UNIT II

HIGH FREQUENCY TRANSMISSION LINES

Transmission line equations at radio frequencies - Line of Zero dissipation - Voltage and current on the dissipation-less line, Standing Waves, Nodes, Standing Wave Ratio - Input impedance of the dissipation-less line - Open and short circuited lines - Power and impedance measurement on lines - Reflection losses - Measurement of VSWR and wavelength.

Parameters of open wire and coaxial lines at radio frequency:

Open wire:

At high frequencies the current is flowing on the surface of the conductor in a skin of very small depth. Since the internal flux and internal inductance are reduced to zero, the inductance of open wire line becomes.

$$L = \frac{\mu_0}{2\pi} \ln\left(\frac{d}{a}\right) \text{ H/m}$$

Where a is the radius of open wire line

d is the distance between the two open wire lines

$\mu_0 = 4\pi * 10^{-7}$ H/m

The value of capacitance of a open wire line is not affected by skin effect or high frequency.

$$C = \frac{\pi\varepsilon_0}{\ln\left(\frac{d}{a}\right)} \text{ F/m}$$

Where $\varepsilon_0 = 8.854 * 10^{-12}$ F/m

If the current flows at high frequency over the surface of the conductor in a thin layer, there is an increase in resistance of the conductor. The skin depth or the effective thickness of the surface layer of current is

$$\delta = \frac{1}{\sqrt{\pi f \mu_0}}$$

for direct current resistance of the open wire line is

$R_{dc} = \frac{K}{\pi a^2}$

For alternating current resistance of the open wire line is

$R_{ac} = \frac{K}{\pi a \delta}$

$R_{ac} = \frac{K\sqrt{\pi f \mu_0}}{\pi a}$

$$R_{ac} = \frac{K}{a}\sqrt{\frac{\mu\sigma}{\pi}}\sqrt{f} \text{ ohm/m}$$

This equation shows that the resistance increases with increasing frequency.

Coaxial cable:

The parameters of the coaxial line are also modified by the presence of high frequency currents on the line. Because of skin effect, the current flows only on the surface of the conductor and it eliminates the flux linkages.

The inductance of the coaxial cable is

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) \text{ H/m}$$

Where a is the outer radius of the inner conductor.

b is the inner radius of the outer conductor.

The capacitance of the coaxial cable is not affected by high frequency current. The value of capacitance of coaxial cable is

$$C = \frac{2\pi\varepsilon}{\ln\left(\frac{b}{a}\right)} \text{ F/m}$$

Due to skin effect, the resistant of the coaxial cable is given by

$R_{ac} = \frac{K}{\pi\delta}\ (\frac{1}{a} + \frac{1}{b})$

$R_{ac} = \frac{K\sqrt{\pi f \mu\sigma}}{\pi}\ (\frac{1}{a} + \frac{1}{b})$

$R_{ac} = K\sqrt{\frac{\mu\sigma}{\pi}}\sqrt{f}\left(\frac{1}{a} + \frac{1}{b}\right)$ ohms/m

Resistance increases with an increase of frequency.

Input impedance of open and short circuited lines.

Input impedance of transmission line:

$V = V_r \cos\beta x + jI_r Z_0 \sin\beta x$

$I = I_r \cos\beta x + j\frac{V_r}{Z_0}\sin\beta x$

$Z_S = \frac{V}{I} = \frac{(V_r \cos\beta x + jI_r Z_0 \sin\beta x)}{(I_r \cos\beta x + j\frac{V_r}{Z_0}\sin\beta x)}$

w.k.t $V_r = Z_r I_r$

$Z_S = \frac{(Z_r I_r \cos\beta x + jI_r R_0 \sin\beta x)}{(I_r \cos\beta x + j\frac{Z_r I_r}{R_0}\sin\beta x)}$

$Z_S = \frac{(Z_r \cos\beta x + jR_0 \sin\beta x)}{(\cos\beta x + j\frac{Z_r}{R_0}\sin\beta x)}$

$Z_S = \frac{R_0(Z_r \cos\beta x + jR_0 \sin\beta x)}{(R_0 \cos\beta x + jZ_r \sin\beta x)}$

Dividing cos βxin numerator and denominator

$Z_S = \frac{R_0(Z_r + jR_0 \tan\beta x)}{(R_0 + jZ_r \tan\beta x)}$

For short circuited line

$Z_R = 0$

$Z_S = \frac{R_0(Z_r + jR_0 \tan\beta x)}{(R_0)} => Z_S = jR_0 \tan\beta x$

$$\beta = \frac{2\pi}{\lambda}$$

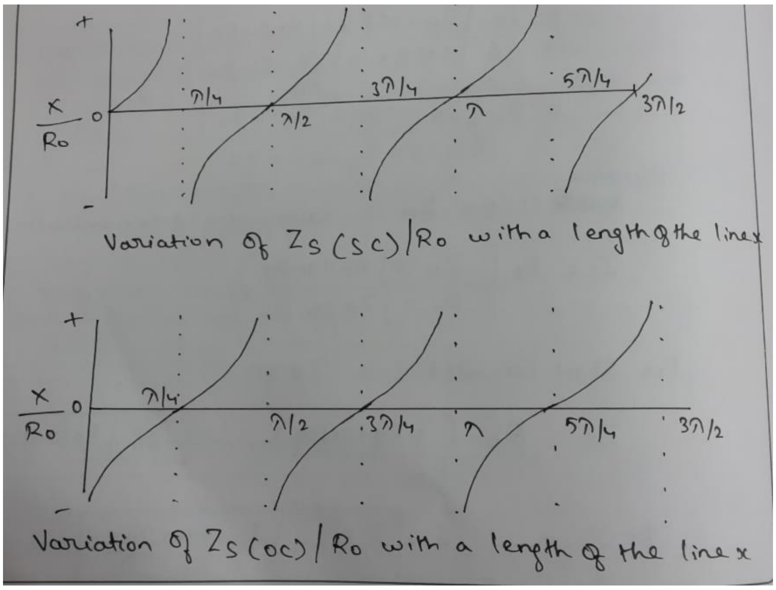
$Z_S = \; jR_0 \tan\frac{2\pi}{\lambda}x$

The input impedance of the line can be written as

$Z_S = \dfrac{\boldsymbol{R_0}(1 \;\; +\frac{\mathbf{j}\boldsymbol{R_0}}{Z_r}\mathbf{tan}\boldsymbol{\beta x})}{(\frac{\boldsymbol{R_0}}{Z_r} + \mathbf{jtan}\boldsymbol{\beta x})}$ by dividing numerator and denominator by Z_r

For open circuited line $Z_r = \infty$

$Z_S = \dfrac{\boldsymbol{R_0}(1)}{(\mathbf{jtan}\boldsymbol{\beta x})}$ =>$\mathbf{Z_S} = \dfrac{\boldsymbol{-j\,R_0}}{(\;\mathbf{tan}\boldsymbol{\beta x})}$



Power and impedance measurement on lines.

The voltage and current on the dissipation less line are given by

$$V = \frac{V_r(Z_r+R_0)}{2Z_r}[e^{j\beta x}+Ke^{-j\beta x}]$$

$$V = \frac{I_r(Z_r+R_0)}{2}[e^{j\beta x}+Ke^{-j\beta x}]$$

$$I = \frac{I_r(Z_r+Z_0)}{2R_0}[e^{j\beta x}-(K)e^{-j\beta x}]$$

A maximum of voltage occurs at which the incident and reflected waves are in phase.

$$V_{max} = \frac{V_r(Z_r+Z_0)}{2Z_r}[1+|K|]$$

$$I_{max} = \frac{I_r(Z_r+Z_0)}{2R_0}[1+|K|]$$

$$\frac{V_{max}}{I_{max}} = R_0 = R_{max}$$

The current minimum and voltage maximum occur at the same point on the line and are in phase.

$$I_{min} = \frac{I_r(Z_r+Z_0)}{2R_0}[1-|K|]$$

$$\frac{V_{max}}{I_{min}} = R_0\frac{[1+|K|]}{[1-|K|]} = S\ R_0$$

Where $S = \frac{[1+|K|]}{[1-|K|]}$ = standing wave ratio

The voltage minimum and current maximum occur at the same point on the line and are in phase.

$$V_{min} = \frac{V_r(Z_r+Z_0)}{2Z_r}[1-|K|]$$

$$\frac{V_{min}}{I_{max}} = R_0\frac{[1-|K|]}{[1+|K|]} = \frac{R_0}{s}$$

Power flowing into resistance is given by

$P = \frac{{V_{max}}^2}{R_{max}}$

$P = \frac{{V_{min}}^2}{R_{min}}$

$$P^2 = \frac{V_{max}^{\ 2}}{R_{max}}\frac{V_{min}^{\ 2}}{R_{min}}$$

Since $R_{max} = R_{min} = R_0$

P = $\frac{|V_{max}||V_{min}|}{R_0}$

Power also can be expressed as

P = $|I_{max}||I_{min}| * R_0$

Using these expressions power flow can be measured.

The unknown value of the load impedance Z_R connected to a transmission line can be determined by standing wave measurements. Bridge circuit is used to measure the unknown impedance.

At the point of voltage minimum, distant x' from the load

Z_S = R_{min} = $\frac{R_0}{S}$

Input impedance at any point on line,

$$Z_S = \frac{\boldsymbol{R_0}(Z_r \ \ + \mathbf{j}\boldsymbol{R_0}\mathbf{tan}\boldsymbol{\beta x})}{(\boldsymbol{R_0} \ + \mathbf{j}Z_r\mathbf{tan}\boldsymbol{\beta x})}$$

Becomes,

$$\frac{R_0}{S} = \frac{\boldsymbol{R_0}(Z_r \ \ + \mathbf{j}\boldsymbol{R_0}\mathbf{tan}\boldsymbol{\beta x})}{(\boldsymbol{R_0} \ + \ \mathbf{j}Z_r\mathbf{tan}\boldsymbol{\beta x})}$$

From this equation, the load impedance Z_R is given by

$$\mathbf{Z_R} = \frac{\boldsymbol{R_0}(\mathbf{1} - \mathbf{j}\boldsymbol{S}\mathbf{tan}\boldsymbol{\beta x\prime})}{(\boldsymbol{S} - \mathbf{j}\ \mathbf{tan}\boldsymbol{\beta x\prime})}$$

voltages and currents on the dissipation less line

The equations for voltage at any point from the receiving end of the transmission line is given by

$$V = \frac{V_r(Z_r+Z_0)}{2Z_r}[e^{\gamma x}+\frac{Z_r-Z_0}{Z_r+Z_0}\,e^{-\gamma x}]$$

$$I = \frac{I_r(Z_r+Z_0)}{2Z_0}\left[e^{\gamma x} - \left(\frac{Z_r-Z_0}{Z_r+Z_0}\right)e^{-\gamma x}\right]$$

For zero dissipation line $Z_0 = R_0$, and $\gamma = j\beta$

$$V = \frac{V_r(Z_r+Z_0)}{2Z_r}\left[e^{j\beta x} + \frac{Z_r-Z_0}{Z_r+Z_0}e^{-j\beta x}\right]$$

$$I = \frac{I_r(Z_r+Z_0)}{2Z_0}\left[e^{j\beta x} - \left(\frac{Z_r-Z_0}{Z_r+Z_0}\right)e^{-j\beta x}\right]$$

$$V = \frac{V_r}{2Z_r}[(Z_r + R_0)e^{j\beta x} + (Z_r - R_0)e^{-j\beta x}]$$

$$V = \frac{V_r}{2Z_r}[Z_r\, e^{j\beta x} + R_0\, e^{j\beta x} + Z_r\, e^{-j\beta x} - R_0 e^{-j\beta x}]$$

$$V = \frac{V_r}{2Z_r}[Z_r(e^{j\beta x} + e^{-j\beta x}) + R_0(e^{j\beta x} - e^{-j\beta x})]$$

$$V = \frac{V_r}{Z_r}[Z_r cos\beta x + jR_0 sin\beta x]$$

$$\mathbf{V = [V_r cos\beta x + jI_r R_0 sin\beta x]}$$

Similarly from 1,

$$I = \frac{I_r}{2R_0}[(Z_r + R_0)\, e^{j\beta x} - (Z_r - R_0)e^{-j\beta x}]$$

$$I = \frac{I_r}{2R_0}[Z_r\, e^{j\beta x} + R_0\, e^{j\beta x} - Z_r\, e^{-j\beta x} + R_0 e^{-j\beta x}]$$

$$I = \frac{I_r}{2R_0}[Z_r(e^{j\beta x} - e^{-j\beta x}) + R_0(e^{j\beta x} + e^{-j\beta x})]$$

$$I = \frac{I_r}{2R_0}[Z_r(e^{j\beta x} - e^{-j\beta x}) + R_0(e^{j\beta x} + e^{-j\beta x})]$$

$$I = \frac{I_r}{2R_0}[R_0 cos\beta x + jZ_R sin\beta x]$$

$$\mathbf{I = [I_r cos\beta x + j(V_r/R_0) sin\beta x]}$$

If line is open circuited $I_r = 0$

$$V = [V_r cos\beta x]$$

$$I = [j(V_r/R_0\,) sin\beta x]$$

If line is short circuited $V_r = 0$

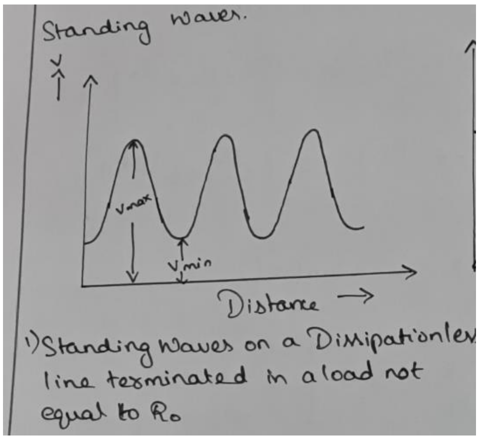
$$V = [jI_r R_0 sin\beta x]$$

$$I = [I_r cos\beta x]$$

Standing waves and derive standing wave ratio.

Standing waves:

When the transmission line is not matched to its load (i.e.) load impedance is not equal to the characteristic impedance $(Z_r \neq Z_0)$, the energy delivered to the load is reflected back to the source. The combination of incident and reflected waves gives rise to the standing wave.



The plot of current variation along the line is the same as that of voltage variation except for a $\frac{\lambda}{4}$ shift in the position of maxima and minima.

Nodes are the points of zero voltage or current and anti-nodes are the points of maximum voltage or current in the standing wave systems.

A line terminated in its characteristic impedance has no standing waves and thus no nodes. It is called smooth line.

For open circuit voltage node occurs at distances of $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$& so on from the open end.

For short circuit voltages node occurs at 0, $\frac{\lambda}{2}$,λ& so on

Whereas current nodes occur at $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$& so on.

The equations for voltage and current at any point on line from receiving end are given below.

$$V = \frac{V_r(Z_r+Z_0)}{2Z_r}[e^{\gamma x}+\frac{Z_r-Z_0}{Z_r+Z_0}\, e^{-\gamma x}]$$

$$I = \frac{I_r(Z_r+Z_0)}{2Z_0}\,[e^{\gamma x}-\left(\frac{Z_r-Z_0}{Z_r+ Z_0}\right) e^{-\gamma x}]$$

For zero dissipation line $\alpha = 0$, K = $\frac{Z_r-Z_0}{Z_r+ Z_0}$

$$V = \frac{V_r(Z_r+Z_0)}{2Z_r}[e^{j\beta x}+Ke^{-j\beta x}]$$

$$I = \frac{I_r(Z_r+Z_0)}{2Z_0}\,[e^{j\beta x}-(K)e^{-j\beta x}]$$

The term involving $e^{j\beta x}$ is the incident wave and $e^{-j\beta x}$ is the reflected wave.

Standing wave ratio:

The ratio of the maximum to minimum magnitudes of voltage or current on a line having standing waves is called standing wave ratio (SWR) or voltage standing wave ratio(VSWR).

S = $\left|\frac{V_{max}}{V_{min}}\right| = \left|\frac{I_{max}}{I_{min}}\right|$

$$V = \frac{V_r(Z_r+Z_0)}{2Z_r}[e^{j\beta x}+Ke^{-j\beta x}]$$

Maxima of voltage occurs at which the incident and reflected waves are in phase.

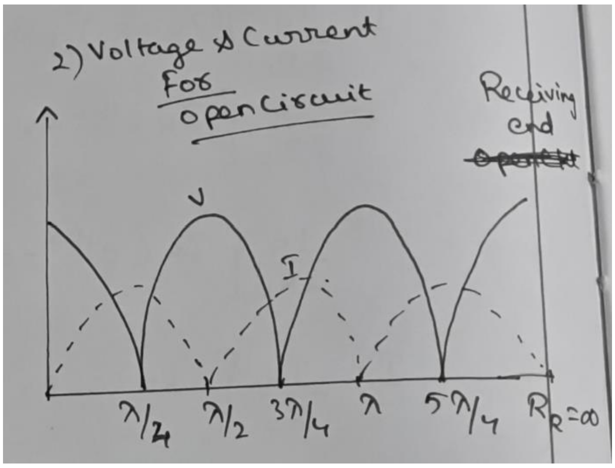
$$V_{max} = \frac{V_r(Z_r+Z_0)}{2Z_r}[1 + |K|]$$

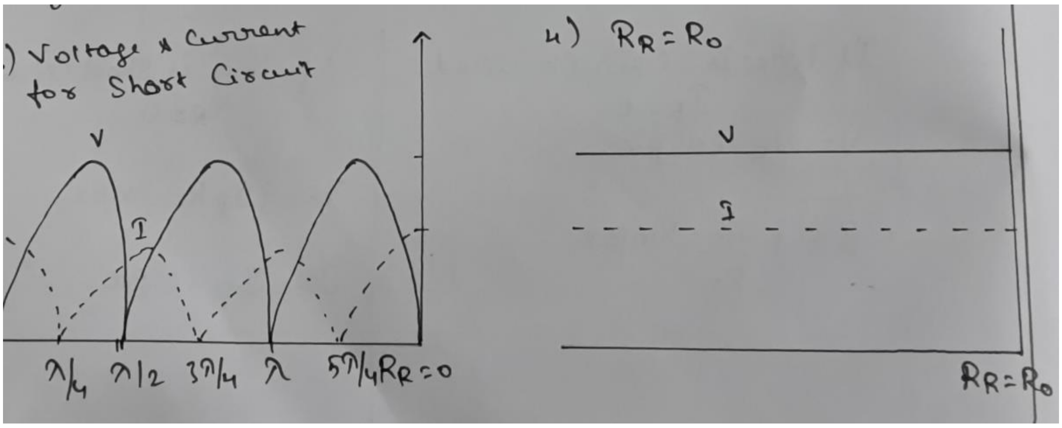
Minima of voltage occurs at which the incident and reflected waves are out of phase.

$$V_{min} = \frac{V_r(Z_r+Z_0)}{2Z_r}[1 - |K|]$$

$$\mathbf{S} = \left|\frac{\boldsymbol{V_{max}}}{\boldsymbol{V_{min}}}\right| = \frac{\mathbf{1} + |\boldsymbol{K}|}{\mathbf{1} - |\boldsymbol{K}|}$$

$$|\boldsymbol{K}| = \frac{|\boldsymbol{V_{max}}| - |\boldsymbol{V_{min}}|}{|\boldsymbol{V_{max}}| + |\boldsymbol{V_{min}}|}$$





PART A

1. What are the assumption to simply the analysis of the line performance at high frequencies?A/M 2018

What are the nature and value of Z_0 for the dissipation less line? N/D 2017

For the line of zero dissipation, what will be the values of attenuation constant and characteristic impedance? N/D 2015, M/J 2016

(i)For a line of zero dissipation R is very small and G is assumed to be zero.

(ii) Attenuation constant $\alpha = 0$

(iii)Phase constant $\beta = \omega\sqrt{Lc}$ rad/m

(iv)Characteristic impedance $Z_o=R_o=\sqrt{\frac{L}{C}}$

$Z_o=R_o$=wholly resistive

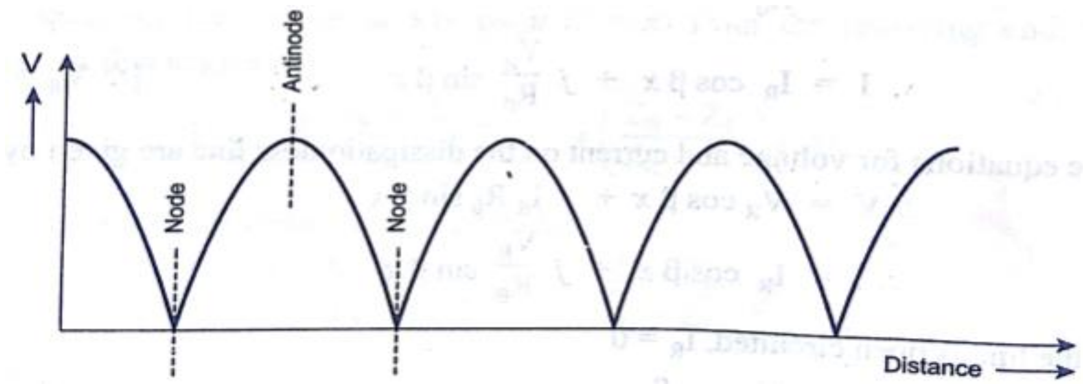
(v)Velocity of propagation $V=\frac{1}{\sqrt{LC}}$m/s

2. Write the expression for the input impedance open and short circuited dissipation less line. A/M 2018,N/D 2016

$$Z_{oc}=-jR_o\cot\beta s = -jRocot\left(\frac{2\pi}{\lambda}\right)s$$

$$Z_{sc}=jR_o\tan\beta s = jRotan\left(\frac{2\pi}{\lambda}\right)s$$

. **3. What are nodes and antinodes on a line? N/D 2017**



Nodes are the points of zero voltage or current and anti-nodes are the points of maximum voltage or current in the standing wave systems.

A line terminated in its characteristic impedance has no standing waves and thus no nodes. It is called smooth line.

For open circuit voltage node occurs at distances of $\frac{\lambda}{4},\frac{3\lambda}{4},\frac{5\lambda}{4}$& so on from the open end.

For short circuit voltages node occurs at 0, $\frac{\lambda}{2}$,λ& so on and current nodes occur at $\frac{\lambda}{4},\frac{3\lambda}{4},\frac{5\lambda}{4}$& so on.

4. Define standing wave ratio. A/M 2017

The ratio of maximum to minimum magnitudes of voltage or current on a line having standing waves is called the standing wave ratio (SWR)

$$S = \left|\frac{Emax}{Emin}\right| = \left|\frac{Imax}{Imin}\right|$$

5. A loss line has a characteristic impedance of 400Ω.Determine the standing wave ratio if the receiving end impedances is 800+j0.0Ω. A/M 2017

Z_o=400Ω

Z_R=800+j0.0Ω

$$K=\frac{Z_R-Z_o}{ZR+Zo}=\frac{800-400}{800+400}=\frac{400}{1200}=0.33$$

$$SWR=\frac{1+|K|}{1-|K|}=\frac{1+0.33}{1-0.33}=1.985$$

6. A transmission line has a characteristic impedance of 400ohm and is terminated by a load impedance of 650-j475 ohm. Determine the reflection co efficient. A/M 2017

Z_o=400Ω

Z_L=650-j475

$$K=\frac{Z_L-Z_O}{Z_L+Z_O}=\frac{650-j475-400}{650+j475+400}=\frac{250-j475}{1050+j475}$$

7.Calculate standing wave ratio and reflection coefficient on a line having the characteristic impedance Z_o=300Ω and terminating impedance in Z_R=300+j400Ω. N/D 2016

$$K=\frac{Z_R-Z_o}{ZR+Zo}=\frac{(300+j400)-300}{(300+j400)+300}=\frac{j400}{600+j400}=\frac{400\angle 90°}{721.11\angle 33.69°}$$

$K=0.5547\angle 56.31^{\circ}$

$$\mathbf{S=\frac{1+|k|}{1-|k|}=\frac{1+0.5547}{1-0.5547} = 3.4913}$$

8. Write the expression for SWR in terms of reflection co efficient. M/J 2016

$$\mathbf{S=\frac{1+|k|}{1-|k|}}$$

Where K=$\frac{Z_R - Z_o}{ZR + Zo}$

9.What is the drawback of using ordinary telephone cable? A/M 2015

In ordinary telephone cable, with increase in frequency, attenuation constant and velocity both are high. Thus the attenuation is very high. Also phase and frequency distortions are dominant.

10. A loss less transmission line has a shunt capacitance of 100pF/m and a series inductance of 4μH/m. Determine the characteristic impedance. N/D 2015

L=4μH/m=4×10^{-6}H/m

C=100pF/m=100×10^{-12}F/m

$$Zo = \sqrt{\frac{L}{C}} = \sqrt{\frac{4\times10^{-6}}{100\times10^{-12}}} = 200\Omega$$

11.List parameters of open wire line at high frequencies. N/D 2014

The line parameters of a transmission line are resistance, inductance, capacitance and conductance.

(i) Resistance (R) is defined as the loop resistance per unit length of the transmission line. Its unit is ohms/km.

(ii) Inductance (L) is defined as the loop inductance per unit length of the transmission line. Its unit is Henries/km.

(iii) Capacitance (C) is defined as the shunt capacitance per unit length between the two transmission lines. Its unit is Farad/km.

(iv) Conductance (G) is defined as the shunt conductance per unit length between the two transmission lines. Its unit is mhos/km

12.A line having characteristics impedance of 50Ω is terminated in load impedance (75+j75).determine the reflection coefficient. **N/D 2014**

$$K=\frac{Z_L - Z_O}{Z_L + Z_O} = \frac{(650 - j475) - 600}{(650 - j475) + 600}$$

$$\frac{50 - j475}{1250 + j475} = \frac{477.6243\angle -83.99°}{1337.2079\angle -20.8°} = 0.3572\angle - 63.19°$$

K=|K|∠Φ=0.3572∠-63.19°

13.**Write the conditions to be satisfied by dissipation less line. N/D 2013**

A line in which there is no phase or frequency distortion and also correctly terminated with the condition$\frac{R}{L} = \frac{G}{C}$**.** The line is said to be distortion less line

14.A 50Ω coaxial cable feeds a 75+j20Ω dipole antenna. Find reflection co efficient and standing wave ratio. N/D 2012

Terminating impedance Z_R=75+j20Ω

Characteristic impedance Z_o=50Ω

Reflection coefficient K=$\frac{Z_L-Z_O}{Z_L+Z_O}$

$=\frac{(75+j20)-50}{(75+j20)+50}$

K=$\frac{25+j20}{125+j20}=\frac{32.0156\angle-38.66°}{126.589\angle90.902°}$

=0.2529∠-52.24°

=|K|=0.2529

$\varphi=-52.24°$

S=$\frac{1+0.2529}{1-0.2529}=\frac{1.2529}{0.7471}=1.677$

S=1.677

15.At a frequency of 80MHz, a lossless transmission line has a characteristic impedance of 300Ω and a wavelength of 2.5 m. Find Land C. N/D 2012

f=80 MHz,Z_0=300Ω,λ=2.5m

For lossless transmission line $Z_0=R_0$=300Ω

λ=$\frac{2\pi}{\beta}=\frac{2\pi}{\omega\sqrt{LC}}=\frac{2\pi}{2\pi f\sqrt{LC}}=\frac{1}{f\sqrt{LC}}$

fλ=$\frac{1}{\sqrt{LC}}$

$R_o=\sqrt{\frac{L}{C}}$

fλ=$80\times10^6\times2.5=\frac{1}{\sqrt{LC}}$; 300=$\sqrt{\frac{L}{C}}$

$\frac{1}{\sqrt{LC}}=200\times10^6$…………… (1)

$\sqrt{\frac{L}{C}}=300$ ………………... (2)

(1)×(2)⟹ $\frac{1}{\sqrt{LC}}\times\sqrt{\frac{L}{C}}=\frac{1}{C}=200\times10^6\times300$

C=0.0166nF

(1)÷ (2) ⇒ L=1.5μH

16. Define skin effect.

Current is flowing essentially on the surface of the conductor in a skin of very small depth.

For coaxial cable at high frequencies, because of skin effect , the current flows on the outer surface of the inner conductor and the inner surface of the outer conductor.

17. If the reflection co-efficient of a line is 0.3<-66°. Calculate the standing wave ratio.

SWR = $\frac{1+k}{1-K} = \frac{1+0.3}{1-0.3}$ = 1.3/0.7 = 1.571

18. What are standing waves?

When a line is not terminated correctly into its characteristic impedance then the part of the energy transmitted returns back to the source as reflected wave. Then the distribution of voltage along the length of the line is not uniform but the minimum or maximum at different lengths. The points of minimum and maximum voltage or current are called nodes and anti-nodes respectively. A wave reflected back from the load consisting nodes and anti-nodes is called standing waves.

19. What are the primary and secondary constants of a transmission line?

The four line parameters resistance (R), inductance (L), capacitance (C), and conductance (G) are termed as primary constants of a transmission line.

Propagation constant (γ) and characteristic impedance (Z_0) are the secondary constants of a transmission line.

20. Draw the equivalent circuit of a transmission line.