{\gamma} \qquad \text{and}$$

$$\sigma^2_{H-S/MRC} = L\left(1 + \sum_{n=L+1}^{N_r} \frac{1}{n^2}\right)\bar{\gamma}^2$$

RAKE RECEIVER CONCEPTS

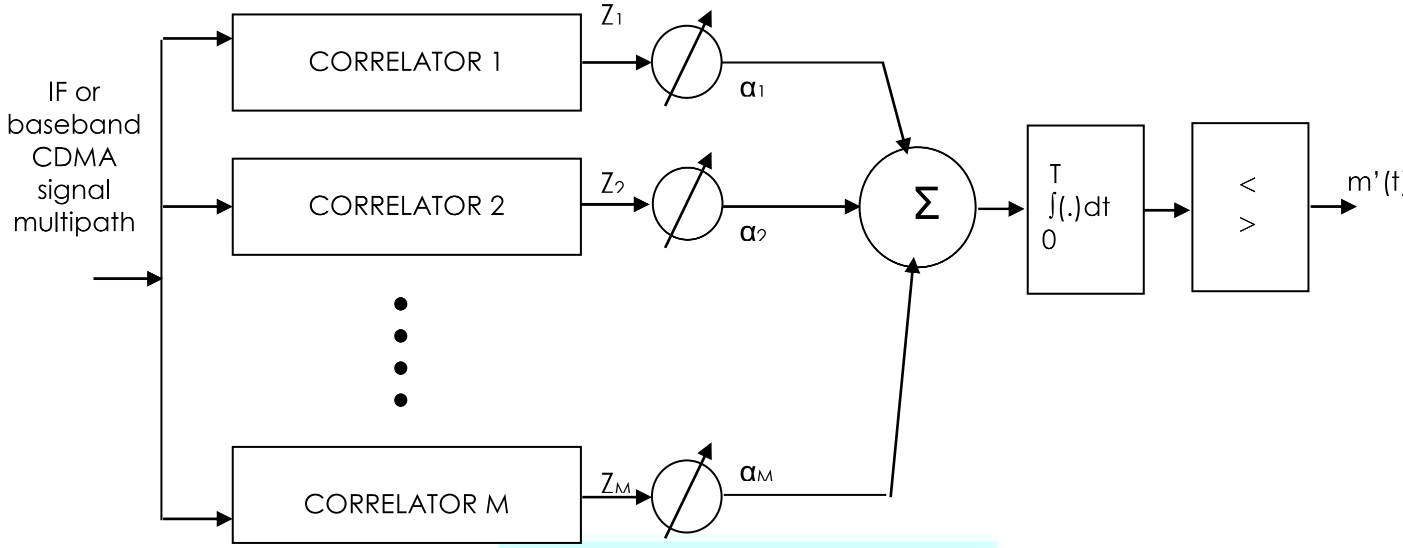
In CDMA spread spectrum systems, the chip rate is typically much greater than the flat-fading bandwidth of the channel. Whereas conventional modulation techniques require an equalizer to undo the intersymbol interference between adjacent symbols, CDMA spreading codes are designed to provide very low correlation between successive chips. Thus, propagation delay spread in the radio channel merely provides multiple versions of the transmitted signal at the receiver.

If these multipath components are delayed in time by more than chip duration, they appear like uncorrelated noise at a CDMA receiver, and equalization is not required. The spread spectrum processing gain makes uncorrelated noise negligible after dispreading.

However, since there is useful information in the multipath components, CDMA receivers may combine the time delayed versions of the original signal transmission in order to improve the SNR at the receiver. A RAKE receiver does just this-it attempts to collect the time-shifted versions of the original signal by providing a separate correlation receiver for each of the multipath signals.

Each correlation receiver may be adjusted in time delay, so that a microprocessor controller can cause different correlation can cause different correlation receivers to search in different time windows for significant multipath. The range of time delays that a particular correlator can search is called a search window.

The RAKE receiver is essentially a diversity receiver designed specifically for CDMA, where the diversity is provided by the fact that the multipath components are practically uncorrelated from one another when their relative propagation delays exceed a chip period.



A RAKE receiver utilizes multiple correlators to separately detect the M strongest multipath components. The outputs of each correlators are then weighted to provide a better estimate of the transmitted signal than is provided by a single component. Demodulation and bit decisions are then based on the weighted outputs of the M correlators.

The basic idea of a RAKE receiver was first proposed by Price and Green. In outdoor environments, the delay between multipath components is usually large and, if the chip rate is properly selected, the low autocorrelation properties of a CDMA spreading sequence can assure that multipath components will appear nearly uncorrelated with each other. However, the RAKE receiver in IS-95 CDMA has been found to perform poorly in indoor environments, which is to be expected since the multipath delay spreads in indoor channels (approximately 100 ns) are much smaller than an IS-95 chip duration (approximately 800 ns). In such cases, a RAKE will not work since multipath is un resolvable, the Rayleigh flat-fading typically occurs within a single chip period.

To explore the performance of the RAKE receiver, assume M correlators are used in a CDMA receiver to capture the M strongest multipath components. A weighting network is used to provide a linear combination of the correlator output for bit detection. Correlator 1 is synchronized to the strongest multipath m_1. Multipath component m_2 arrives τ_1 later than component m_1 . Where $\tau_2 - \tau_1$ is assumed to be greater than chip duration. The second Correlator is synchronized to m_2. It correlates strongly with m_2, but has low correlation with m_1. Note that if only a single correlator is used in the receiver, once the output of the single correlator is corrupted by fading, the receiver cannot correct the value. Bit decision based on only a single correlation may produce large bit error . in a RAKE receiver, if the output from one from is corrupted by fading, the others may not be, and the corrupted signal may be discounted through the weighting process. Decisions based on the combination of the M, separate decision statistics offered by the RAKE provide a form of the diversity which can overcome fading and thereby improves CDMA reception.

The M decision statistics are weighted to form an overall decision statistics as shown in figure. The outputs of the M correlators are denoted as Z_1, Z_2, …………Z_M. They are weighted by α_1,

α_2, α_3 ……….α_M respectively. The weighting coefficients are based on the power or the SNR from each correlator output. If the power or SNR is small out of a particular correlator, it will be assigned a small weighting factor just as in the case of a maximal ratio combining diversity scheme, the overall signal Z' is given by

$$\alpha_M = \frac{Z_M^2}{\sum_{m=1}^{M} Z_M^2} \qquad Z' = \sum_{m=1}^{M} \alpha_M Z_M$$

The weighting coefficients,α_M are normalized to the output signal power of the correlator in such a way the coefficients sum to unity,

As in the case of adaptive equalizer and diversity combining there are many ways to generate the weighting coefficients. However, due to multiple access interference, RAKE fingers with strong multipath amplitudes will not necessarily provide strong output after correlation. Choosing weighting coefficients based on the actual outputs of the correlators yields better RAKE performance.

EQUALIZERS

INTRODUCTION

Wireless channels can exhibit delay dispersion. Delay dispersion leads to Inter-Symbol Interference (ISI). **Equalization** is a technique used to combat Inter-Symbol Interference.

Equalizers are RX structures that reduce or eliminate ISI, and at the same time exploit the delay diversity inherent in the channel.

The operational principle of an equalizer can be visualized either in the time domain or the frequency domain.

In radio channels, a variety of adaptive equalizers can be used to cancel interference while providing diversity. Since the mobile fading channel is random and time varying, equalizers must track the time varying characteristics of the mobile channel, and thus are called **adaptive equalizers.**

The general operating modes of an adaptive equalizer include training and tracking. First, a known, fixed-length training sequence is sent by the transmitter so that the receiver's equalizer may average to a proper setting. The training sequence is typically a pseudorandom binary signal or a fixed, prescribed bit pattern. Immediately following this training sequence, the user data (which may or may not include coding bits) is sent, and the adaptive equalizer at the receiver utilizes a recursive algorithm to evaluate the channel and estimate filter coefficients to compensate for the channel. The training sequence is designed to permit an equalizer at the receiver to acquire the proper filter coefficients in the worst possible channel conditions so that when the training sequence is finished, the filter coefficients are near the optimal values for reception of user data. As user data are received, the adaptive algorithm of the equalizer tracks the changing channel. As a consequence, the adaptive equalizer is continually changing its filter characteristics over time.

Time division multiple access (TDMA) wireless systems are particularly well suited for equalizers. TDMA systems send data in fixed-length time blocks, and the training sequence is usually sent at the beginning of a block. Each time a new data block is received, the equalizer is retrained using the same training sequence.

An equalizer is usually implemented at baseband or at IF in a receiver. Since the baseband complex envelope expression can be used to represent bandpass waveforms, the channel response, demodulated signal, and adaptive equalizer algorithms are usually simulated and implemented at baseband.

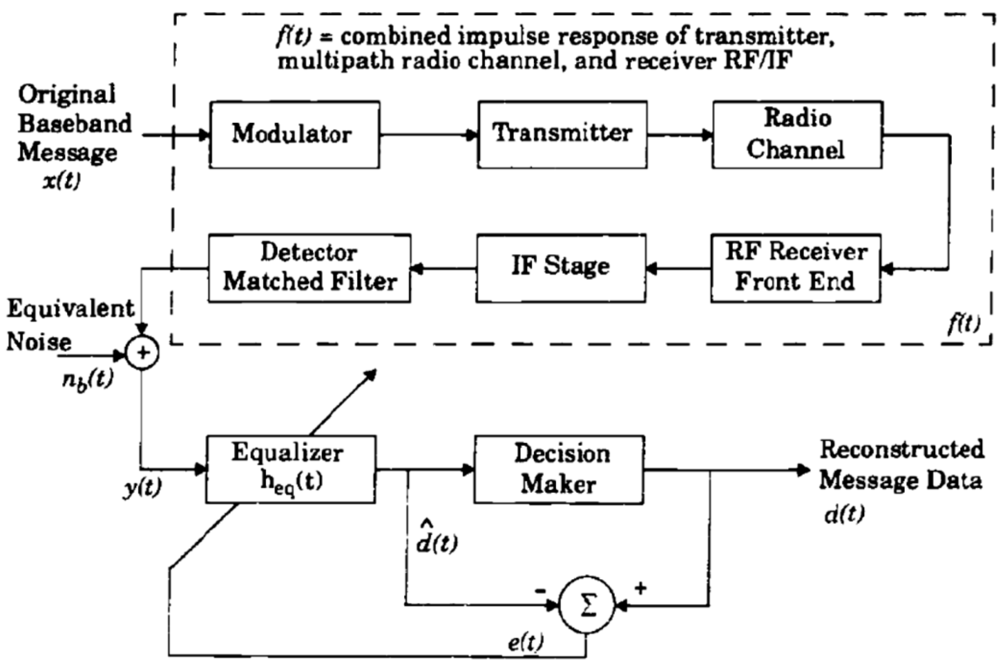


Figure shows a block diagram of a communication system with an adaptive equalizer in the receiver.

If x (t) is the original information signal, and f(t) is the combined complex baseband impulse response of the transmitter, channel, and the RF/IF sections of the receiver, the signal received by the equalizer may be expressed as

$$\boldsymbol{y(t) = x(t) \otimes f^{*}(t) + n_b(t)}$$

where f* (t) is the complex conjugate of f(t). $n_b(t)$ is the baseband noise at the input of the equalizer, and ⊗ denotes the convolution operation. If the impulse response of the equalizer is $h_{eq}(t)$, then the output of the equalizer is

$$\delta(t) = x(t) \otimes f^*(t) \otimes h_{eq}(t) + n_b(t) \otimes h_{eq}(t)$$

$$= x(t) \otimes g(t) + n_b(t) \otimes h_{eq}(t)$$

Where g (t) is the combined impulse response of the transmitter, channel, RF/IF sections of the receiver, and the equalizer. The complex baseband impulse response of a transversal filter equalizer is given by

$$h_{eq}(t) = \sum_n c_n \delta(t - nT)$$

where c_n are the complex filter coefficients of the equalizer. The desired output of the equalizer is x(t), the original source data. Assume that $\boldsymbol{n_b(t)} = 0$ Then, in order to force $\hat{d}(t) = x(t)$ in equation, g (t) must be equal to

$$\boldsymbol{g(t) = f^*(t) \otimes h_{eq}(t) = \delta(t)}$$

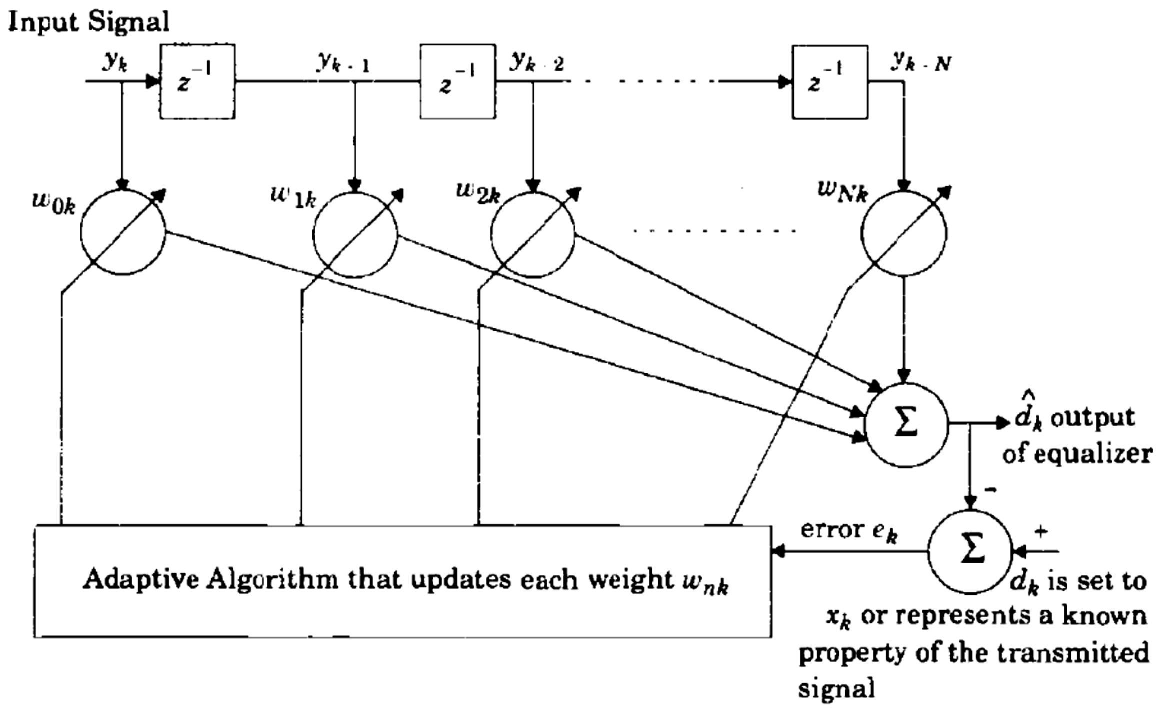
$$H_{eq}(f)F^*(-f) = 1$$

Where $\boldsymbol{H_{eq}(f)}$ and F (f) are Fourier transforms of h_{eq} (t) and f (t), respectively.

Equation indicates that an equalizer is actually an inverse filter of the channel. If the channel is frequency selective, the equalizer enhances the frequency components with small amplitudes and attenuates the strong frequencies in the received frequency spectrum in order to provide a flat, composite, received frequency response and linear phase response. For a time-varying channel, an adaptive equalizer is designed to track the channel variations so that equation is approximately satisfied.

A Generic Adaptive Equalizer

An adaptive equalizer is a time-varying filter which must constantly be retuned. The basic structure of an adaptive equalizer is shown in Figure, where the subscript k is used to denote a discrete time index



Notice in that there is a single input Y_k at any time instant. The value of Y_k depends upon the instantaneous state of the radio channel and the specific value of the noise. As such, Y_k is a random process. The adaptive equalizer structure shown above is called a transversal filter, and in this case has N delay elements, N + 1 taps, and N + 1 tunable complex multipliers, called weights.

The weights of the filter are described by their physical location in the delay line structure, and have a second subscript, k, to explicitly show they vary with time. These weights are updated continuously by the adaptive algorithm, either on a sample by sample basis (i.e., whenever k is incremented by 1) or on a block by block basis (i.e., whenever a specified number of samples have been clocked into the equalizer).

The adaptive algorithm is controlled by the error signal e_k. This error signal is derived by comparing the output of the equalizer, dk. with some signal which is either an exact scaled replica of the transmitted signal x_k or which represents a known property of the transmitted signal. The adaptive algorithm uses C_k to minimize a cost function and updates the equalizer weights in a manner that iteratively reduces the cost function. For example, the least mean squares (LMS) algorithm searches for the optimum or near-optimum filter weights by performing the following iterative operation:

New weights = Previous weights + (constant) x (Previous error) x (Current input vector)

where

Previous error = Previous desired output — Previous actual output

and the constant may be adjusted by the algorithm to control the variation between filter weights on successive iterations. This process is repeated rapidly in a programming loop while the equalizer attempts to converge, and many techniques (such as gradient or steepest decent algorithms) may be used to minimize the error. Upon reaching convergence, the adaptive algorithm freezes the filter weights until the error signal exceeds an acceptable level or until a new training sequence is sent.

Based on classical equalization theory, the most common cost function is the mean square error (MSE) between the desired signal and the output of the equalizer. The MSE is denoted by E[e(k) e*(k)]. and when a replica of the transmitted signal is required at the output of the equalizer (i.e., when dk is set equal to Xk and is known a priori), a known training sequence must be periodically transmitted. By detecting the training sequence, the adaptive algorithm in the receiver is able to compute and minimize the cost function by driving the tap weights until the next training sequence is sent.

A more recent class of adaptive algorithms are able to exploit characteristics of the transmitted signal and do not require training sequences. These modern algorithms are able to acquire equalization through property restoral techniques of the transmitted signal, and are called **blind algorithms** because they provide equalizer convergence without burdening the transmitter with training overhead. These techniques include algorithms such as the **constant modulus algorithm (CMA) and the spectral coherence restoral algorithm (SCORE)**. is used for constant envelope modulation, and forces the equalizer weights to maintain a constant envelope on the received signal, whereas SCORE exploits spectral redundancy or cyclostationarity in the transmitted signal.

To study the adaptive equalizer, it is helpful to use vector and matrix algebra.

Define the input signal to the equalizer as a vector Y_k where

$$Y_k = [Y_k\ Y_{k-1}\ Y_{k-2} \dots \dots .. Y_{k-N}]^T$$

It should be clear that the output of the adaptive equalizer is a scalar given by

$$\dot{d}_k = \sum_{n=0}^{N} W_{nk} y_{k-n}$$

A weight vector can be written as

$$w_k = [w_{0k}\ w_{1k}\ w_{2k} \dots \dots .. w_{Nk}]^T$$

May be written in vector notation as

$$\overline{d_k} = {y_k}^T w_k = {w_k}^T y_k$$

It follows that when the desired equalizer output is known (i.e., $d_k = x_k$), the error signal e_k is given by

$$e_k = d_k - \overline{d_k} = x_k - \overline{d_k}$$

and from

$$e_k = x_k - {y_k}^T w_k = x_k - {W_k}^T y_k$$

To compute the mean square error $|e_k|^2$ at time instant k ,then equation becomes

$$|e_k|^2 = {x_k}^2 + {W_k}^T y_k\, {y_k}^T w_k - 2x_k\, {y_k}^T w_k$$

Taking the expected value of $|e_k|^2$ over k (which in practice amounts to computing a time average) yields

$$E\,[|e_k|^2] = E[{x_k}^2] + {W_k}^T\, E\,[y_k\, {y_k}^T]\, w_k - 2E\,[x_k\, {y_k}^T]\, w_k$$

Notice that the filter weights W_k are not included in the time average since, for convenience, it is assumed that they have converged to the optimum value and are not varying with time.

Equation would be trivial to simplify if X_k and Y_k were independent. However, this is not true in general since the input vector should be correlated with the desired output of the equalizer (otherwise, the equalizer would have a very difficult time extracting the desired signal.). Instead, the cross correlation vector p between the desired response and the input signal is defined as

$$P = E\,[X_k\, Y_k] = E[x_k y_k \quad x_k y_{k-1}\; x_k y_{k-2} \quad \ldots \ldots x_k y_{k-N}]^T$$

and the input correlation matrix is defined as the (N + 1) x (N + 1) square matrix R where

$$R = E\,[Y_k\, {Y_k}^*] = \begin{bmatrix} {Y_k}^2 & y_k y_{k-1} \cdots & y_k y_{k-N} \\ y_{k-1} y_k & y_{k-1} y_{k-1} \cdots & y_{k-1} y_{k-N} \\ \vdots\, y_{k-N} y_k & \vdots\, y_{k-N} y_{k-1} \cdots & \vdots\, y_{k-N} y_{k-N} \end{bmatrix}$$

The matrix R is sometimes called the input covariance matrix. The major diagonal of R contains the mean square values of each input sample, and the cross terms specify the autocorrelation terms resulting from delayed samples of the input signal.

If X_k and Yk are stationary, then the elements in R and p are second order statistics which do not vary with time. Using equations

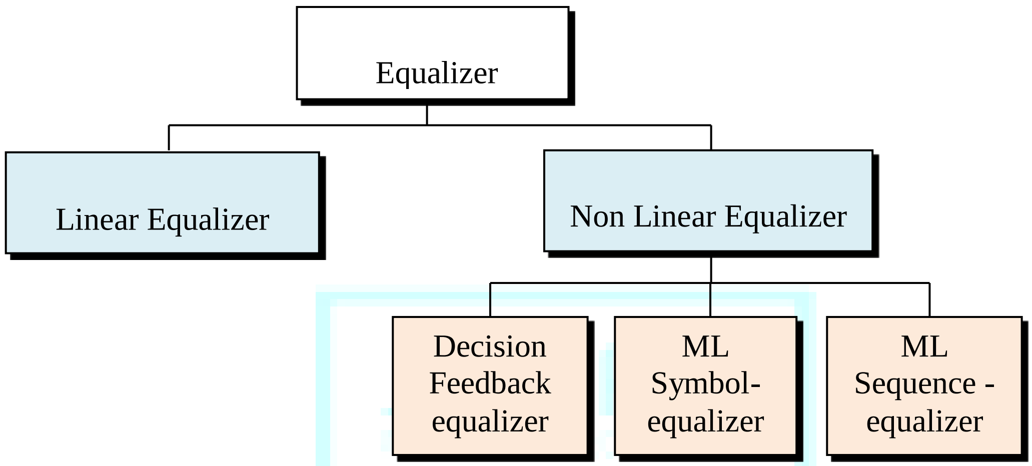
$$Mean\ Square\ Error \equiv \varepsilon = E\,[{x_k}^2] + W^T R w - 2P^T W$$

By minimizing equation in terms of the weight vector W_k, it becomes possible to adaptively tune the equalizer to provide a flat spectral response (minimal ISI) in the received signal. This is due to the fact that when the input signal Y_k and the desired response X_k are the mean square error (MSE) is quadratic on Wk, and minimizing the MSE leads to optimal solutions for $W_{k.}$

TYPES OF EQUALIZERS

Equalizer can be classified as

1. Linear equalizer
2. Non linear equalizer



Classification of Equalizers

Linear Equalizer: Linear equalizer is usually a tapped-delay-line filter with coefficients that are adapted to the channel state. A linear equalizer can be implemented as an FIR filter. In such an equalizer, the current and past values of the received signal are linearly weighted by the filter coefficient and summed to produce the output.

Advantage: This type of equalizer is the simplest type available.
Disadvantage: The linear equalizer places too much gain near the spectral null, thereby enhancing the noise present in those frequencies.

Nonlinear Equalization: A Nonlinear equalizers are used in applications where the channel distortion is too severe for a linear equalizer to handle.

Three very effective nonlinear methods offer improvements over linear equalization techniques. These are

1. Decision Feedback Equalization (DFE)
2. Maximum Likelihood Symbol Detection
3. Maximum Likelihood Sequence Estimation (MLSE)

Decision feedback equalizer: The ISI created by past symbols can be computed (and subtracted) from the received signal.

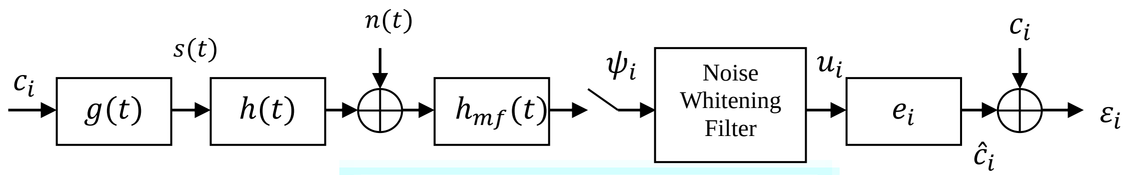
Maximum Likelihood Sequence Estimation: The optimum way of detection in a delay-dispersive channel is the Maximum Likelihood Sequence Estimation.

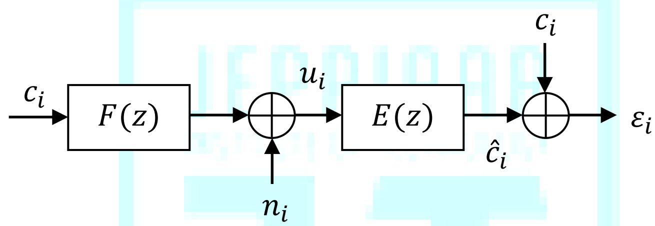
LINEAR EQUALIZERS

Linear equalizers are simple linear filter structures. Linear equalizers try to invert the channel in the sense that the product of the transfer functions of channel and equalizer fulfills a certain criterion. This criterion can either be

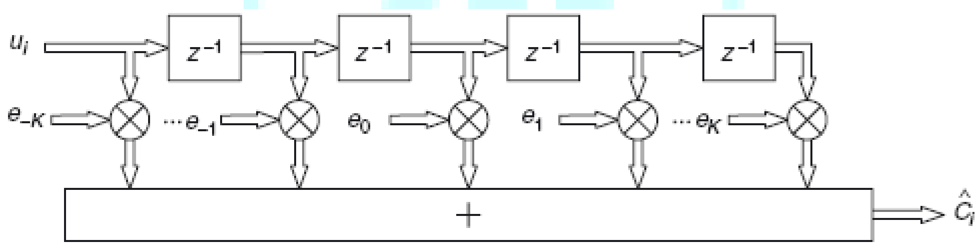
1. flat transfer function of the channel–filter concatenation, or
2. minimizing the mean-squared error at the filter output.

The basic structure of a linear equalizer is sketched in Figure.





A transmit sequence $\{c_i\}$ is sent over a dispersive, noisy channel, so that the sequence $\{u_i\}$ is available at the equalizer input. We now need to find the coefficients of a Finite Impulse Response (FIR) filter (transversal filter, with $2K + 1$ taps.



This filter should convert sequence $\{u_i\}$ into sequence $\{\hat{c}_i\}$ that should be as close as possible to the sequence $\{c_i\}$. Defining the deviation ε_i as

$$\varepsilon_i = c_i - \hat{c}_i$$

we aim to find a filter so that

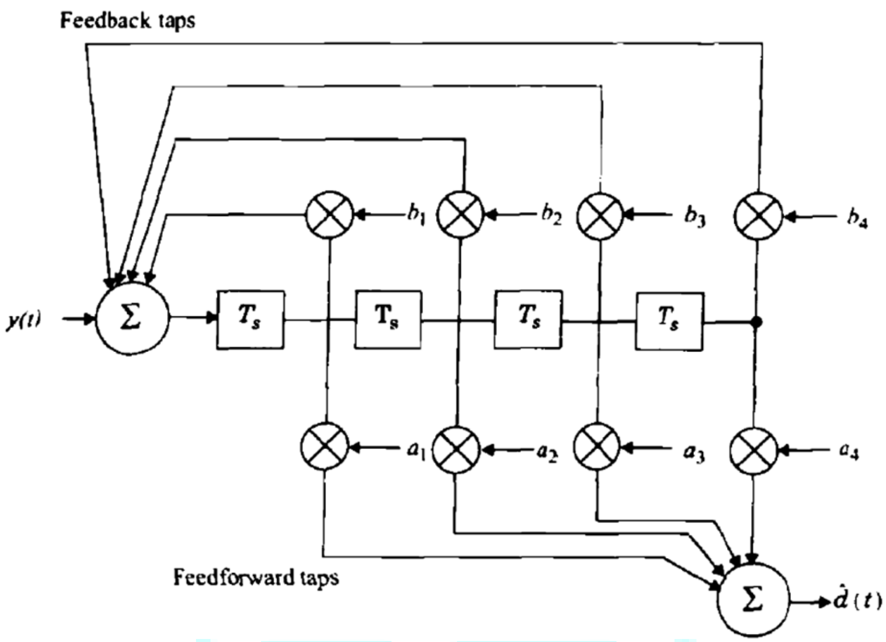
$$\varepsilon_i = 0 \quad \text{for } N_0 = 0$$

which gives the **ZF equalizer**, or that

$$E\left\{|\varepsilon_i|^2\right\} \to \min \quad \text{for } N_0 \text{ having a finite value}$$

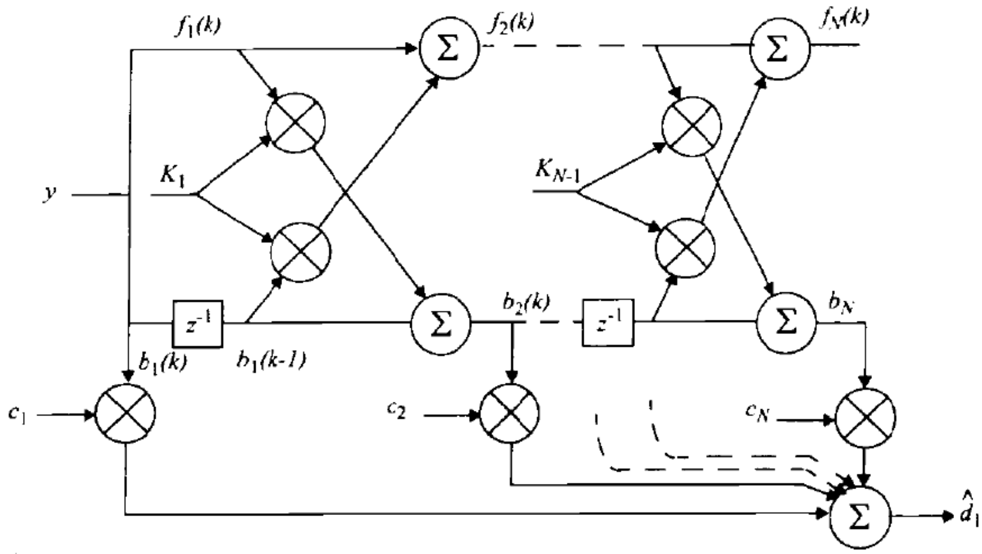
which gives the **Minimum Mean Square Error (MMSE) equalizer**.

Transversal Linear Equalizer (Explain the theory of adaptive equalizer)



Lattice Equalizer

Two main advantages of the lattice equalizer is its numerical stability and faster convergence. Also, the unique structure of the lattice filter allows the dynamic assignment of the most effective length of the lattice equalizer. Hence, if the channel is not very time dispersive, only a fraction of the stages are used. When the channel becomes more time dispersive, the length of the equalizer can be increased by the algorithm without stopping the operation of the equalizer. The structure of a lattice equalizer, however, is more complicated than a linear transversal equalizer.



Zero-forcing equalizer

The ZF equalizer can be interpreted in the frequency domain as enforcing a completely flat (constant) transfer function of the combination of channel and equalizer by choosing the equalizer transfer function as

$$E(z)=\frac{1}{F(Z)}$$

In the time domain, this can be interpreted as minimizing the maximum ISI (**peak distortion criterion**). Zero-forcing equalizer is based on the peak distortion criterion, where the equalizer forces the ISI to zero.

The ZF equalizer is optimum for elimination of ISI. However, channels also add noise, which is amplified by the equalizer. At frequencies where the transfer function of the channel attains small values, the equalizer has a strong amplification, and thus also amplifies the noise.

Let the Fourier transform of sample ACF ζ_i be $\Xi(e^{j\omega T_S})$ and the Fourier transform of $\eta(t)$ be $\hat{\Xi}(e^{j\omega T})$

$$\Xi(e^{j\omega T_S})=\frac{1}{T_S}\sum_{n=-\infty}^{\infty}\left|\hat{\Xi}\left(\omega+\frac{2\pi n}{T_S}\right)\right|^2 \qquad |\omega|\le\frac{\pi}{T_S}$$

The noise power at the detector is

$$\sigma^2_{n-\text{LE-ZF}}=N_0\frac{T_S}{2\pi}\int_{-\frac{\pi}{T_S}}^{\frac{\pi}{T_S}}\frac{1}{\Xi(e^{j\omega T_S})}\,d\omega$$

The Minimum Mean Square Error Equalizer

The ultimate goal of an equalizer is minimization of the bit error probability and not ISI. Noise enhancement makes the ZF equalizer not suitable for this purpose. A better criterion is minimization of the **Mean Square Error** (MSE) between the transmit signal and the output of the equalizer.

We are thus searching for a filter that minimizes:

$$MSE=E\left\{|\varepsilon_i|^2\right\}=E\left\{\varepsilon_i\varepsilon_i^*\right\}$$

This can be achieved with a filter whose coefficients **e**opt are given by

$$\mathbf{e}_{opt}=\mathbf{R}^{-1}\mathbf{p}$$

where

$\mathbf{R}=E\{\mathbf{u}_i^*\mathbf{u}_i^T\}$ is the correlation matrix of the received signal

$\mathbf{p}=E\{\mathbf{u}_i^*c_i\}$ is the crosscorrelation between the received signal and the transmit signal.

Considering the frequency domain, cascade of the noise-whitening filter with the equalizer $E(z)$ has the transfer function:

$$\tilde{E}(z)=\frac{1}{\Xi(Z)+\frac{N_0}{\sigma_S^2}}$$

The MSE is then

$$\sigma^2_{n-\text{LE-ZF}} = N_0 \frac{T_S}{2\pi} \int_{-\pi/T_S}^{\pi/T_S} \frac{1}{\Xi(e^{j\omega T_S}) + \frac{N_0}{\sigma_S^2}} \, d\omega$$

The noise power of an MMSE equalizer is smaller than that of a ZF equalizer.

Adaptation Algorithms for Mean Square Error Equalizers

Least Mean Square Algorithm

The LMS algorithm; also known as the **stochastic gradient method**, consists of the following steps:

1. Initialize the weights with values $\mathbf{e}_0$.
2. With this value, compute an approximation for the gradient of the MSE. We are using an estimate for **R** and **p**:

$$\hat{\mathbf{R}}_n = \mathbf{u}_n^* \mathbf{u}_n^T$$
$$\hat{\mathbf{p}}_n = \mathbf{u}_n^* c_n$$

where subscript n indexes the iterations. The gradient is estimated as

$$\hat{\nabla}_n = -2\hat{\mathbf{p}}_n + 2\hat{\mathbf{R}}_n \mathbf{e}_n$$

3. We next compute an updated estimate of the weight vector **e** by adjusting weights in the direction of the negative gradient:

$$\mathbf{e}_{n+1} = \mathbf{e}_n - \mu \hat{\nabla}_n$$

4. If the stop criterion is fulfilled – e.g., the relative change in weight vector falls below a predefined threshold – the algorithm has converged. Otherwise, we return to step 2.

It can be shown that the LMS algorithm converges if

$$0 < \mu < \frac{2}{\lambda_{\max}}$$

Here λ_{max} is the largest eigenvalue of the correlation matrix **R**

The Recursive Least Squares Algorithm

In most cases, the LMS algorithm converges very slowly. Furthermore, the use of this algorithm is justified only when the statistical properties of the received signal fulfill certain conditions.

The general Least Squares (LS) criterion does not require such assumptions. It just analyzes the N subsequent errors ε_i , and chooses weights such that the sum of the squared errors is minimized. This general LS problem can be solved by a recursive algorithm as well – known as *RLS.*

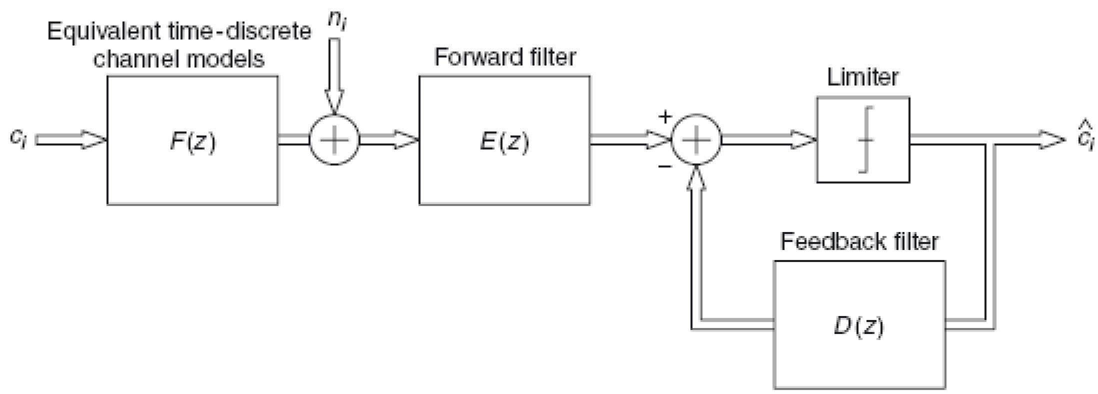
Comparison of Algorithms

LMS Algorithm	RLS Algorithm
The LMS algorithm usually converges too slowly.	The RLS algorithm converges faster, but has a larger residual error.
The LMS algorithm requires fewer (complex) operations than RLS algorithm.	The RLS algorithm requires more (complex) operations than LMS algorithm.

DECISION FEEDBACK EQUALIZERS

In decision feedback equalization, once a bit is correctly detected, the effect this bit on subsequent bit is determined. The ISI caused by each bit is then subtracted from these later samples.

The block diagram of a DFE is shown in Figure below.
The DFE consists of a **forward filter** with transfer function $E(z)$, which is a conventional linear equalizer, as well as a **feedback filter** with transfer function $D(z)$. As soon as the RX has decided on a received symbol, its impact on all future samples (**postcursor ISI**) can be computed, and (via the feedback) subtracted from the received signal.

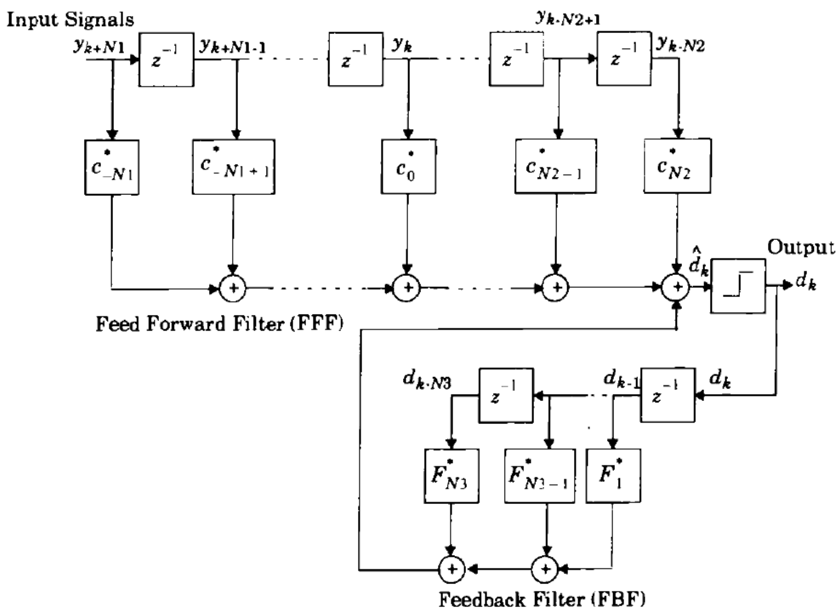


The ISI is computed based on the signal after the hard decision. This eliminates additive noise from the feedback signal. Therefore, a DFE results in a smaller error probability than a linear equalizer.

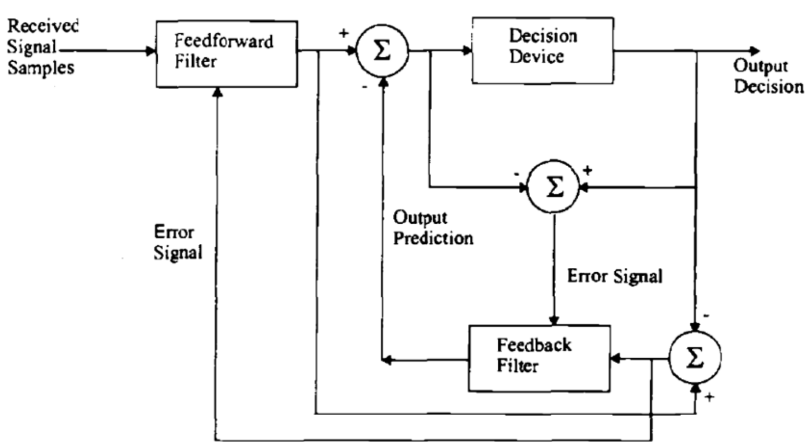
One possible source of problems is **error propagation**. If the RX decides incorrectly for one bit, then the computed postcursor ISI is also erroneous, so that later signal samples arriving at the decision device are even more afflicted by ISI than the unequalized samples.

The basic idea behind decision feedback equalization is that once an information symbol has been detected and decided upon, the ISI that it induces on future symbols can be estimated and subtracted out before detection of subsequent symbols. The DFE can be realized in either the direct transversal form or as a lattice filter.

It consists of a feedforward filter (FFF) and a feedback filter (FBF). The FBF is driven by decisions on the output of the detector, and its coefficients can be adjusted to cancel the ISI on the current symbol from past detected symbols. The equalizer has N1 + N2 + I taps in the feed forward filter and N3 taps in the feedback filter. The lattice implementation of the DFE is equivalent to a transversal DFE having a feed forward filter of length N1 and a feedback filter of length N2, where N1 > N2.



Another form of DFE proposed by Belfiore and Park is called a predictive DFE, and is shown in below Figure. It also consists of a feed forward filter (FFF) as in the conventional DFE. However, the feedback filter (FBF) is driven by an input sequence formed by the difference of the output of the detector and the output of the feed forward filter. Hence, the FBF here is called a noise predictor because it predicts the noise and the residual ISI contained in the signal at the FFF output and subtracts from it the detector output after some feedback delay. The predictive DFE performs as well as the conventional DFE as the limit in the number of taps in the FFF and the FBF approach infinity. The FEF in the predictive DFE can also be realized as a lattice structure. The RLS lattice algorithm can be used in this case to yield fast convergence.



MMSE Decision Feedback Equalizer

The goal of the MMSE DFE is again minimization of the MSE. We now aim to minimize the sum of noise and (average) **precursor** ISI.

The coefficients of the feedforward filter can be computed from the following equation:

$$\sum_{n=-K_{ff}}^{0} e_n \left(\sum_{m=0}^{-l} f_m^* f_{m+l-n} + N_0 \delta_{nl} \right) = -f_{-l}^* \quad \text{for } n,l = -K_{ff} \text{ to } 0$$

where K_{ff} is the number of taps in the feedforward filter.

The coefficients of the feedback filter are then

$$d_n = -\sum_{m=-K_{fb}}^{0} e_m f_{n-m} \quad \text{for } n = 1 \text{ to} K_{fb}$$

where $K_{fb}$is the number of taps in the feedback filter.

The MSE at the equalizer output is

$$\sigma_n^2(DFE - MMSE) = N_0 \exp\left(\frac{T_S}{2\pi} \int_{-\pi/T_S}^{\pi/T_S} \ln\left[\frac{1}{\Xi(e^{j\omega T}) + N_0} \right] d\omega \right)$$

Zero-Forcing Decision Feedback Equalizer

In ZF DFE, the noise-whitening filter eliminates all precursor ISI, such that the resulting effective channel is purely causal. Postcursor ISI is subtracted by the feedback branch. The effective noise power at the decision device is

$$\sigma_n^2(DFE - ZF) = N_0 \exp\left(\frac{T_S}{2\pi} \int_{-\pi/T_S}^{\pi/T_S} \ln\left[\frac{1}{\Xi(e^{j\omega T})} \right] d\omega \right)$$

This equation demonstrates that noise power is larger than it is in the unequalized case, but smaller than that for the linear ZF equalizer.

Comparison of Equalizer Structures

When selecting an equalizer for a practical system, we have to consider the following criteria:

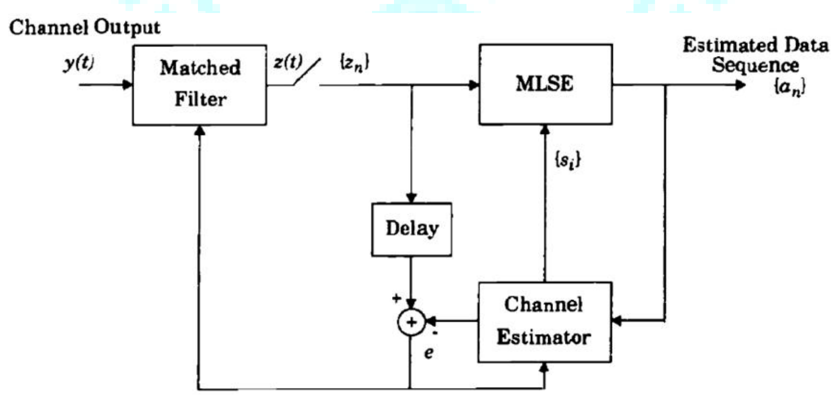
1. ***Minimization of the BER:*** here MLSE is superior to all other structures. DFEs, though worse than MLSE estimators, are better than linear equalizers.
2. ***Computational effort:*** The effort for linear equalizers and DFEs is not significantly different. Depending on the adaptation algorithm, the number of operations increases linearly, quadratically, or cubically with equalizer length (number of weights). For MLSE, the computation effort increases exponentially with length of the impulse response of the channel.
3. ***Sensitivity to channel misestimation*:** Due to the error propagation effect, DFE equalizers are more sensitive to channel estimation errors than linear equalizers. Also, ZF equalizers are more sensitive than MMSE equalizers.

Maximum Likelihood Sequence Estimation (MLSE) Equalizer

The MSE-based linear equalizers described previously are optimum with respect to the criterion of minimum probability of symbol error when the channel does not introduce any amplitude distortion. Yet this is precisely the condition in which an equalizer is needed for a mobile communications link. This limitation on MSE-based equalizers led researchers to investigate optimum or nearly optimum nonlinear structures. These equalizers use various forms of the classical maximum likelihood receiver structure. Using a channel impulse response simulator within the algorithm, the MLSE tests all possible data sequences (rather than decoding each received symbol by itself), and chooses the data sequence with the maximum probability as the output. An MLSE usually has a large computational requirement, especially when the delay spread of the channel is large.

Using the MLSE as an equalizer was first proposed by Forney in which he set up a basic MLSE estimator structure and implemented it with the Viterbi algorithm. This algorithm, was recognized to be a maximum likelihood sequence estimator (MLSE) of the state sequences of a finite state Markov process observed in memoryless noise. It has recently been implemented successfully for equalizers in mobile radio channels.

The MLSE can be viewed as a problem in estimating the state of a discrete time finite state machine, which in this case happens to be the radio channel with coefficients f_k and with a channel state which at any instant of time is estimated by the receiver based on the L most recent input samples. Thus the channel has states, where M is the size of the symbol alphabet of the modulation. That is, an trellis is used by the receiver to model the channel over time. The Viterbi algorithm then tracks the state of the channel by the paths through the trellis and gives at stage k a rank ordering of the M^L most probable sequences terminating in the most recent L symbols.



The block diagram of a MLSE receiver based on the DFE is shown in Figure. The MLSE is optimal in the sense that it minimizes the probability of a sequence error. The MLSE requires knowledge of the channel characteristics in order to compute the metrics for making decisions. The MLSE also requires knowledge of the statistical distribution of the noise corrupting the signal. Thus, the probability distribution of the noise determines the form of the metric for optimum demodulation of the received signal. Notice that the matched filter operates on the continuous time signal, whereas the MLSE and channel estimator rely on discretized (nonlinear) samples.

Algorithms for Adaptive Equalization

The performance of an algorithm is determined by various factors which include:

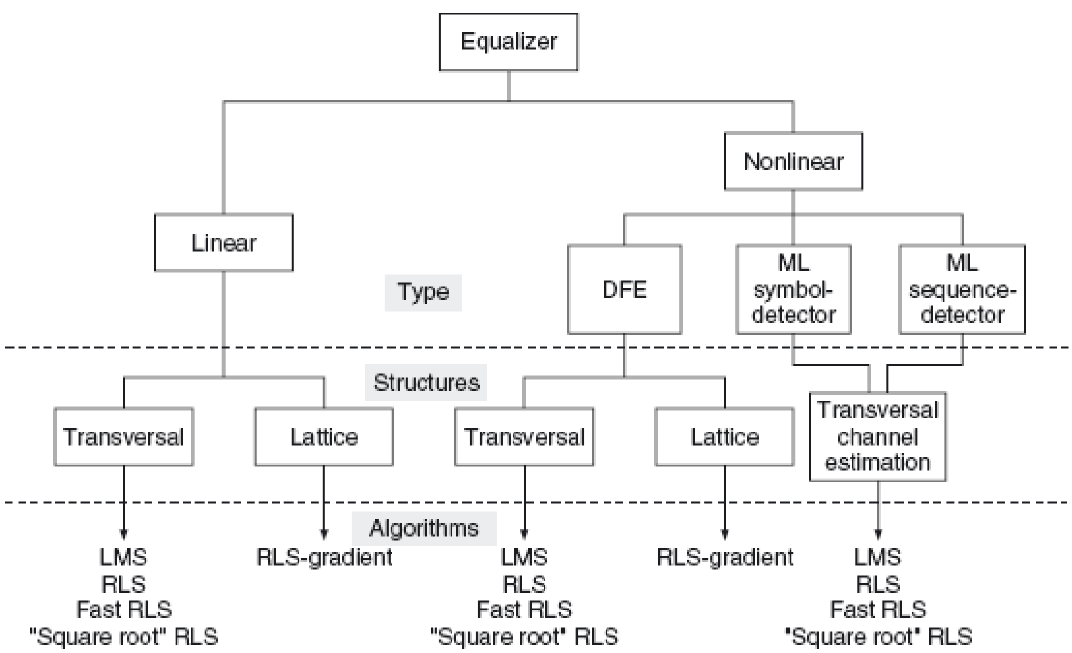
Rate of convergence — This is defined as the number of iterations required for the algorithm, in response to stationary inputs, to converge close enough to the optimum solution. A fast rate of convergence allows the algorithm to adapt rapidly to a stationary environment of unknown statistics. Furthermore, it enables the algorithm to track statistical variations when operating in a nonstationary environment.

Misadjustment For an algorithm of interest, this parameter provides a quantitative measure of the amount by which the final value of the mean square error, averaged over an ensemble of adaptive filters, deviates from the optimal minimum mean square error.

Computational complexity — This is the number of operations required to make one complete iteration of the algorithm.

Numerical properties — When an algorithm is implemented numerically, inaccuracies are produced due to round-oft' noise and representation errors in the computer. These kinds of errors influence the stability of the algorithm.

Figure shows a taxonomy of equalizer structures.



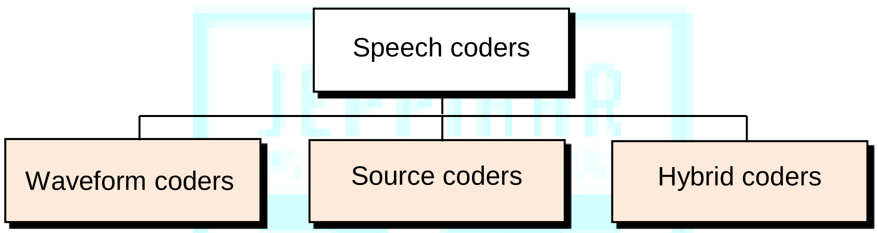
SPEECH CODING

Speech coding is the process for reducing the bit rate of digital speech representation for transmission or storage, while maintaining a speech quality that is acceptable for the application.

Speech coding is used to save bandwidth and improve bandwidth efficiency whereas channel coding is employed to improve signal quality and reduce bit-error-rate (BER).

Speech coders are classified as

1. Waveform coders
2. Source coders
3. Hybrid coders



Error Probability in Fading Channels with Diversity Reception

Error Probability in Flat-Fading Channels

Classical Computation Method

The error probability of diversity systems by averaging the conditional error probability (conditioned on a certain SNR) over the distribution of the SNR:

Symbol Error Rate:

$$\overline{SER} = \int_0^\infty pdf_\gamma(\gamma) SER(\gamma)\, d\gamma$$

As an example, let us compute the performance of BPSK with N_r diversity branches with MRC. The SER of BPSK in AWGN is

$$SER(\gamma) = Q(\sqrt{2\gamma})$$

Let us apply this principle to the case of MRC.

$$pdf_\gamma(\gamma) = \frac{1}{(N_r - 1)!} \frac{\gamma^{N_r - 1}}{\bar{\gamma}^{N_r}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$$

$$\overline{SER} = \left(\frac{1-b}{2}\right)^{N_r} \sum_{n=0}^{N_r-1} \binom{N_r - 1 + n}{n} \left(\frac{1+b}{2}\right)^n$$

where b is defined as

$$b = \sqrt{\frac{\overline{\gamma}}{1+\overline{\gamma}}}$$

For large values of γ , this can be approximated as

$$\overline{SER} = \left(\frac{1}{4\overline{\gamma}}\right)^{N_r} \binom{2N_r - 1}{N_r}$$

From this, we can see that (with *N*r diversity antennas) the BER decreases with the *N*r-th power of the SNR.

Computation via the Moment-Generating Function

In the previous section, we averaged the BER over the distribution of SNRs, using the "classical" representation of the Q-function. There is an alternative definition of the Q-function, which can easily be combined with the moment-generating function *Mγ (s)* of the SNR. Let us start by writing the SER conditioned on a given SNR in the form

$$SER(\gamma) = \int_{\theta_1}^{\theta_2} f_1(\theta) \exp(-\gamma_{MRC} f_2(\theta))\, d\theta$$

$$\gamma_{MRC} = \sum_{n=1}^{N_r} \gamma_n$$

$$SER(\gamma) = \int_{\theta_1}^{\theta_2} f_1(\theta) \prod_{n=1}^{N_r} \exp(-\gamma_n f_2(\theta))\, d\theta$$

Averaging over the SNRs in different branches then becomes

$$\overline{SER} = \int d\gamma_1 pdf_{\gamma_1}(\gamma_1) \int d\gamma_2 pdf_{\gamma_2}(\gamma_2) \cdots \int d\gamma_{N_r} pdf_{\gamma_{N_r}}(\gamma_{N_r}) \int_{\theta_1}^{\theta_2} d\theta f_1(\theta) \prod_{n=1}^{N_r} \exp(-\gamma_n f_2(\theta))$$

$$= \int_{\theta_1}^{\theta_2} d\theta f_1(\theta) \prod_{n=1}^{N_r} \int d\gamma_n pdf_{\gamma_n}(\gamma_n) \exp(-\gamma_n f_2(\theta))$$

$$= \int_{\theta_1}^{\theta_2} d\theta f_1(\theta) \prod_{n=1}^{N_r} M_\gamma(-f_2(\theta))$$

$$= \int_{\theta_1}^{\theta_2} d\theta f_1(\theta) [M_\gamma(-f_2(\theta))]^{N_r}$$

we can write the error probability for BPSK in Rayleigh fading as

$$\overline{SER} = \frac{1}{\pi}\int_0^{\pi/2}\left[\frac{\sin^2(\theta)}{\sin^2(\theta)+\overline{\gamma}}\right]^{N_r} d\theta$$

Symbol Error Rate in Frequency-Selective Fading Channels

We now determine the SER in channels that suffer from time dispersion and frequency dispersion.
We assume here FSK with differential phase detection. The analysis uses the correlation coefficient ρ_{XY} between signals at two sampling times that was discussed in Chapter 12.
For binary FSK with selection diversity:

$$\overline{SER} = \frac{1}{2} - \frac{1}{2}\sum_{n=1}^{N_r}\binom{N_r}{n}(-1)^{n+1}\frac{b_0\,\mathrm{Im}\{\rho_{XY}\}}{\sqrt{(\mathrm{Im}\{\rho_{XY}\})^2 + n(1-|\rho_{XY}|^2)}}$$

where b_0 is the transmitted bit. This can be approximated as

$$\overline{SER} = \frac{(2N_r-1)!!}{2}\left(\frac{1-|\rho_{XY}|^2}{2(\mathrm{Im}\{\rho_{XY}\})^2}\right)^{N_r}$$

where $(2N_r - 1)!! = 1\cdot 3\cdot 5\ldots(2N_r-1)$.
For binary FSK with MRC:

$$\overline{SER} = \frac{1}{2} - \frac{1}{2}\frac{b_0\mathrm{Im}\{\rho_{XY}\}}{\sqrt{1-(\mathrm{Re}\{\rho_{XY}\})^2}}\sum_{n=0}^{N_r-1}\frac{(2n-1)!!}{(2n)!!}\left(1-\frac{(\mathrm{Im}\{\rho_{XY}\})^2}{1-(\mathrm{Re}\{\rho_{XY}\})^2}\right)^n$$

$$\overline{SER} = \frac{(2N_r-1)!!}{2(N_r!)}\left(\frac{1-|\rho_{XY}|^2}{2(\mathrm{Im}\{\rho_{XY}\})^2}\right)^{N_r}$$

This formulation shows that MRC improves the SER by a factor Nr! compared with selection diversity.