UNIT III

IMPEDANCE MATCHING IN HIGH FREQUENCY LINES

Impedance matching: Quarter wave transformer - Impedance matching by stubs - Single stub and double stub matching - Smith chart - Solutions of problems using Smith chart - Single and double stub matching using Smith chart.

One eighth wave line, quarter wave line, and half wave line.

For the transmission line the voltage and current at any point distant x from the receiving end is

$$V = \frac{V_r(Z_r+Z_0)}{2Z_r}[e^{\gamma x}+Ke^{-\gamma x}]$$

$$I = \frac{I_r(Z_r+Z_0)}{2Z_r}[e^{\gamma x}-Ke^{-\gamma x}]$$

The term $e^{\gamma x}$ indicates incident wave = source -> load

The term $e^{-\gamma x}$ indicates incident wave = load -> source

For the line of zero dissipation, α=0, $\gamma = j\beta, Z_0 = R_0$

$$V = V_r \cos\beta x + j\, I_r R_0 \sin\beta x$$

$$I = I_r \cos\beta x + j\,\frac{V_r}{R_0}\sin\beta x$$

$$Z_s = \frac{V}{I} = \frac{V_r \cos\beta x + j\, I_r R_0 \sin\beta x}{I_r \cos\beta x + j\,\frac{V_r}{R_0}\sin\beta x}$$

$$= \frac{Z_r \cos\beta x + j\, R_0 \sin\beta x}{\cos\beta x + j\,\frac{Z_r}{R_0}\sin\beta x}, \; V_r = I_r Z_r$$

$$= R_0\left(\frac{Z_r \cos\beta x + j\, R_0 \sin\beta x}{R_0 \cos\beta x + j\, Z_r \sin\beta x}\right)$$

Divide by $\cos\beta x$

$$Z_s = R_0\left(\frac{Z_r + j\, R_0 \tan\beta x}{R_0 + j\, Z_r \tan\beta x}\right)$$

i) one eighth wave line:

For one eighth wave line $x = \frac{\lambda}{8}$

$\beta x = \frac{2\pi}{\lambda} * \frac{\lambda}{8} = \frac{\pi}{4}$

$Z_s = R_0 \left(\frac{Z_r + \mathrm{j}\, R_0 \tan\frac{\pi}{4}}{R_0 + \mathrm{j}\, Z_r \tan\frac{\pi}{4}}\right)$

$$\tan\frac{\pi}{4} = 1$$

$Z_s = R_0 \left(\frac{Z_r + \mathrm{j}\, R_0}{R_0 + \mathrm{j}\, Z_r}\right)$

If such a line is terminated with pure resistanceR_R (i.e.) $Z_R = R_R$

$Z_s = R_0 \left(\frac{R_R + \mathrm{j}\, R_0}{R_0 + \mathrm{j}\, R_R}\right)$

Take magnitude on both sides

$|Z_s| = R_0 \frac{\sqrt{R_R^2 + R_0^2}}{\sqrt{R_0^2 + R_R^2}}$

Therefore $|Z_s| = R_0$

Thus an eighth wave line may be used to transfer any resistance to impedance with magnitude equal toR_0.

It is used to obtain a magnitude match between a resistance of any value &R_0, the internal resistance of the source.**Quarter wave line:**

For quarter wave line x = $\frac{\lambda}{4}$

$\beta x = \frac{2\pi}{\lambda} * \frac{\lambda}{4} = \frac{\pi}{2}$

$Z_s = R_0 \left(\frac{Z_r + \mathrm{j}\, R_0 \tan\frac{\pi}{2}}{R_0 + \mathrm{j}\, Z_r \tan\frac{\pi}{2}}\right)$

$Z_s = R_0 \left(\frac{\frac{Z_r}{\tan\frac{\pi}{2}} + \mathrm{j}\, R_0}{\frac{R_0}{\tan\frac{\pi}{2}} + \mathrm{j}\, Z_r}\right)$

$$\tan\frac{\pi}{2} = \infty$$

$Z_s = R_0 \left(\frac{\mathrm{j}\, R_0}{\mathrm{j}\, Z_r}\right) = \frac{{R_0}^2}{Z_r}$

A quarter wave line may be considered as a transformer to match a load of $Z_r$to a source ofZ_s .

A quarter wave transformer may also be used if the load is not a pure resistance.

A quarter wave transformer is also a single frequency or narrow band device.

It is also used as an impedance inverter.

Important application of the quarter wave section is to couple a transmission line to a resistive load such as an antenna.

ii) Half wave line:

$$\beta x = \frac{2\pi}{\lambda} * \frac{\lambda}{2} = \pi$$

$$Z_s = R_0 \left(\frac{Z_r + \text{j}\, R_0 \tan \pi}{R_0 + \text{j}\, Z_r \tan \pi} \right)$$

$$Z_s = R_0 \left(\frac{Z_r}{R_0} \right)$$

$$Z_s = Z_r$$

A half wavelength of line is considered as one to one transformer.

It is used in connecting a load to a source when the load and source cannot be made adjacent.

Single stub matching and double stub matching.

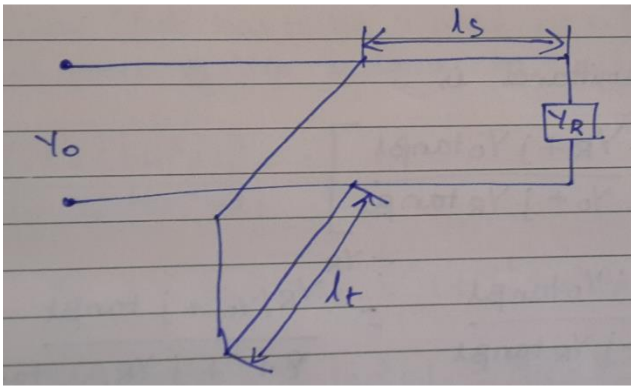
If the load impedance is not equal to the complex conjugate of the input impedance, the maximum power transfer will not take place. This is known as mismatching.

The impedance matching is accomplished by using an open or short circuited line of suitable length called stub at a designated distance from the load. This is called stub matching. There are two types of stub matching.

They are i) single stub matching

ii) Double stub matching.

i) Single stub matching:



- A transmission line with a characteristic admittance Y_0 terminated with load conductance Y_R is considered.
- $Y_R = Y_S$, standing waves are setup in between source and load.
- The input admittance is changing from maximum to minimum conductance, minimum to maximum conductance for every $\frac{\lambda}{2}$.
- When the line is transverse along the conductance there will be a p[point at which real part of admittance is equal to the characteristic admittance($R = Z_S$)
- An appropriate length of a short circuited / open circuited line with the transmission line is shunted at this point to achieve the impedance matching.

ii) Double stub matching:

The limitations of the smith chart are

i) Single stub impedance matching requires the stub to be located at a definite point on the line. This requirement frequently calls for placement of the stub at a desirable place from a mechanical view point.

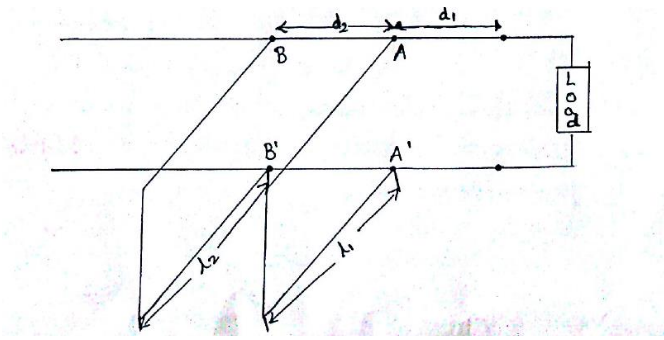
ii) For a coaxial line, it is not possible to determine the location of a voltage minimum without a slotted line section, so that placement of a stub at the exact required point is difficult.

iii) In the case of the single stub it was mentioned that two adjustments were required, these being location and length of the stub.

Double stub matching is preferred over single stub due to these limitations.

Definition:

In this method of impedance matching two stubs are used in which the locations of the stub are arbitrary, the two stub lengths furnishing the required adjustments. The spacing is frequently made.



In this method two different short circuited stubs are used. Let stub 1 of length l_1 located at point AA' at a distance d_1 from load. Let stub 2 of length l_2 be located are point BB' at a distance d_2 from stub 1.

In single stub matching the stub is connected in such a way to make the normalized admittance, $Y_S/G_0 = 1$(i.e.) $Y_S = G_0$ (i.e. susceptance value is zero).Hence line behaves as a smooth line.

But in general, the line admittance is $Y_S = G_S + jB_S$

When the stub is connected, the line admi9ttance becomes

$Y_S = G_S + jB_S'$

Since the stub has admittance, $Y_S = 0 \pm jB$

Y_S becomes $Y_S = G_S + jB_S'$ where $B_S' = B_S + B$

The length of the stub is adjusted in such a way to cancel the susceptance (i.e) $B_S' = 0$, so that $Y_S = G_S$

Similarly in double stub matching, the admittance of line at point BB'must be equal to G_0. The point BB' must be located such that the normalized admittance at that point becomes

$Y_B' = Y_S/G_0 = 1 \pm jB_2$.

The length should be adjusted such that its conductance is equal to 1 and susceptance = $\pm jB_2$

The distance between two stubs, must be greater than $\frac{\lambda}{2}$. since the impedance repeats after every $\frac{\lambda}{2}$ distance. $\frac{\lambda}{4}$ or $\frac{3\lambda}{8}$ separation between two stubs is preferred. $\frac{\lambda}{2} = 360$, $\lambda = 720$, so, $\frac{\lambda}{4} = \frac{720}{4} = 180$, $\frac{3\lambda}{8} = 270$

There is a chance of having reflections after the point BB_1. To avoid this, studs must be located very close to the load. Distance of 0.1λ to 0.15λ between load and the first stub is maintained to avoid reflctions. Spacing between stubs equal to $\frac{\lambda}{4}$ and $\frac{3\lambda}{8}$.

Determine the following a) SWR, b) load admittance c) impedance of the transmission line at the maximum and minimum of the stationary waves along the line d) distance between the load and first voltage maximum for a transmission line with characteristic impedance of 50 Ω with a receiving impedance of 100 + 121j . The wavelength of the electrical signal along the line is 2.5m and explains the steps followed for single stub matching (16)

GIVEN :

Characteristic impedance, Zo = 50 ohm

Load impedance, Zl = 100+j121 ohm

Wavelength, = 2.5m

Steps:

i) The normalized impedance is determined by dividing the load impedance by the characteristic impedance of the transmission line.

$z_l = \frac{Zl}{Zo} = \frac{100+j121}{50}$ = 2+ j 2.42 ohm

ii) The normalized impedance zl is plotted on the smith chart. The real part of zl is seen on the x axis. the imaginary part is seen on the inner circumference of the circle. If the imaginary part is +ve it is plotted on upper half of the circle or it is plotted on the lower half of the circle. The intersection of curves of real and imaginary parts gives the exact point zl .Name it as A.

iii) With '1' as centre, distance between 1 and zl as radius draw a circle. This circle is called impedance circle.

Draw a diameter through point A. the point at which it crosses the circle on the opposite side gives the normalized admittance, name it as point B.

Load admittance, $\frac{y}{G_0}$= y *Zo = (0.22-j0.25) 50 = 11 – j 12.5 mho

iv) Dark the circle which passes through '1' on the real axis.

The unit circle passes through impedance circle at two points. The point which lies on the upper half of the circle is denoted as centre C. the admittance value is 1 + j 1.7

v) Impedance at the first voltage maximum from load = 4.8 * Zo = 240 ohm

Impedance at the first voltage minimum = 0.2 * Zo = 10 ohm

vi) Distance between load and first voltage maximum = 0.042λ

= 0.042 * 2.5 = 0.105 m = 105 cm.

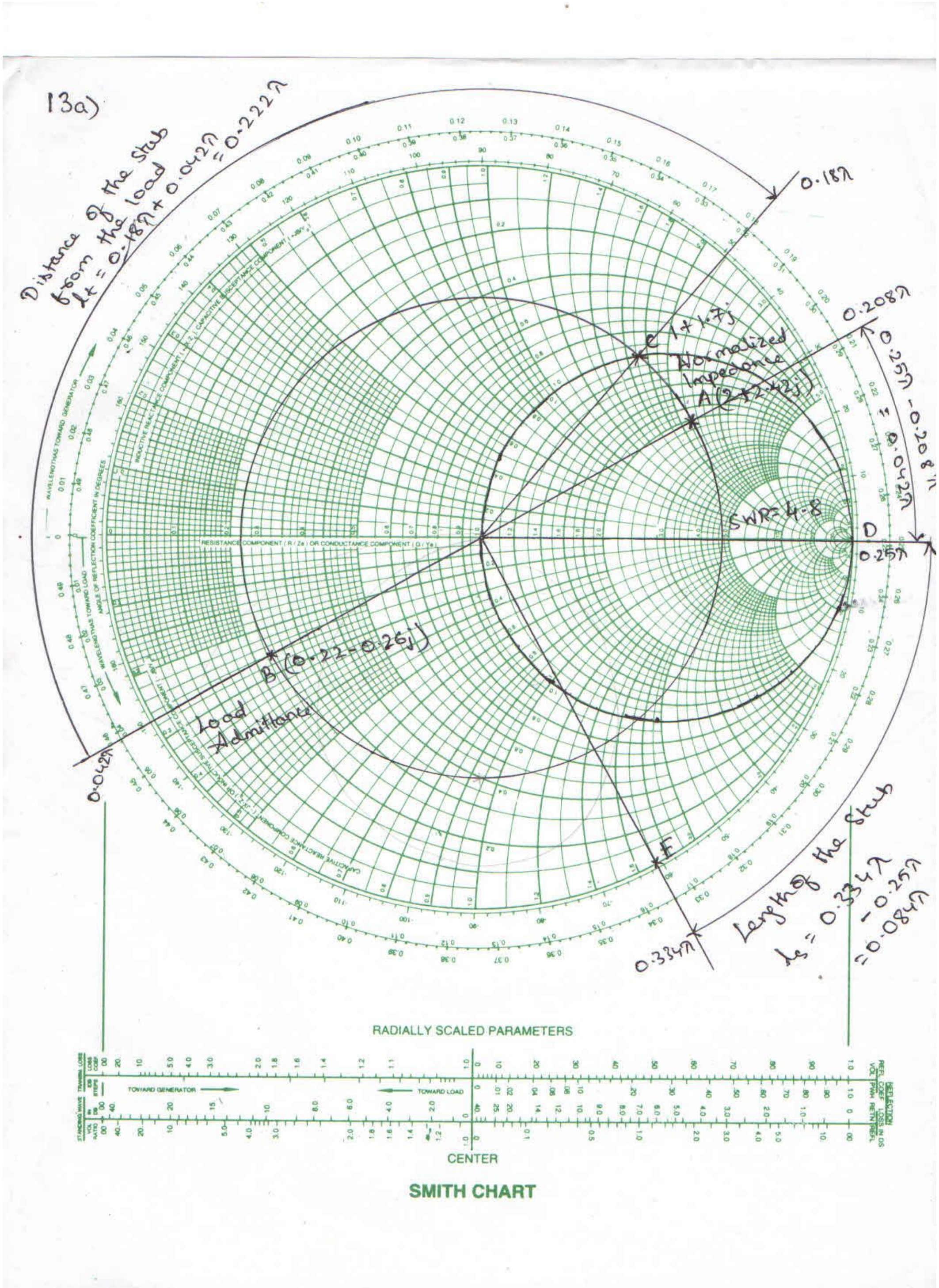
vii) The imaginary value of point C is the value for point E. it is plotted on the inner circumference of lower half of the circle.

viii) The distance between point B and C gives the distance of the stub from the load.

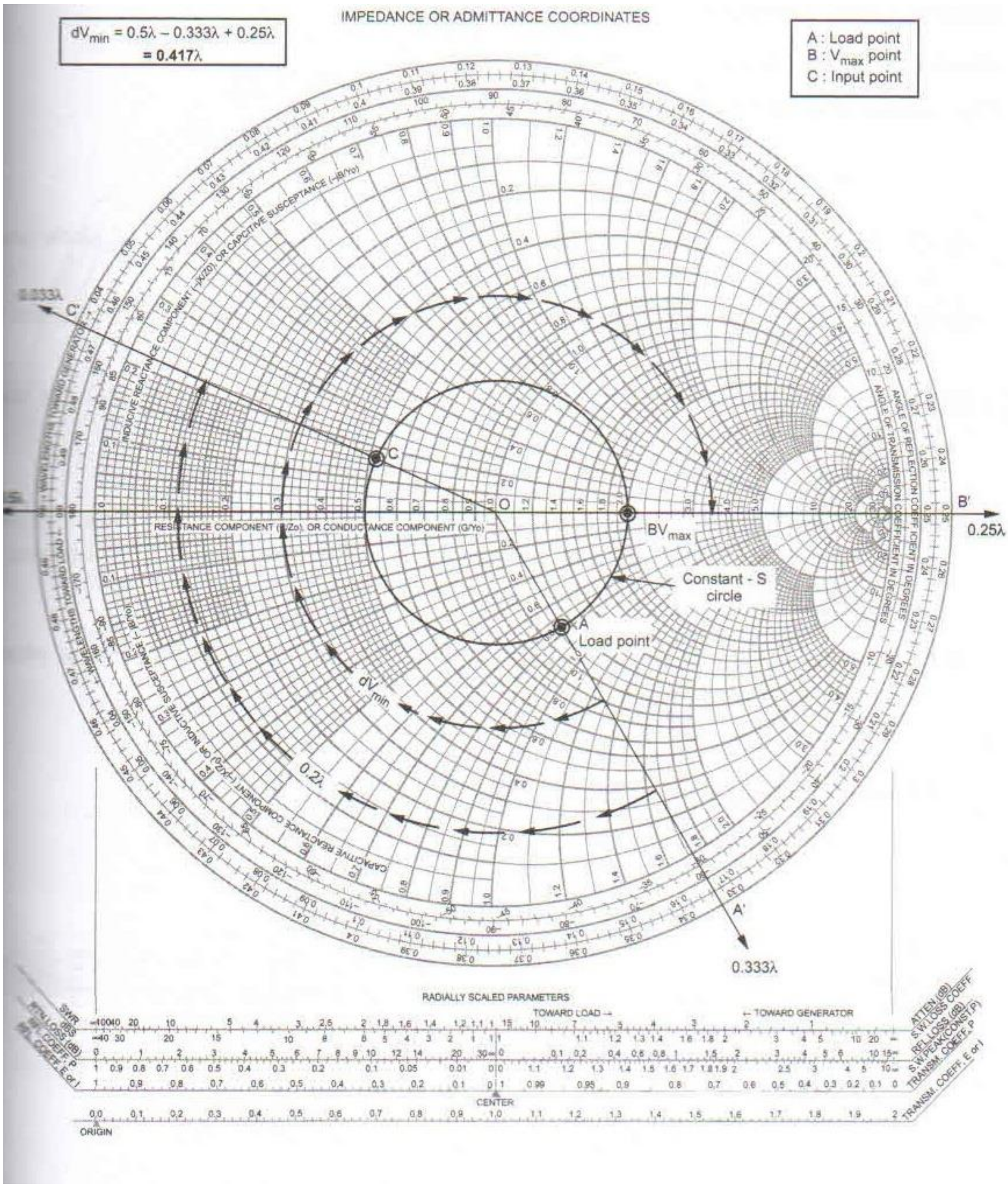
$L_t = 0.222 * 2.5m = 0.555$ m

ix) The distance between point D and E gives the length of the stub,

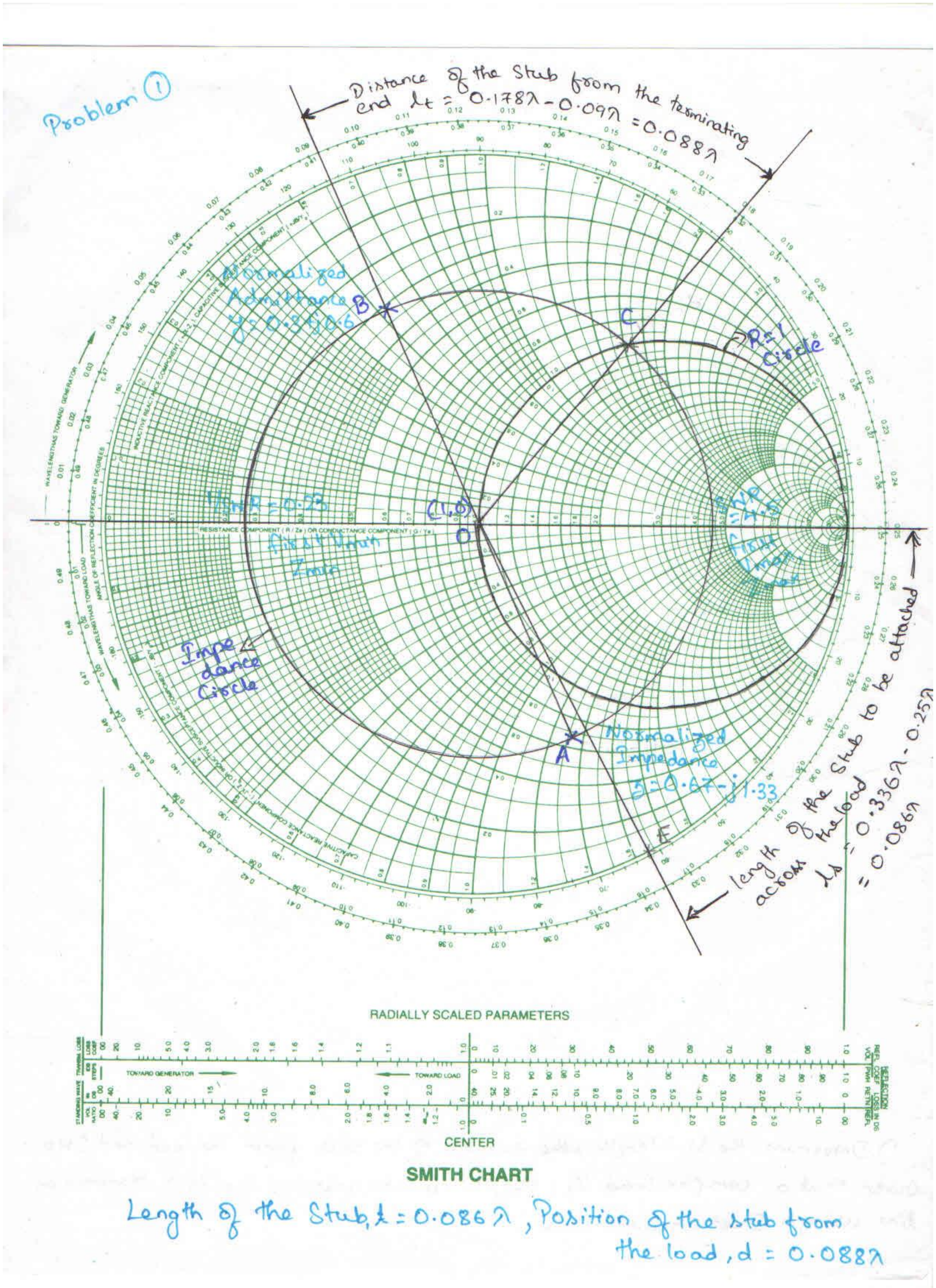
$L_s = 0.084\lambda = 0.084 * 2.5$ m $= 0.21$m


13a)
Distance of the stub from the load
dt = 0.18λ + 0.042λ = 0.222λ
0.18λ
0.208λ
C 1+1.7j
Normalized Impedance
0.25λ - 0.208λ = 0.042λ
SWR=4.8
D
0.25λ
B (0.22-0.26j)
Load Admittance
0.042λ
E
0.334λ
Length of the stub
ds = 0.334λ - 0.25λ = 0.084λ
RADIALLY SCALED PARAMETERS
TOWARD GENERATOR
TOWARD LOAD
CENTER
SMITH CHART

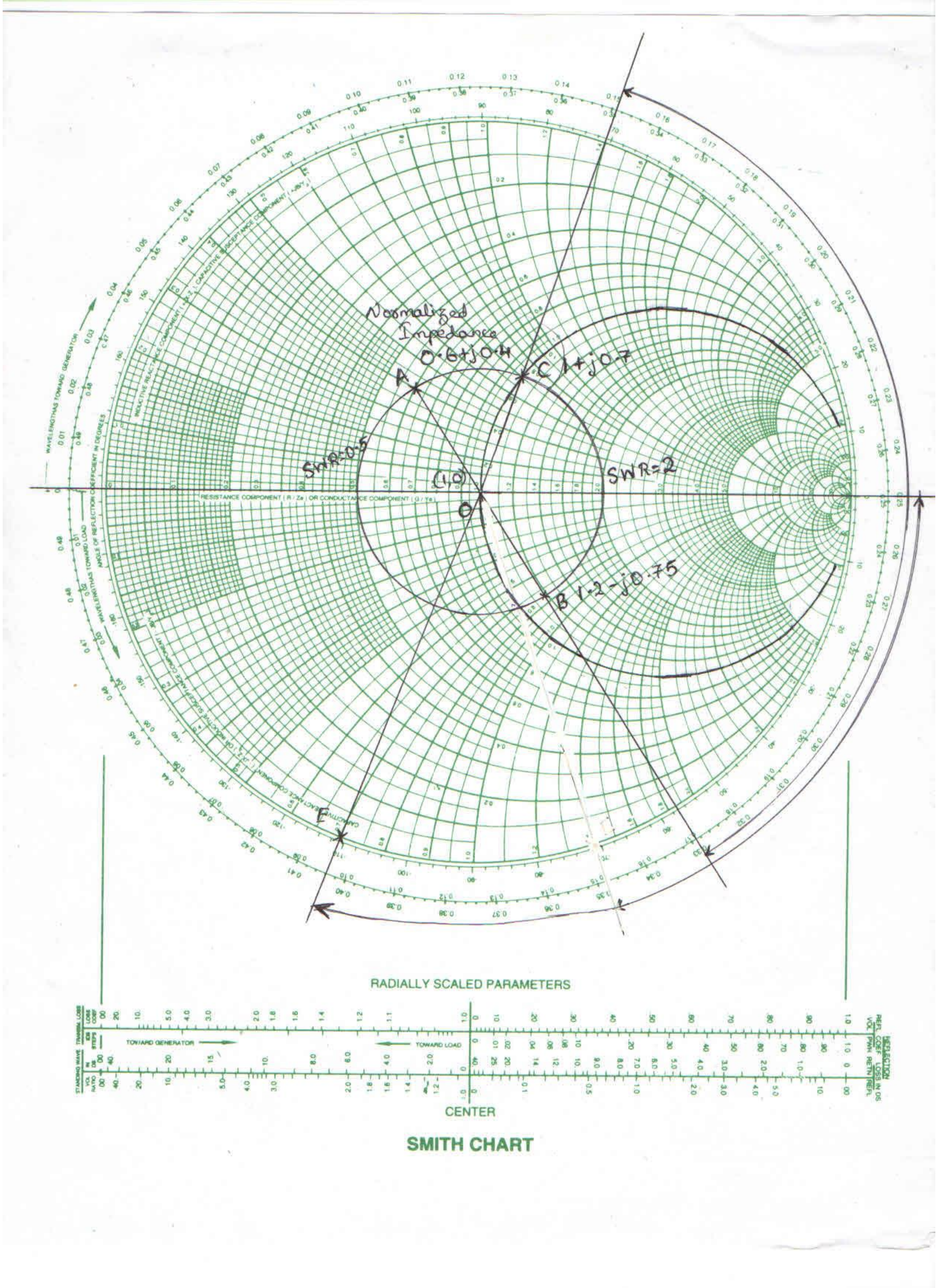
A load of Z_r = 115 – j75 Ω is used to terminate a lossless line of 100Ω. Determine standing wave ratio and distance from load to the first voltage maximum. Also determine input impedance if the line is 0.2 λ long.



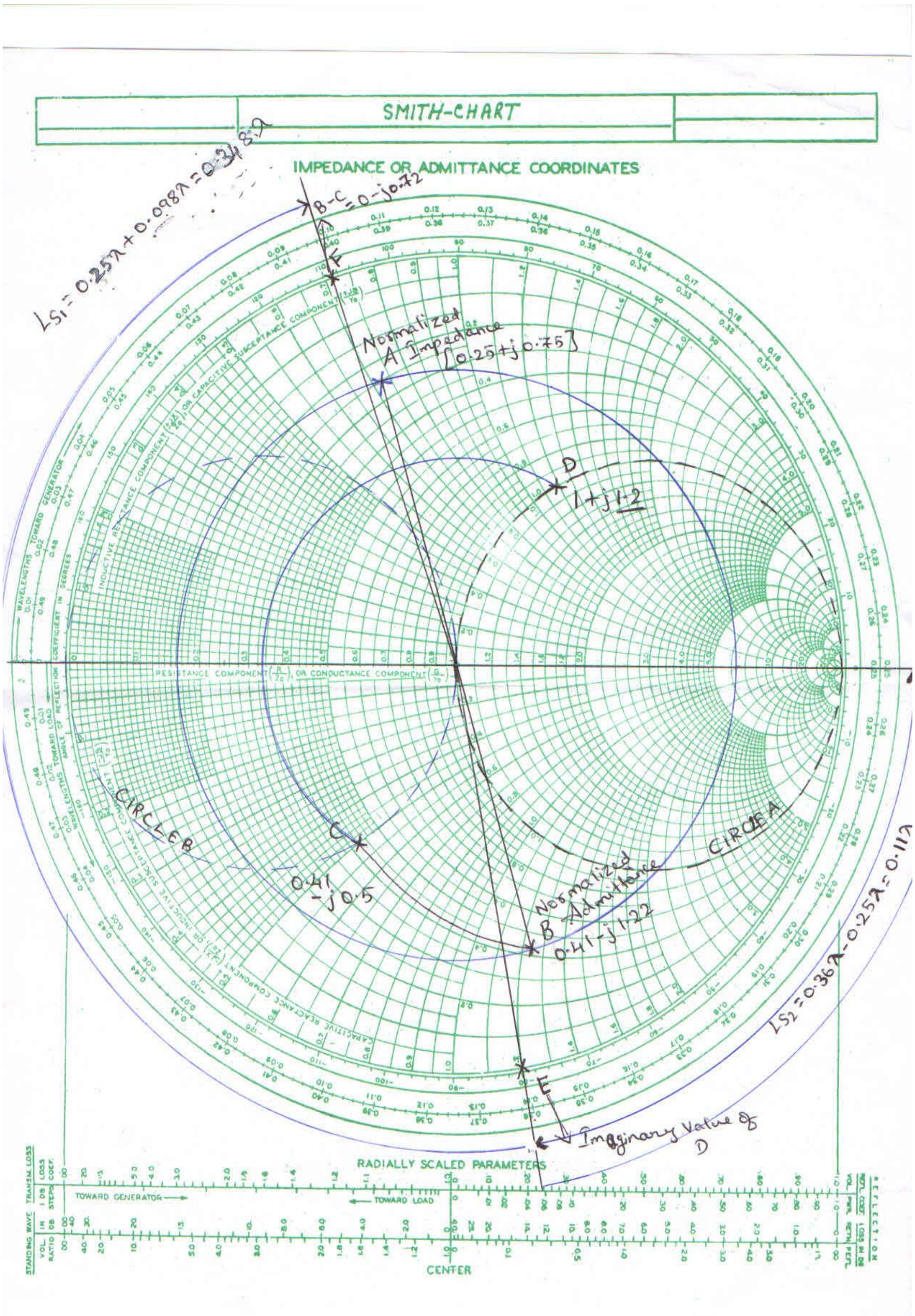
Determine the following a) SWR, b) load admittance c) impedance of the transmission line at the maximum and minimum of the stationary waves along the line d) distance between the load and first voltage maximum for a transmission line with characteristic impedance of 75 Ω with a receiving impedance of 50- 100j . explains the steps followed for single stub matching.



Determine the following a) SWR, b) load admittance c) impedance of the transmission line at the maximum and minimum of the stationary waves along the line d) distance between the load and first voltage maximum for a transmission line with characteristic impedance of 50 Ω with a receiving impedance of 30+ 20j . explains the steps followed for single stub matching.



Using double stub matching , match a complex load of Zl = 18.75 + j 56.25 to a line with characteristic impedance Zo = 75 ohm.Determine the stub length, assuming a quarter wavelength spacing is maintained between the two short circuited stubs.



PART A

1. **What is an impedance matching in stub? N/D 2017,A/M 2018**

In the method of impedance matching using stub, an open or closed stub line of suitable length is used as a reactance shunted across the transmission line at a designed distance from the load, to tune the length of the line and the load to resonance with an anti-resonant resistance equal to R_0

2. **What are the uses of smith chart? A/M 2018,A/M 2015**

The uses of the smith chart are,

(i) Determination of SWR, sending end impedance and load admittance.

(ii) The solution of the stub matching problem may be easily carried out using smith chart.

3. **Why quarter wave line is considered as an impedance inverter? Justify N?D 2017, M/J 2016**

   A quarter wave lines may be considered as an impendence inverter because it can transform a low impendence in to a high impendence and vice versa.

4. **Why a short circuited stub is ordinarily preferred to an open circuited stub? A/M 2017**

A short circuited stub is preferred to open circuited stub because of the
Following reason:

i. Easy in constructions

ii. Lower loss of energy due to radiation

iii. Effectively stopping all field propagation

5.List the applications of a quarter wave line. A/M 2017 N/D 2015 A/M 2017

(i)A quarter wave line may be used as a transformer for impedance matching of load Zr with input impedance Zin= Zr

(ii)A quarter wave lines can transform low impedance into high impedance and vice versa, thus it can be considered as an impedance inverter.

(iii)Couple a transmission line to a resistive load such as antenna

(iv)Serve as an insulator to support an open wire line.

6. Express standing wave ratio in ratio in terms of reflection coefficient. A/M 2017

The relationship between standing wave ratio and reflection coefficient is given by

$$S=\frac{1+|k|}{1-|k|}$$

7.Distinguish between single stub and double stub. N/D 2016, N/D 2015

Single stub	Double stub
(i) It has one stub to match the transmission line impedance	It requires two stub for impedance matching
(ii) It necessities both length and location of stub to be altered for matching	It requires only to alter the length of stubs for matching
(iii) It requires stub should be placed on a definite place on a line	The location of the stub is arbitrary

8.What is stub? Why it is used in between transmission lines? M/J 2016

For the maximum transfer of power , the sending end impedance and the receiving end impedance of a transmission line should be matched perfect .In practical cases, this impedance matching is not perfect, So a stub is placed on the transmission line and its length position is adjusted for maximum power transfer**.**

9.Give the application of eight wave line. N/D 2016

The eighth wave line is generally used to transform any resistance to an impedance having its magnitude equal to the characteristic resistance of the line.

10.How will you make standing wave measurements on coaxial line? A/M 2015

For coaxial lines it is necessary to use a length of line in which a longitudinal slot, one half wavelength or more long has been cut. A wire probe is inserted into the air dielectric of the line as a pickup device, a vacuum tube voltmeter or other detector being connected between probe and sheath as an indicator. If the meter provides linear indications, S is readily determined. If the indicator is non linear, corrections must be applied to the readings obtained.

11.Define SWR. **M/J 2013**

Standing wave ratio is the ratio of the maximum to minimum magnitude of voltages or currents over a line.

$$S = \left|\frac{Emax}{Emin}\right| = \left|\frac{Imax}{Imin}\right|$$

12.Design a quarter wave transformers to match a load of 200ohm to a source resistance 500 ohm. The operating frequency is 200MHz. M/J 2013

$$\boldsymbol{\lambda = \frac{c}{f} = \frac{3\times10^8}{200\times10^6}}$$

$$\lambda = 1.5m$$

Length of quater wave line $\frac{\lambda}{4} = \frac{1.5}{4} = 37.5\text{cm}$

Characteristic impedance $Z_o=\sqrt{Z_s Z_R} = \sqrt{200 \times 500}$

$Z_o == 316.22\Omega$

13. Write the expression for VSWR in terms of (a) the reflection coefficient (b) VSWR in terms of Z_L and Z_o. N/D 2012

(a) $S=\frac{1+|k|}{1-|k|}$

(b) $S=Z_L/Z_0$

14.Mention the significance of λ/2 line. N/D 2012

A half wavelength line may be considered as a one- to – one transformer. It is used to connect load to a source in cases where the load source cannot be made adjacent

15.List the advantages of smith chart.

The advantages of the smith chart are,

i) It is used to find the input impedance and input admittance of the line.

ii) The position of voltage minima and maxima can be easily found out.

iii) The position and location of stub in stub matching problem can be solved using smith chart.

16. What are the difficulties in single stub matching?

The difficulties of the single stub matching are

iv) Single stub impedance matching requires the stub to be located at a definite point on the line.

v) For a coaxial line, it is not possible to determine the location of a voltage minimum without a slotted line section, so that placement of a stub at the exact required point is difficult.

vi) In the case of the single stub it was mentioned that two adjustments were required, these being location and length of the stub.

17.What is double stub matching?

Another possible method of impedance matching is to use two stubs in which the locations of the stub are arbitrary. The spacing is frequently made. This is called double stub matching.

18.Give reason for an open line not frequently employed for impedance matching.

An open line is rarely used for impedance matching because of radiation losses from the open end and capacitance effects and the difficulty of a smooth adjustment of length.

19.Why double stub matching is preferred over single stub matching?

Double stub matching is preferred over single stub due to following disadvantages of single stub.

i) Single stub impedance matching requires the stub to be located at a definite point on the line.

ii) For a coaxial line, it is not possible to determine the location of a voltage minimum without a slotted line section, so that placement of a stub at the exact required point is difficult.

iii) In the case of the single stub it was mentioned that two adjustments were required, these being location and length of the stub.

20.Write down the expression to find the length of the stub?

$$L_t = \frac{\lambda}{2\pi} \tan^{-1} \frac{\sqrt{Z_R Z_O}}{Z_R - Z_O}$$

K is the reflection constant

$\lambda$is the wavelength

$Z_R$is the load impedance

$Z_O$is the characteristic impedance