UNIT IV WAVEGUIDES

General Wave behaviour along uniform guiding structures – Transverse Electromagnetic Waves, Transverse Magnetic Waves, Transverse Electric Waves – TM and TE Waves between parallel plates. Field Equations in rectangular waveguides, TM and TE waves in rectangular waveguides, Bessel Functions, TM and TE waves in Circular waveguides.

Perfectly conducting planes:

The electromagnetic waves that are guided along or over conducting or dielectric surfaces are called guided waves.

Consider an electromagnetic wave propagating between a pair of parallel perfectly conducting planes of infinite extent in the y and z directions.

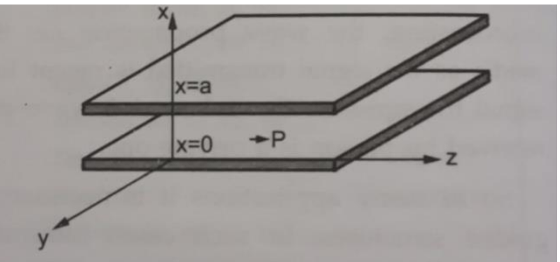


Fig. 5.2.1 An electromagnetic wave between parallel conducting planes

Maxwell's equations will be solved to determine the electromagnetic field configuration in the rectangular region.

Maxwell's equations for a non-conducting rectangular region are given as

$$\nabla \times H = j\omega\varepsilon E$$

$$\nabla \times E = j\omega\mu H$$

$$\nabla \times H = \begin{bmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_X & H_Y & H_Z \end{bmatrix}$$

$$= \overline{a_x}\left(\frac{\partial H_Z}{\partial y} - \frac{\partial H_y}{\partial z}\right) + \overline{a_y}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_Z}{\partial x}\right) + \overline{a_z}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

$$= j\omega\varepsilon\ [E_X\overline{a_x} + E_Y\overline{a_Y} + E_z\overline{a_Z}]$$

Equating x, y, and z components on both sides

$\frac{\partial H_Z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon E_X$ ---------------------------- 1.1

$\frac{\partial H_x}{\partial z} - \frac{\partial H_Z}{\partial x} = j\omega\varepsilon E_y$----------------------------- 1.2

$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$ ----------------------------- 1.3

$$\nabla \times E = \begin{bmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_X & E_Y & E_Z \end{bmatrix}$$

$= \overline{a_x}\left(\frac{\partial E_Z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \overline{a_y}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_Z}{\partial x}\right) + \overline{a_z}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$

$= -j\omega\mu\ [H_X\overline{a_x} + H_Y\overline{a_Y} + H_z\overline{a_Z}]$

Equating x, y, and z components on both sides

$\frac{\partial E_Z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_X$ ---------------------------- 2.1

$\frac{\partial E_x}{\partial z} - \frac{\partial E_Z}{\partial x} = -j\omega\mu H_Y$----------------------------- 2.2

$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_Z$ ----------------------------- 2.3

It is assumed that the propagation is in the z direction and the variation of field components are expressed in the form $e^{-\gamma z}$

Where $\gamma = \alpha + j\beta$

If α= 0, wave propagation without attenuation

If α is real i.e. $\beta = 0$ there is no wave motion but onlu an exponential decrease in amplitude.

$H_Y = H_y^0\ e^{-\gamma z}$

$\frac{\partial H_y}{\partial z} = -\gamma\ H_y^0\ e^{-\gamma z} = -\gamma H_Y$

$\frac{\partial H_y}{\partial z} = -\gamma H_Y$

Similarly

$\frac{\partial H_x}{\partial z} = -\gamma H_x$, $\frac{\partial E_y}{\partial z} = -\gamma E_Y$, $\frac{\partial E_x}{\partial z} = -\gamma E_x$

Similarly

$\frac{\partial^2 E}{\partial Z^2} = -\gamma \frac{\partial E}{\partial z} = -\gamma\ (-\gamma E\ ) = \gamma^2 E$

$\frac{\partial^2 H}{\partial Z^2} = \gamma^2 H$

The wave equation is given by

$\frac{\partial E_x}{\partial z} = -j\omega\mu H_Y$ (Ampere's law)

$\frac{\partial H_y}{\partial z} = (\sigma + j\omega\varepsilon)\ E_X$ (faraday's law)

$\frac{\partial^2 E_x}{\partial Z^2} = -j\omega\mu \frac{\partial H_y}{\partial z} = j\omega\mu(\sigma + j\omega\varepsilon)\ E_X$

$\frac{\partial^2 E_x}{\partial Z^2} = \gamma^2 E_x$ where $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

Similarly

$\frac{\partial^2 H_y}{\partial Z^2} = \gamma^2 H_y$

For a non-conducting medium, consider only real terms

$\nabla^2 E$ = - $\omega^2 \mu\varepsilon E$

$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial Z^2}$ = - $\omega^2 \mu\varepsilon E$ --------------------------3.1

$\nabla^2 H$ = - $\omega^2 \mu\varepsilon H$

$\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial Z^2}$ = - $\omega^2 \mu\varepsilon H$ --------------------------3.2

There is no variation in the y direction (i.e.) derivative of y is zero.

Substituting the values of z derivatives and y derivatives in the equation 1, 2 and 3

$\gamma H_Y = j\omega\varepsilon E_X$ ---------------------------- 4.1

-$\gamma H_x - \frac{\partial H_Z}{\partial x} = j\omega\varepsilon E_y$----------------------------- 4.2

$\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z$ ----------------------------- 4.3

$\gamma E_Y = -j\omega\mu H_X$ ---------------------------- 5.1

$-\gamma E_x - \frac{\partial E_Z}{\partial x} = -j\omega\mu H_Y$----------------------------- 5.2

$\frac{\partial E_y}{\partial x} = -j\omega\mu H_Z$ ----------------------------- 5.3

$\frac{\partial^2 E}{\partial x^2} + \gamma^2 E_x$ = - $\omega^2\mu\varepsilon E$ --------------------------6.1

$\frac{\partial^2 H}{\partial x^2} + \gamma^2 H_y$ = - $\omega^2\mu\varepsilon H$ --------------------------6.2

To find H_x *and* E_y

From 4.2 and 5.1

-$\gamma H_x - \frac{\partial H_Z}{\partial x} = j\omega\varepsilon E_y$----------------------------- 4.2

$\gamma E_Y = -j\omega\mu H_X$ ---------------------------- 5.1

$E_Y = -\frac{j\omega\mu}{\gamma} H_X$ ---- 7

Substitute 7 in 4.2

-$\gamma H_x - \frac{\partial H_Z}{\partial x} = -\frac{\omega^2\mu\varepsilon}{\gamma} H_X$

$- \frac{\partial H_Z}{\partial x} = H_x \left[\frac{\omega^2\mu\varepsilon}{\gamma} + \gamma\right]$

$H_x = \left[\frac{-\gamma}{\omega^2\mu\varepsilon+\gamma^2}\right]\frac{\partial H_Z}{\partial x}$

$$\boldsymbol{H_x = \left[\frac{-\gamma}{h^2}\right]\frac{\partial H_Z}{\partial x}}$$

Where $h^2 = \omega^2\mu\varepsilon + \gamma^2$

1.1 =>$H_X = \frac{-\gamma}{j\omega\mu} E_Y$ ----------------------- 8

Sub 8 in 4.2

$-\gamma(\frac{-\gamma}{j\omega\mu}E_Y) - \frac{\partial H_Z}{\partial x} = j\omega\varepsilon E_y$

$$(\frac{\gamma^2}{j\omega\mu}E_Y) - j\omega\varepsilon E_y = \frac{\partial H_Z}{\partial x}$$

$$E_Y\left[\frac{\omega^2\mu\varepsilon + \gamma^2}{j\omega\mu}\right] = \frac{\partial H_Z}{\partial x}$$

$$\mathbf{E_Y} = [\frac{\mathbf{j\omega\mu}}{\mathbf{h^2}}]\frac{\mathbf{\partial H_Z}}{\mathbf{\partial x}},$$

Where $h^2 = \omega^2\mu\varepsilon + \gamma^2$

To find H_Y and E_x

From 5.2 and 4.1

$\gamma E_x + \frac{\partial E_Z}{\partial x} = +j\omega\mu H_Y$------------------------------ 5.2

$\gamma H_Y = j\omega\varepsilon E_X$ ----------------------------- 4.1

$E_X = \frac{\gamma}{j\omega\varepsilon}H_Y$ ------------------------------ 9

Sub 9 in 5.2

$$\frac{\gamma^2}{j\omega\varepsilon}H_Y - \frac{\partial E_Z}{\partial x} = -j\omega\mu H_Y$$

$\frac{\gamma^2}{j\omega\varepsilon}H_Y - j\omega\mu H_Y = -\frac{\partial E_Z}{\partial x}$

$H_Y[\frac{\omega^2\mu\varepsilon+\gamma^2}{j\omega\varepsilon}] = -\frac{\partial E_Z}{\partial x}$

$$\boldsymbol{H_Y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_Z}{\partial x}}$$

1.1 =>$H_Y = \frac{j\omega\varepsilon}{\gamma}E_X$ ------------------- 10

Sub 10 in 5.2

$$\gamma E_x + \frac{\partial E_Z}{\partial x} = j\omega\mu\frac{j\omega\varepsilon}{\gamma}E_X$$

$$\frac{\partial E_Z}{\partial x} = -E_X\left(\frac{\omega^2\mu\varepsilon + \gamma^2}{\gamma}\right)$$

$$\boldsymbol{E_X = -\frac{\gamma}{h^2}\frac{\partial E_Z}{\partial x}}$$

The components of electric and magnetic field strengths E_X, E_y, H_X, H_Y are expressed in terms of E_Z and H_Z.

It is observed that there must be a Z component of either E or H, otherwise all the components would be zero.

In general case both E_Z and H_Z may be present at the same time, It is convenient to divide the solutions into two cases.

In the first case, there is a component of E in the direction of propagation (E_Z) but no component of H in this direction. Such waves are called TM waves.

In the second case, there is a component of H in the direction of propagation (E_Z) but no component of E in this direction. Such waves are called TE waves.

TE waves between parallel planes.

TE waves are waves in which the electric field strength E is entirely transverse. It has a magnetic field strength H_z in the direction of propagation and no component of electric field E_z in the same direction $E_z = 0$

$$E_z = 0,\ \boldsymbol{E_X = -\frac{\gamma}{h^2}\frac{\partial E_Z}{\partial x}},\ \boldsymbol{H_Y = \frac{-j\omega\varepsilon}{h^2}\frac{\partial E_Z}{\partial x}}$$

Therefore $E_X = 0, H_Y = 0$

Then the wave equation for the component

$$\frac{\partial^2 E_y}{\partial x^2} + \gamma^2 E_y = -\omega^2\mu\varepsilon E_y$$

$$\frac{\partial^2 E_y}{\partial x^2} = -\omega^2\mu\varepsilon E_y - \gamma^2 E_y$$

$$\frac{\partial^2 E_y}{\partial x^2} = -(\omega^2\mu\varepsilon E_y + \gamma^2)\ E_y$$

$$\frac{\partial^2 E_y}{\partial x^2} = -h^2\ E_y \qquad \text{Where } h^2 = \omega^2\mu\varepsilon + \gamma^2$$

$$\frac{\partial^2 E_y}{\partial x^2} + h^2\ E_y = 0$$

$$(D^2 + h^2)E_y = 0$$

$$m^2 + h^2 = 0, m^2 = -h^2,\ \text{m} = \pm jh$$

Therefore

$$E_y = e^{0.x}[c_1\sin(hx) + c_2\cos(hx)]$$

$E_y = [c_1\sin(hx) + c_2\cos(hx)]$ where c_1 and c_2 are arbitrary constants.

If E_y is expressed in time and direction

$E_y{}^0 = [c_1 \sin(hx) + c_2 \cos(hx)]$

$E_y = E_y{}^0 e^{-\gamma z} = [c_1 \sin(hx) + c_2 \cos(hx)]\, e^{-\gamma z}$

The boundary conditions for the parallel planes are

$E_y = 0$ at x = 0,

$E_y = 0$ at x = a

Applying the first boundary condition

$E_y = 0$ at x = 0

$0 = [c_1 \sin(ha)] e^{-\gamma z}$

$$e^{-\gamma x} \neq 0$$

$c_1$cannot be zero

Therefore sin (ha) = 0

ha = $m\pi$ h =$\frac{m\pi}{a}$

$$\boldsymbol{E_y = [c_1 \sin(\frac{m\pi}{a} x)] e^{-\gamma z}}$$

$\frac{\partial E_y}{\partial x} = [\frac{m\pi}{a} c_1 \cos(\frac{m\pi}{a} x)] e^{-\gamma z}$

W.k.t from Maxwell equation of parallel planes

$$\gamma E_Y = -j\omega\mu H_X$$

$$\frac{\partial E_y}{\partial x} = -j\omega\mu H_Z$$

$H_X = -\frac{\gamma}{j\omega\mu} E_Y$

$$\boldsymbol{H_X = -\frac{\gamma}{j\omega\mu} [c_1 \sin(\frac{m\pi}{a} x)] e^{-\gamma z}}$$

$$H_Z = \frac{-1}{j\omega\mu}\frac{\partial E_y}{\partial x}$$

$$\boldsymbol{H_Z = \frac{-1}{j\omega\mu}[\frac{m\pi}{a}c_1\cos(\frac{m\pi}{a}x)]}e^{-\gamma z}$$

The field strength for TE waves between parallel planes are

$$\boldsymbol{E_y = [c_1\sin(\frac{m\pi}{a}x)]}e^{-\gamma z}$$

$$\boldsymbol{H_X = -\frac{\gamma}{j\omega\mu}[c_1\sin(\frac{m\pi}{a}x)]}e^{-\gamma z}$$

$$\boldsymbol{H_Z = \frac{-1}{j\omega\mu}[\frac{m\pi}{a}c_1\cos(\frac{m\pi}{a}x)]}e^{-\gamma z}$$

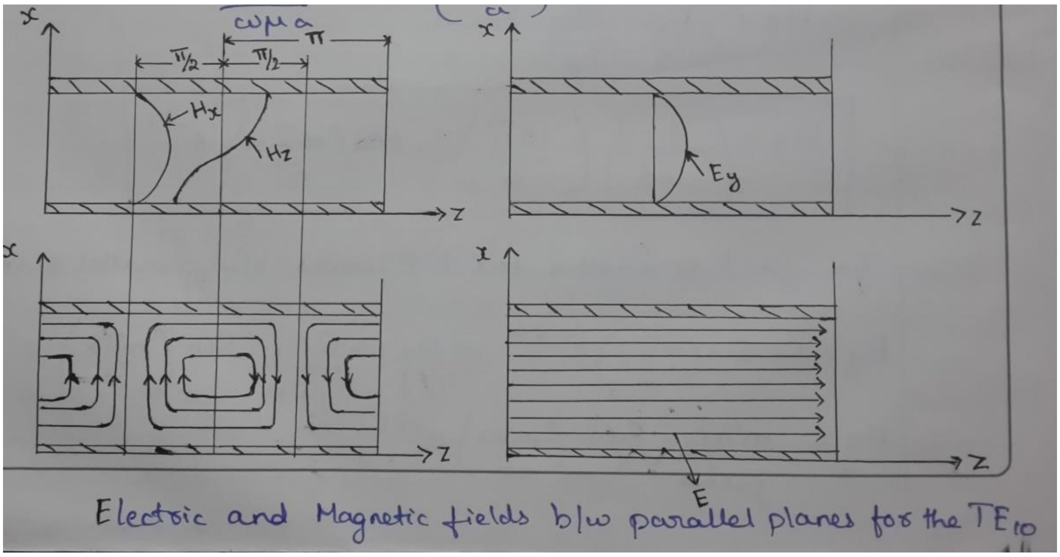
Each value of m specifies a particular field of configuration or mode of the wave designated as TE_{m0} wave or TE_{m0} mode.

If m= $0 E_y = 0, H_X = 0, H_Z = 0$

Therefore the lowest value of m is 1

The lowest order mode is $\boldsymbol{TE_{10}}$

This is called dominant mode in TE waves.



Transverse magnetic waves

Transverse magnetic waves are waves in which the magnetic field strength H is entirely transverse. If has an electric field strength E_z in the direction of propagation and no comkponent of magnetic field H_z in the same direction.

i.e. H_z =0 thenH_x =0 and E_y = 0

To find E_x, E_z, H_y

The wave equation for the component H_y

$$\frac{\partial^2 H_y}{\partial x^2} + \gamma^2 H_y = - \omega^2 \mu\varepsilon H_y$$

$$\frac{\partial^2 H_y}{\partial x^2} = -+ \gamma^2 H_y - \omega^2 \mu\varepsilon H_y$$

$$\frac{\partial^2 H_y}{\partial x^2} = - h^2 H_y \quad \text{Where } h^2 = \omega^2 \mu\varepsilon + \gamma^2$$

$$\frac{\partial^2 H_y}{\partial x^2} + h^2 H_y = 0$$

$$(D^2 + h^2) H_y = 0$$

$m^2 + h^2 = 0, m^2 = -h^2$, m = $\pm jh$

Therefore

$H_y = e^{0.x}$[$c_3$sin (hx) + $c_4$cos(hx)]

$H_y =$ [$c_3$sin (hx) + $c_4$cos(hx)] where c_3 and c_4 are arbitrary constants.

If H_y is expressed in time and direction

${H_y}^0 =$ [$c_3$sin (hx) + $c_4$cos(hx)]

H_y = ${H_y}^0 e^{-\gamma x}$ = [c_3 sin (hx) + $c_4$cos(hx)] $e^{-\gamma z}$

The boundary conditions cannot be applied directly to H_y because the magnetic field is not zero at the surface of a conductor.

$E_z$can be obtained in terms of H_y

$$\frac{\partial H_y}{\partial x} = j\omega\varepsilon E_z$$

$$E_z = \frac{1}{j\omega\varepsilon}\frac{\partial H_y}{\partial x}$$

$$E_z = \frac{h}{j\omega\varepsilon}[\text{c3 cos (hx)} - \text{ c4 sin}(hx)]\, e^{-\gamma z}$$

The boundary conditions for the parallel planes are

$E_z = 0$ at x = 0,

$E_z = 0$ at x = a

Applying the first boundary condition

$E_z = 0$ at x = 0

$$0 = \frac{h}{j\omega\varepsilon}[\text{c3} \cos 0 - \text{ c4 sin0}]e^{-\gamma z}$$

$$0 = \frac{h}{j\omega\varepsilon}[\text{c3}]e^{-\gamma z}$$

$\frac{h}{j\omega\varepsilon}$ *and* $e^{-\gamma z}$cannot be zero because it eliminates the entire wave equation

Therefore c3 $= 0$

$$E_z = \frac{h}{j\omega\varepsilon}[-\text{ c4 sin}(hx)]\, e^{-\gamma z}$$

Applying the second boundary condition

$E_z = 0$at x = a

$$0 = \frac{-h}{j\omega\varepsilon}[\text{c4 sin}(ha)]\, e^{-\gamma z}$$

$$\frac{-h}{j\omega\varepsilon}\text{and } e^{-\gamma z} \neq 0$$

C4 cannot be zero because c3 is already equal to zero.

Therefore sin (ha) = 0

ha = $m\pi$ h $= \frac{\text{m}\pi}{a}$

$$\boldsymbol{E_z} = \frac{\mathbf{m}\boldsymbol{\pi}}{\boldsymbol{j\omega\varepsilon a}}[-\ \mathbf{c4}\ \mathbf{sin}(\frac{\mathbf{m}\boldsymbol{\pi x}}{\boldsymbol{a}})]\ \boldsymbol{e^{-\gamma z}}$$

$$H_y = \ [\ \mathbf{c4}\ \mathbf{cos}(\frac{\mathbf{m}\boldsymbol{\pi x}}{\boldsymbol{a}})]\ \boldsymbol{e^{-\gamma z}}$$

$$\gamma H_Y = \ j\omega\varepsilon E_X$$

$E_X = \frac{\gamma}{j\omega\varepsilon} H_Y$

$$\boldsymbol{E_X} = \frac{\boldsymbol{\gamma}}{\boldsymbol{j\omega\varepsilon}}[\ \mathbf{c4}\ \mathbf{cos}(\frac{\mathbf{m}\boldsymbol{\pi x}}{\boldsymbol{a}})]\ \boldsymbol{e^{-\gamma z}}$$

The field strengths for TM waves between parallel planes are

$$\boldsymbol{E_z} = \frac{\mathbf{m}\boldsymbol{\pi}}{\boldsymbol{j\omega\varepsilon a}}[-\ \mathbf{c4}\ \mathbf{sin}(\frac{\mathbf{m}\boldsymbol{\pi x}}{\boldsymbol{a}})]\ \boldsymbol{e^{-\gamma z}}$$

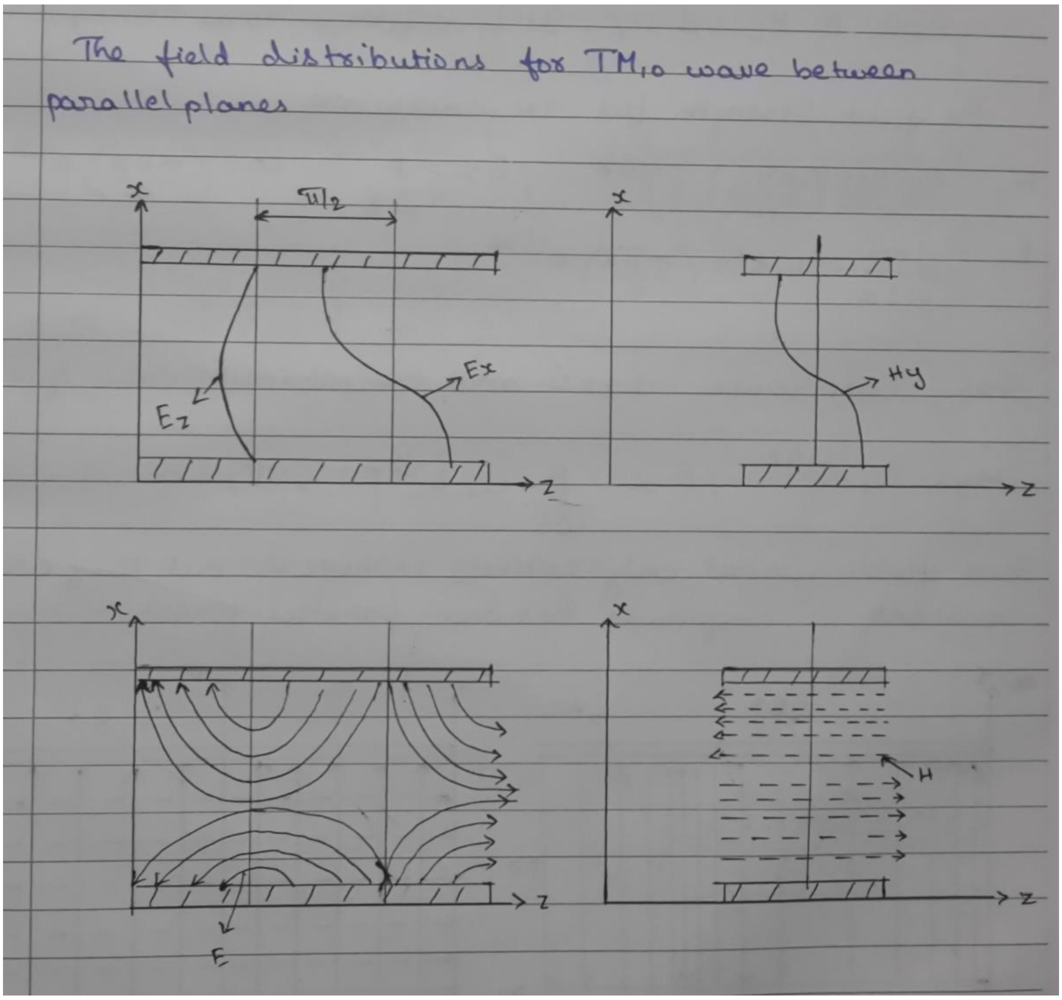
$$\boldsymbol{H_y} = \ [\ \mathbf{c4}\ \mathbf{cos}(\frac{\mathbf{m}\boldsymbol{\pi x}}{\boldsymbol{a}})]\ \boldsymbol{e^{-\gamma z}}$$

$$\boldsymbol{E_X} = \frac{\boldsymbol{\gamma}}{\boldsymbol{j\omega\varepsilon}}[\ \mathbf{c4}\ \mathbf{cos}(\frac{\mathbf{m}\boldsymbol{\pi x}}{\boldsymbol{a}})]\ \boldsymbol{e^{-\gamma z}}$$

If m = 0 $\boldsymbol{E_X}$ and $\boldsymbol{H_Y}$exists only $\boldsymbol{E_z} = \mathbf{0}$

In this case of TM waves there is a possibility of m =0

TM_{10} is the dominant mode.



Transverse electromagnetic waves:

It is a special type of transverse magnetic wave in which electric field E along the direction of propagation is also zero.

TEM waves are waves in which both electric and magnetic fields are transverse entirely but has no component of E_z and H_z. it is also referred as principle waves.

The field strengths for TM waves between parallel planes are

$$\boldsymbol{E_z = \frac{m\pi}{j\omega\varepsilon a}[-\,c4\,\sin(\frac{m\pi x}{a})]\,e^{-\gamma z}}$$

$$\boldsymbol{H_y = \,[\,c4\,\cos(\frac{m\pi x}{a})]\,e^{-\gamma z}}$$

$$\boldsymbol{E_X = \frac{\gamma}{j\omega\varepsilon}[\,c4\,\cos(\frac{m\pi x}{a})]\,e^{-\gamma z}}$$

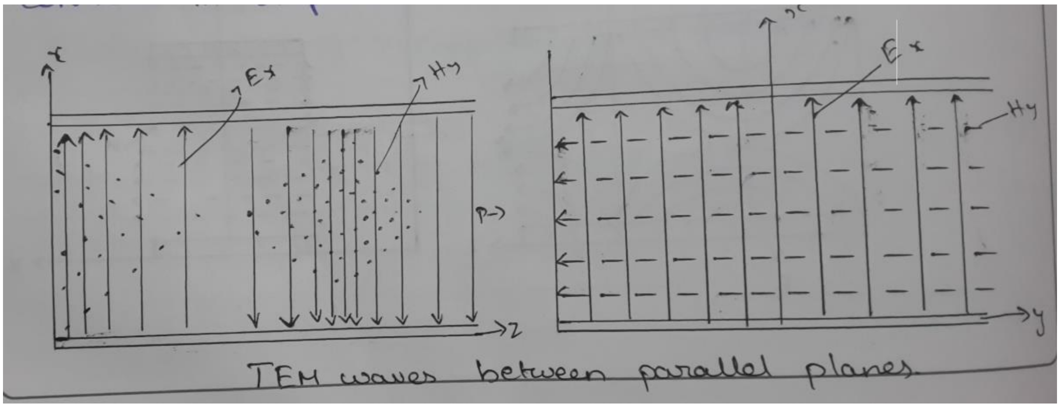
For TEM waves E_z = 0 and the minimum value of m = 0

$$H_y = c4e^{-\gamma z}$$

$$E_X = \frac{\gamma}{j\omega\varepsilon} c4e^{-\gamma z}$$

$$E_z = 0$$

These fields are not only entirely transverse but they are constant in amplitude between parallel planes



Rectangular Wave guide.

A hollow conducting metallic tube of uniform cross section is used for propagating electromagnetic waves, waves that are guided along the surface of the tube is called a wave guide.

Waveguides usually in the form of rectangular or circular cylinders

Propagation of waveguide can be considered as a phenomenon in which the waves are reflected from wall to wall and hence pass down the waveguide in a zigzag fashion.

To determine the electromagnetic field configuration within the guide, Maxwell's equation are solved subject to the appropriate boundary conditions at the walls of the guide.

Maxwell's equations for a non-conducting rectangular region are given as

$$\nabla \times H = j\omega\varepsilon E$$

$$\nabla \times E = j\omega\mu H$$

$$\nabla \times H = \begin{bmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_X & H_Y & H_Z \end{bmatrix}$$

$$= \overline{a_x}\left(\frac{\partial H_Z}{\partial y} - \frac{\partial H_y}{\partial z}\right) + \overline{a_y}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_Z}{\partial x}\right) + \overline{a_z}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

$= j\omega\varepsilon\ [E_X\overline{a_x} + E_Y\overline{a_Y} + E_z\overline{a_Z}]$

Equating x, y, and z components on both sides

$\frac{\partial H_Z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\varepsilon E_X$ ---------------------------- 1.1

$\frac{\partial H_x}{\partial z} - \frac{\partial H_Z}{\partial x} = j\omega\varepsilon E_y$---------------------------- 1.2

$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$ ---------------------------- 1.3

$$\nabla \times E = \begin{bmatrix} \overline{a_x} & \overline{a_y} & \overline{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_X & E_Y & E_Z \end{bmatrix}$$

$= \overline{a_x}\left(\frac{\partial E_Z}{\partial y} - \frac{\partial E_y}{\partial z}\right) + \overline{a_y}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_Z}{\partial x}\right) + \overline{a_z}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)$

$= -j\omega\mu\ [H_X\overline{a_x} + H_Y\overline{a_Y} + H_z\overline{a_Z}]$

Equating x, y, and z components on both sides

$\frac{\partial E_Z}{\partial y} - \frac{\partial E_y}{\partial z} = -j\omega\mu H_X$ ---------------------------- 2.1

$\frac{\partial E_x}{\partial z} - \frac{\partial E_Z}{\partial x} = -j\omega\mu H_Y$---------------------------- 2.2

$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_Z$ ---------------------------- 2.3

It is assumed that the propagation is in the z direction and the variation of field components are expressed in the form $e^{-\gamma z}$

Where $\gamma = \alpha + j\beta$

If α= 0, wave propagation without attenuation

If α is real i.e. β = 0 there is no wave motion but onlu an exponential decrease in amplitude.

$H_Y = H_y^0\ e^{-\gamma z}$

$\frac{\partial H_y}{\partial z} = -\gamma\ H_y^0\ e^{-\gamma z} = -\gamma H_Y$

$\frac{\partial H_y}{\partial z} = -\gamma H_Y$

Similarly

$\frac{\partial H_x}{\partial z} = -\gamma H_x$, $\frac{\partial E_y}{\partial z} = -\gamma E_Y$, $\frac{\partial E_x}{\partial z} = -\gamma E_x$

$\frac{\partial H_Z}{\partial y} + \gamma H_Y = j\omega\varepsilon E_X$ ---------------------------- 3

$\gamma H_x + \frac{\partial H_Z}{\partial x} = -j\omega\varepsilon E_y$----------------------------- 4

$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon E_z$ ----------------------------- 5

$\frac{\partial E_Z}{\partial y} + \gamma E_Y = -j\omega\mu H_X$ ---------------------------- 6

$\gamma E_x + \frac{\partial E_Z}{\partial x} = j\omega\mu H_Y$----------------------------- 7

$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_Z$ ----------------------------- 8

To find E_x and H_Y

$\frac{\partial H_Z}{\partial y} + \gamma H_Y = j\omega\varepsilon E_X$ ---------------------------- 3

$\gamma E_Y + \frac{\partial E_Z}{\partial x} = j\omega\mu H_Y$----------------------------- 7

$E_X = \frac{1}{j\omega\varepsilon}(\frac{\partial H_Z}{\partial y} + \gamma H_Y)$ -------------------------- 9

Sub 9 in 7

$$\frac{\gamma}{j\omega\varepsilon}(\frac{\partial H_Z}{\partial y} + \gamma H_Y) + \frac{\partial E_Z}{\partial x} = j\omega\mu H_Y$$

$$\frac{\gamma}{j\omega\varepsilon}\frac{\partial H_Z}{\partial y} + \frac{\gamma^2}{j\omega\varepsilon}H_Y + \frac{\partial E_Z}{\partial x} = j\omega\mu H_Y$$

$$\frac{\gamma}{j\omega\varepsilon}\frac{\partial H_Z}{\partial y} + \frac{\partial E_Z}{\partial x} = H_Y\left[j\omega\mu - \frac{\gamma^2}{j\omega\varepsilon}\right]$$

$$\frac{\gamma}{j\omega\varepsilon}\frac{\partial H_Z}{\partial y} + \frac{\partial E_Z}{\partial x} = H_Y\left[\frac{-(\omega^2\mu\varepsilon + \gamma^2)}{j\omega\varepsilon}\right]$$

$H_Y = \frac{-j\omega\varepsilon}{h^2}\left(\frac{\gamma}{j\omega\varepsilon}\frac{\partial H_Z}{\partial y} + \frac{\partial E_Z}{\partial x}\right)$

$$\boldsymbol{H_Y} = \left(-\frac{\boldsymbol{\gamma}}{\boldsymbol{h^2}}\frac{\boldsymbol{\partial H_Z}}{\boldsymbol{\partial y}} - \frac{\boldsymbol{j\omega\varepsilon}}{\boldsymbol{h^2}}\frac{\boldsymbol{\partial E_Z}}{\boldsymbol{\partial x}}\right) \text{----------------------- A}$$

Sub Ain 9

$E_X = \frac{1}{j\omega\varepsilon}(\frac{\partial H_Z}{\partial y} + \gamma\left(-\frac{\boldsymbol{\gamma}}{\boldsymbol{h^2}}\frac{\boldsymbol{\partial H_Z}}{\boldsymbol{\partial y}} - \frac{\boldsymbol{j\omega\varepsilon}}{\boldsymbol{h^2}}\frac{\boldsymbol{\partial E_Z}}{\boldsymbol{\partial x}}\right))$

$E_X = \frac{1}{j\omega\varepsilon}\frac{\partial H_Z}{\partial y} - \frac{\gamma^2}{\boldsymbol{h^2}(j\omega\varepsilon)}\frac{\boldsymbol{\partial H_Z}}{\boldsymbol{\partial y}} - \frac{\boldsymbol{\gamma}}{\boldsymbol{h^2}}\frac{\boldsymbol{\partial E_Z}}{\boldsymbol{\partial x}}$

$E_X = (\frac{\boldsymbol{h^2} - \gamma^2}{\boldsymbol{h^2}(j\omega\varepsilon)})\ \frac{\boldsymbol{\partial H_Z}}{\boldsymbol{\partial y}} - \frac{\boldsymbol{\gamma}}{\boldsymbol{h^2}}\frac{\boldsymbol{\partial E_Z}}{\boldsymbol{\partial x}}$

$E_X = (\frac{\omega^2\mu\varepsilon}{\boldsymbol{h^2}(j\omega\varepsilon)})\ \frac{\boldsymbol{\partial H_Z}}{\boldsymbol{\partial y}} - \frac{\boldsymbol{\gamma}}{\boldsymbol{h^2}}\frac{\boldsymbol{\partial E_Z}}{\boldsymbol{\partial x}}$

$$\mathbf{E_X} = \frac{\mathbf{-j\omega\mu}}{\mathbf{h^2}}\frac{\mathbf{\partial H_Z}}{\mathbf{\partial y}} - \frac{\boldsymbol{\gamma}}{\mathbf{h^2}}\frac{\mathbf{\partial E_Z}}{\mathbf{\partial x}} \ \textbf{------------------------------------ B}$$

To find $\mathbf{E_Y}$ **and** $\mathbf{H_X}$

$\gamma H_x + \frac{\partial H_Z}{\partial x} = -j\omega\varepsilon E_y$----------------------------- 4

$\frac{\partial E_Z}{\partial y} + \gamma E_Y = -j\omega\mu H_X$ ---------------------------- 6

$E_y = \frac{-1}{j\omega\varepsilon}(H_x + \frac{\partial H_Z}{\partial x})$ ---------------------------- 10

Sub 10 in 6

$$\frac{\partial E_Z}{\partial y} + \gamma\left(\frac{-1}{j\omega\varepsilon}\ (\gamma H_x + \frac{\partial H_Z}{\partial x})\right) = -j\omega\mu H_X$$

$$\frac{\partial E_Z}{\partial y} - \frac{\gamma^2}{j\omega\varepsilon}H_x - \frac{\gamma}{j\omega\varepsilon}\frac{\partial H_Z}{\partial x} = -j\omega\mu H_X$$

$$\frac{\partial E_Z}{\partial y} - \frac{\gamma}{j\omega\varepsilon}\frac{\partial H_Z}{\partial x} = \left(\frac{\gamma^2}{j\omega\varepsilon} - j\omega\mu\right)H_X$$

$$\left(\frac{(\omega^2\mu\varepsilon+\gamma^2)}{j\omega\varepsilon}\right)H_X = \frac{\partial E_Z}{\partial y} - \frac{\gamma}{j\omega\varepsilon}\frac{\partial H_Z}{\partial x}$$

$$H_X = \frac{j\omega\varepsilon}{h^2}\left(\frac{\partial E_Z}{\partial y} - \frac{\gamma}{j\omega\varepsilon}\frac{\partial H_Z}{\partial x}\right)$$

$$\boldsymbol{H_X = \frac{j\omega\varepsilon}{h^2}\frac{\partial E_Z}{\partial y} - \frac{\gamma}{h^2}\frac{\partial H_Z}{\partial x}} \text{---------------------- } \mathbf{C}$$

Substitute C in 10

$$E_y = \frac{-1}{j\omega\varepsilon}\left(\gamma\left(\frac{j\omega\varepsilon}{h^2}\frac{\partial E_Z}{\partial y} - \frac{\gamma}{h^2}\frac{\partial H_Z}{\partial x}\right) + \frac{\partial H_Z}{\partial x}\right)$$

$$E_y = -\frac{\gamma}{h^2}\frac{\partial E_Z}{\partial y} + \frac{\gamma^2}{h^2(j\omega\varepsilon)}\frac{\partial H_Z}{\partial x} - \frac{1}{j\omega\varepsilon}\frac{\partial H_Z}{\partial x}$$

$$E_y = -\frac{\gamma}{h^2}\frac{\partial E_Z}{\partial y} + \left[\frac{\gamma^2}{h^2(j\omega\varepsilon)} - \frac{1}{j\omega\varepsilon}\right]\frac{\partial H_Z}{\partial x}$$

$$E_y = -\frac{\gamma}{h^2}\frac{\partial E_Z}{\partial y} + \left[\frac{\gamma^2 - h^2}{h^2(j\omega\varepsilon)}\right]\frac{\partial H_Z}{\partial x}$$

$$E_y = -\frac{\gamma}{h^2}\frac{\partial E_Z}{\partial y} + \left[\frac{\omega^2\mu\varepsilon}{h^2(j\omega\varepsilon)}\right]\frac{\partial H_Z}{\partial x}$$

$$\boldsymbol{E_y = -\frac{\gamma}{h^2}\frac{\partial E_Z}{\partial y} - \frac{j\omega\mu}{h^2}\frac{\partial H_Z}{\partial x}}$$

Rectangular cavity resonator:

The rectangular waveguides are constructed from closed sections of the waveguide, as the waveguide is the type of the transmission line. Usually the rectangular waveguide are short circuited at both the ends to avoid the radiation losses from open end of the waveguide. Due to short circuited ends of the waveguide, a cavity or closed box is formed. Within this cavity, both the energies, electric and magnetic are stored. The power dissipation is observed at the metallic conducting walls of the waveguides as well as in the dielectric inside the cavity. Through a small aperture or a small probe or a loop such resonators are coupled.

The geometry of the rectangular cavity resonator is as shown in figure

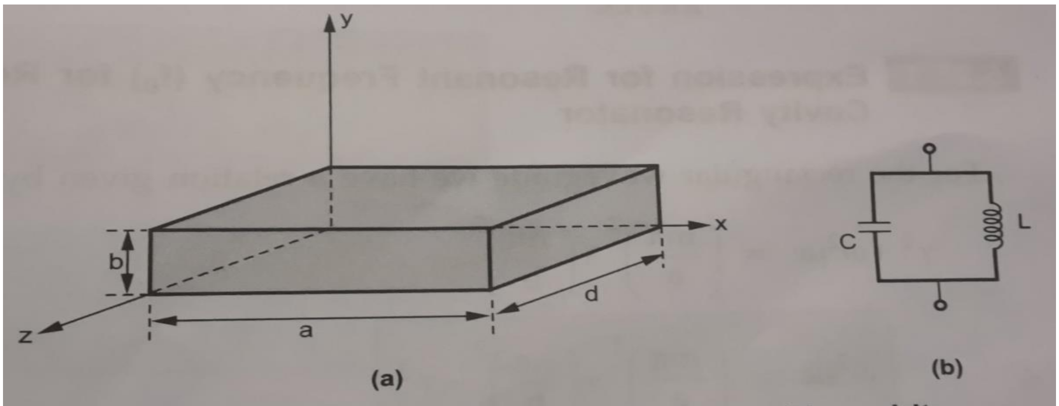


Fig. 5.25.1 Geometry of the rectangular waveguide and its equivalent tank circuit

Consider a rectangular waveguide cavity shorted at both the ends as shown in the figure.

The guide wavelength is given by,

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2}} \text{ --------------------------- } 1$$

The most dominating mode in the rectangular waveguide is TE_{10} mode. Basically for the dominant mode, the resonant frequency of the field configuration is lowest.

For mode,λ_c = 2a. Hence guide wavelength is given by,

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}} \text{ ---------------------------------- } 2$$

From equation 2 it is clear that, dimension a is fixed for the resonator. So also the guide wavelength λ_g is fixed.

But in general the frequency is given by,

F = $\frac{c}{\lambda_0}$ = f_0 ----------------------------- 3

As λ_0 is fixed and cis velocity of light which is also constant, for the given mode the frequency has fixed value denoted by f_0. thus the rectangular resonant cavity supports only one frequency for a given mode. This frequency is called resonant frequency and thus the cavity formed is called resonant cavity. This cavity resonator behaves similar to parallel LC resonant circuit commonly called tank circuits.

The parallel resonant circuit or equivalent tank circuit is as shown in the figure. The resonant frequency for such equivalent parallel circuit is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Expression for resonant frequency for rectangular cavity resonator:

For the rectangular waveguide we have a relation given by,

$\gamma^2 + \omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

$\omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \gamma^2$ ------------------------ 4

But for a condition of wave propagation, we can write,

$$\gamma = j\beta$$

Hence equation 4 can be written as,

$\omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - (j\beta)^2$

$\omega^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \beta^2$ --------------------------------- 5

But the condition for the cavity resonator is given as the cavity must be an integer multiple of a half guide wavelength long at the resonant frequency.
Hence we can write,
$\beta = \frac{p\pi}{d}$Where p = 1, 2, 3 ……………..
Hence p is known as number of half wavelength variations of either electric or magnetic fields along z direction.
Thus depending on the value of p, the general wave mode through the cavity resonators are denoted by TE_{mnp} for the transverse electric (TE) wave and TM_{mnp} for the transverse magnetic (TM) wave.
To have a resonator resonating at a fixed frequency,$\omega_0 = f_0$, substituting value of β from equation 6 in equation 5, we can write,

$\omega_0^2\mu\varepsilon = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2$

$\omega_0^2 = \frac{1}{\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]$

$\omega_0 = \sqrt{\frac{1}{\mu\varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]}$rad/s

But $\omega_0 = 2\pi f_0$

$f_0 = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]}$ Hz ----------------------------------- 7

We can modify this expression by taking $(\pi)^2$ out of the radical term asπ, we can write,

$f_0 = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2\right]}$ Hz -------------------------------------- 8

If the resonator cavity is filled with an air, then we can write,

$\frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{1}{c}$ where c = 3* 10^8 m/s = velocity of light

Thus for free space within the cavity, the frequency of resonance is given by,

$f_0 = \frac{c}{2}\sqrt{\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2\right]}$ Hz --------------------------------------- 9

Equations 8 and 9 indicate the resonant frequency of a rectangular cavity resonator with dimensions a, b, and d for both TE_{mnp} and TM_{mnp} modes in it.

Circular cavity resonator:

The rectangular cavity resonator is constructed from the rectangular waveguide shorted at both the ends. Similarly circular cavity resonator can be constructed from circular wave guide cutting into a section and shorting both the ends of it. The circular cavity resonators are mainly used in microwave frequency meters. The mechanical tuning of the resonant frequency is done with the help of movable top wall. The cavity is coupled to the waveguide through a small aperture. The dominant mode of circular mode is TE_{11}. The circular cavity resonator modes are specified as TE_{mnp} for the transverse electric wave and TM_{mnp} modes for the transverse magnetic wave.

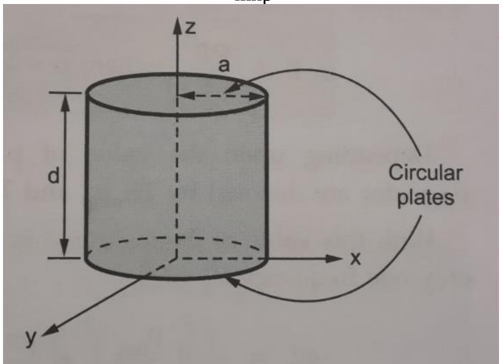


Fig. 5.26.1 Geometry of circular cavity resonator

Consider a circular cavity resonator constructed from the circular waveguide with uniform circular cross section with radius a. the geometry of the circular cavity resonator is shown in figure.

Note that both the ends of the section of circular cavity resonator of length d are shorted with the help of circular shorting plates.

Expression for resonant frequency (f_0) circular cavity resonator:

For a circular waveguide, we have already derived the expression given by

$\gamma^2 + \omega^2\mu\varepsilon = \left(\frac{P_{nm}}{a}\right)^2$ ------------------------- 1

Where P_{nm} is the Eigen value and a is the radius of the circular cylinder. But for the wave propagation, the condition can be written as,

$\gamma = +j\beta$ --------------------------------------- 2

Substituting value of γ in equation 1, we get,

$$-\beta^2 + \omega^2\mu\varepsilon = \left(\frac{P_{nm}}{a}\right)^2$$

$$\omega^2\mu\varepsilon = \left(\frac{P_{nm}}{a}\right)^2 + \beta^2$$

$$\omega^2 = \frac{1}{\mu\varepsilon}\left[\left(\frac{P_{nm}}{a}\right)^2 + \beta^2\right] \text{------------------ 3}$$

But the condition for the circular cavity resonator remains same as the condition in rectangular cavity resonator which is given by,

$\beta = \frac{p\pi}{d}$ Where p = 1, 2, 3 …………….. -------------------- 4

Depending upon the value of p, the general modes through the circular cavity resonator are denoted by TE_{mnp} and TM_{mnp}.

With this value of β substituted in the expression for ω, for the cavity resonator supports only one frequency ω_0 or f_0

$$\omega_0{}^2 = \frac{1}{\mu\varepsilon}\left[\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]$$

$$\omega_0 = \sqrt{\frac{1}{\mu\varepsilon}\left[\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]} \text{ rad/sec ------------------------ 5}$$

For TM_{mnp} wave :

$$f_0 = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left[\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]} \text{ Hz -------------------- 6}$$

For a free space as a dielectric within the circular cavity, we can write,

$\frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ = c, where c = 3* 10^8 m/s = velocity of light

Hence for a free space, the expression for the resonant frequency of circular cavity resonator can be modified as,

$$f_0 = \frac{c}{2\pi}\sqrt{\left[\left(\frac{P_{nm}}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]} \text{ Hz ---------------------------- 7}$$

Expression for the resonant frequency given by the equations 6, 7 is for TM_{mnp} mode. For the TE_{mnp} mode, the expression for f_0 are given as follows.

For TE_{mnp} wave:

$$f_0 = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left[\left(\frac{P_{nm}'}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]} \text{ Hz --------------------------- 8}$$

For free space within the circular cavity, the expression for the resonant frequency for TE_{mnp} is given by,

$$f_0 = \frac{c}{2\pi}\sqrt{\left[\left(\frac{P_{nm}'}{a}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]} \text{ Hz ------------------------------ 9}$$

PART A

1. **What is dominant mode? What is dominant TE and TM mode in rectangular waveguide? A/M 2018, N/D 2017,N/D 2016, M/J 2016, A/M 2015**

   The dominant mode of wave is defined as the mode in which the wave has lowest cutoff frequency.

   For parallel plate waveguides -TE1or TM1

   For Rectangular waveguides- TE01, TM11

   For Circular waveguides- TE11, TM01

2. **What are the application of cavity resonator? Mention the application of resonant cavities. A/M 2018, A/M 2017, M/J 2016, N/D 2015, N/D 2014, M/J 2013, N/D 2013**

   (i)Cavity resonators are tunable circuits used in microwave oscillators, amplifiers, wave meters and filters.

   (ii) They are widely used in light house tube, which is used for VHF range of frequencies.

   (iii) It is used in duplexers in the RADAR system.

3. **Write the expression for the cutoff wavelength of the wave which is propagated in between two parallel planes. N/D 2017**

   Guide wave length

$$\lambda_g = \frac{\lambda}{\sqrt{1-(\frac{fc}{f}^2)}}$$

   Where f_c is the cut off frequency

4. **A wave is propagated in the dominant mode in a parallel plane waveguide. The frequency is 6GHz and the plane separation is 4cm .Calculate the cut off wavelength in the waveguide. A/M 2017**

$$\lambda c = \frac{2a}{m} = \frac{2\times 0.04}{1} = 0.08m$$

5. **Give the equations for the propagation constant and wavelength for TEM waves between parallel planes. A/M 2017**

$$\gamma = \sqrt{(\frac{m\pi}{a}^2 - \omega^2\mu_0\varepsilon_0}$$

$$\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu_0\varepsilon_0}}$$

6. A rectangular wave guide with a 5cm×2cm cross is used to propagate TM_{11} mode at 10GHz.Determine the cut off wave length. A/M 2017, N/D 2015, N/D 2014

a=5cm,b=2cm

$TM_{11mode,m=1,n=1}$

$$\lambda c = \frac{2}{\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}} = 3.714cm$$

7. Calculate the cut off frequency of a rectangular waveguide whose inner dimensions are a=2.5 cm and b=1.5cm operating at TE_{10} mode. A/M 2017

A=2.5cm=2.5$\times 10^{-2}m$

B=1.5cm=1.5$\times 10^{-2}m$

TE_{10} mode : $f_c = \frac{c}{2a} = \frac{3\times10^8}{2\times2.5\times10^{-2}} = 6 \times 10^9 Hz$

f_c=6GHz

8. Enumerate the parameters describing the performance of a cavity resonator. A/M 2017

Parameters describing the performance of a cavity resonator are,

- Field in the direction of propagation
- Field expression
- Wave impedance
- Resonant frequency
- Cut off wave length
- Phase velocity and group velocity

9. What is the need for attenuator? N/D 2016

An attenuator is a device that reduces the amplitude or power of a signal without distorting its waveform .In transmission equipment, it is required to suppress or reduces the level of current and voltage at certain points.

10. How to design an air filled cubical cavity to have its dominant resonant frequency at 3GHz? N/D 2016

$$F_0 = c/2\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2 + (\frac{p}{d})^2}$$

$$= \frac{3\times10^8}{2}\sqrt{(\frac{1}{a})^2 + (\frac{0}{a})^2 + (\frac{1}{a})^2}$$

a=0.0707m

11. How a cavity resonator is formed? M/J 2016

When one end of the waveguide is terminated in a shorting plate, there will be complete reflection of waves. When one more shorting plate is kept at a distance of multiple of $\lambda_s/2$ from first shorting plate resonant cavities are formed

12. .Justify, why TM_{01} and TM_{10} modes in rectangular waveguide do not exit. N/D 2016

It has no axial component of either E or H. so it cannot propagate within a single conductor waveguide.

13. An air filled rectangular waveguide of inner dimensions 2.286×1.016 in centimeters operates in the dominant TE_{10} modes. Calculate the cut off frequency and phase velocity of a wave in the guide at a frequency of 7GHz. N/D 2016

$$F_c=\frac{C}{2}\sqrt{(\frac{m}{a})^2+(\frac{n}{b})^2}$$

$$=\frac{3\times10^8}{2}\sqrt{1913.58}=65.6\times10^8$$

Phase velocity $V_{ph}=\frac{V}{\sqrt{1-\frac{fc^2}{f}}}=\frac{3\times10^8}{\sqrt{1-(\frac{6.56\times10^9}{7\times10^9}^2)}}=8.62\times10^8 m/s$

14. Define the term phase velocity and group velocity. A/M 2015

(i)Free Space Velocity-It is the velocity of propagation of an EM wave in free space. Vo = C =3× 10^8 m/sec.

(ii)Phase Velocity Vp -The phase Velocity is defined as the rate at which wave changes its phase as the wave propagates inside the region between parallel planes.

(iii)Group Velocity Vg - It is defined as the actual velocity with which the wave propagates inside the region between two parallel planes.

15. What are the characteristics of TEM wave? A/M 2015, M/J 2013

1) The fields are entirely transverse.

2) Along the direction normal to the direction of propagation, the amplitude of the field components are constant.

3) Velocity of TEM wave is independent of frequency.

4) The cutoff frequency of the wave is Zero

16. A rectangular wave guide has the following dimensions l=2.54cm,b=1.27cm and thickness=0.127cm.Calculate the cut off frequency for TE_{11}mode. A/M 2015

L=a=2.54cm

B=1.27cm

M=1,n=1

$$f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{(\frac{m\pi^2}{a}) + (\frac{n\pi^2}{b})}$$

$1.5\times 10^8 \times 88.03$

$f_c = 13.20GHz$

17.Why TEM mode is not supported by waveguide? N/D 2014

It has no axial component of either E or H. so it cannot propagate within a single conductor waveguide.

18.State the significance of dominant mode of propagation. N/D 2014

Based on the values of m and n, there can be infinite mode existing in the waveguide. So the input energy to guide the waveguide is shared by all there mode. But this leads to losses as the energy is diverted during propagation. So to avoid loss of energy because of divergence the dominant mode is propagated through the waveguide

19.What is degenerate mode in rectangular waveguide? M/J 2013

Some of the higher order modes, having the same cut off frequency, are called degenerate modes. In rectangular waveguide, TE_{mn} and TM_{mn} modes (both

m$\neq$ 0 and n $\neq$ 0) are always degenerate

20.Write Bessel's function of first kind of order zero M/J 2013

$$J_0(\rho)=c\sum_{r=0}^{\infty}(-1)^r \frac{(\frac{1}{2}\rho)2r}{r!^2}$$

20.A wave is propagated in a parallel plane waveguide with the frequency is 6GHz and the plane separation is 3cm.Determine the group and phase velocities for the dominant mode. N/D 2013

a=3cm

f=6GHz

m=1

Cutoff frequency fc=$\frac{m}{2a\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}}$

fc=$\frac{1}{2\times3\times10^{-2}\sqrt{(4\pi\times10^{-7}\times1)(8.854\times10^{-12}\times1)}}$ = 4.996GHz

Phase velocity $v_p = \frac{V}{\sqrt{1-(\frac{fc^2}{f})}} = \frac{3\times10^8}{\sqrt{1-(\frac{4.996\times10^9}{6\times10^9})^2}}$

$=5.4179\times 10^8$m/sec

20.Define TEM waves. N/D 2013

When the components of the electric and magnetic fields in the wave, both are transverse to the direction of propagation of wave is called TEM wave or Principal waves.

21.A rectangular wave guide with a=7cm and b=3.5 cm is used to propagate TM_{10} at 3.5 GHz. Determine the guided wavelength. N/D 2013

a=7cm

f=3.5 GHz

$=\lambda_c=2a=2\times 7\times 10^{-2} = 0.14m$

$\lambda = c/f = \frac{3\times10^8}{3.5\times10^9} = 0.0857m$

Guide wavelength $\lambda g = \frac{\lambda}{\sqrt{1-(\frac{\lambda}{\lambda c})^2}}$ =0.108m

22.Compare TE and TM mode N/D 2012

TE Mode	TM Mode
TE wave has magnetic field component in the direction of propagation.	TM wave has electric field component in the direction of propagation.
The waves are called M waves or H waves	The waves are called E waves