JEPPIAAR INSTITUTE OF TECHNOLOGY

"Self-Belief | Self Discipline | Self Respect"

DEPARTMENT

OF

ELECTRONICS & COMMUNICATION ENGINEERING

LECTURE NOTES

EC8651-TRANSMISSION LINES AND RF SYSTEM

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UNIT I

TRANSMISSION LINE THEORY

General theory of Transmission lines - the transmission line - general solution - The infinite line - Wavelength, velocity of propagation - Waveform distortion - the distortion-less line - Loading and different methods of loading - Line not terminated in Z_0 - Reflection coefficient - calculation of current, voltage, power delivered and efficiency of transmission - Input and transfer impedance - Open and short circuited lines - reflection factor and reflection loss.

General theory of Transmission lines

The various types of the transmission lines are,

1. **Open wire line:**

These lines are the parallel conductors open to air hence called open wire line. The conductors are separated by air as the dielectric and mounted on the posts or the towers. The telephone lines and the electrical power transmission lines are the best example of the open wire lines. Advantages and disadvantages of open wire line the open wire line easy to construct. It is comparatively cheaper. Since insulation between the conductors is air, the dielectric loss is very small. This line is balanced to the earth. The main disadvantage of this line is that there is significant energy loss due to radiations. So it is unsuitable at higher frequencies. The open wire lines are requirement of telephone posts and towers hence high initial cost, affected by atmospheric conditions like wind, air ice etc., maintenance is difficult and possibility of shorting due to flying objects and birds. But less capacitance compared to underground cable is the advantages of open wire line.

2. Cables:

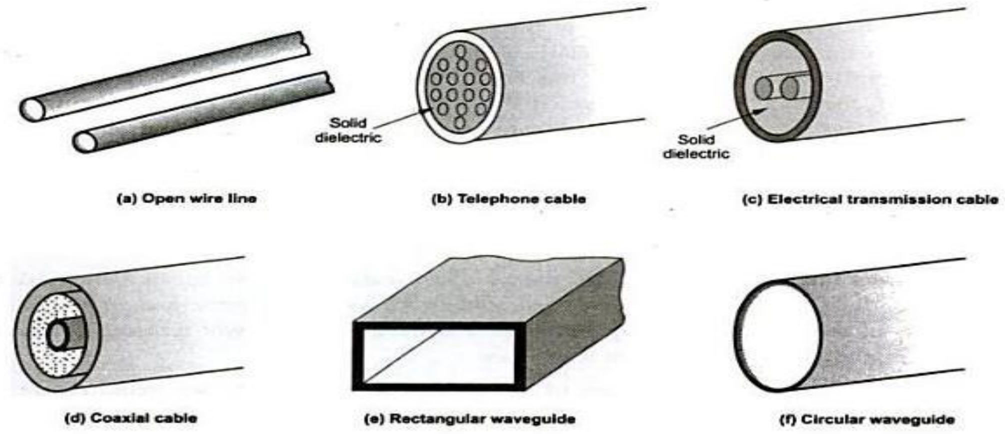
These are underground lines. The telephone cables consists of hundred of conductors which are individually insulated with paper. These are twisted in pairs and combined together and placed inside a protective lead or plastic sheath. While underground electrical transmission cables consists of two or three large conductors which are insulated with oil impregnated paper or other solid dielectric and placed inside protective lead sheath. Both these types are still considered as parallel conductors separated by a dielectric.

3. Coaxial line:

As name suggests there are two conductors which are co axially placed. One conductor is hollow and other is placed co axially inside the first conductor. The dielectric may be solid or gaseous. These lines are used for high voltage levels. Advantages and disadvantages of coaxial cable. The main advantages of the coaxial cable are that the electromagnetic fields cannot leak into the free space; hence radiation losses are totally absent. Outer conductor provides very effective shielding to the external electromagnetic fields. The coaxial cable transmission line is costlier. The losses in the dielectric increase as the frequency of the signal increase. Hence above 1 GHz this line cannot be used.

4. Waveguides:

These types of transmission lines are used to transmit to transmit the electrical waves at microwave frequencies. Constructionally these are the hollow conducting tubes having uniform cross section. The energy is transmitted from inner walls of the tube by the phenomenon of total internal reflection. Different types of transmission lines are shown in figure.



Types of the transmission line

Waveguides are practical only for signals of extremely high frequency, where the wavelength approaches the cross-sectional dimensions of the waveguide.

The electric line used for transmission of telephone messages or for the transmission of power is a common example of an electric circuit with **distributed parameters.**

Distributed parameters imply that the **resistance, inductance, and capacitance** are distributed along the circuit, each elemental length of the circuit having its own values, and concentration of the individual parameters is not possible.

How do we account for the effect of these distributed parameters?

(i) **Resistance** is uniformly distributed along the length of the conductors.

(ii) Since current will be present, the conductors will be surrounded and linked by magnetic flux, and this phenomena will demonstrate its effect in distributed **inductance** along the line.

(iii) The conductors are separated by insulating dielectric, so that **capacitance** will be distributed along the conductor length.

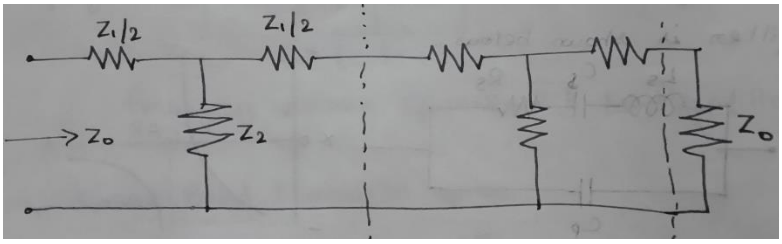
(iv) This dielectric of the open-wire line may not be perfect, and a leakage current will flow and leakage **conductance** will exist between the conductors.

These four parameters, called as **primary constants,** all distributed along the line, are known by the symbols R, L, C and G, where usually quantities per unit length of line are meant.

Transmission Line as Cascaded T Sections:

To study the behavior of transmission line, a transmission can be considered to be made up of a number of identical symmetrical T sections connected in series as in figure shown below.

If the last section is terminated with its characteristic impedance, the input impedance at the first section is Z_0. Each section is terminated by the input impedance of the following section.



The characteristic impedance for a T section is

$Z_{0T} = \sqrt{Z_1 Z_2 \left(1 + \frac{Z_1}{4Z_2}\right)}$

IF 'n' number of T sections are cascaded and if the sending and receiving currents are I_S and I_R respectively.

$$I_S = I_R e^{n\gamma}$$

Where is the propagation constant for one T section.

$$\boldsymbol{\gamma = \alpha + i\beta}$$

$$e^{\gamma} = e^{\alpha + j\beta} = 1 + \frac{Z_1}{2Z_2} + \sqrt{\frac{Z_1}{Z_2}\left(1 + \frac{Z_1}{4Z_2}\right)} \text{ ------------------------- } 1$$

One T section representing an incremental length Δ_x of the line has a series impedance $Z_1 = Z\Delta x$ and the shunt impedance $Z_2 = \frac{1}{Y\Delta x}$. The characteristic impedance of any small T section is that of the line as a whole.

$$Z_0 = \sqrt{Z_1 Z_2\left(1+\frac{Z_1}{4Z_2}\right)} \text{---------------------- } 2$$

Substituting the values of Z_1 and Z_2

$$Z_0 = \sqrt{\frac{Z\Delta x}{Y\Delta x}\left(1+\frac{Z(\Delta x)^2}{4}\right)} = \sqrt{\frac{Z}{Y}\left(1+\frac{Z(\Delta x)^2}{4}\right)}$$

When (Δx) = 0

$$Z_0 = \sqrt{\frac{Z}{Y}} \text{-------------------------------------- } 3$$

From 1 consider

$$\sqrt{\frac{Z_1}{Z_2}\left(1+\frac{Z_1}{4Z_2}\right)} = \sqrt{\frac{Z_1}{Z_2}}\left(1+\frac{Z_1}{4Z_2}\right)^{\frac{1}{2}}$$

By binomial theorem

$$\sqrt{\frac{Z_1}{Z_2}\left(1+\frac{Z_1}{4Z_2}\right)} = \sqrt{\frac{Z_1}{Z_2}}\left(1+\frac{1}{2}\left(\frac{Z_1}{4Z_2}\right)-\frac{1}{8}\left(\frac{Z_1}{4Z_2}\right)^2+\cdots\ldots\right)$$

Substituting this value in 1

$$e^{\gamma} = 1+\frac{Z_1}{2Z_2}+\sqrt{\frac{Z_1}{Z_2}\left(1+\frac{Z_1}{4Z_2}\right)}$$

$$= 1+\frac{Z_1}{2Z_2}+\sqrt{\frac{Z_1}{Z_2}}\left(1+\frac{1}{2}\left(\frac{Z_1}{4Z_2}\right)-\frac{1}{8}\left(\frac{Z_1}{4Z_2}\right)^2+\cdots\ldots\right)$$

$$= \left[1+\frac{Z_1}{2Z_2}+\left(\sqrt{\frac{Z_1}{Z_2}}+\frac{1}{8}\sqrt{\frac{Z_1}{Z_2}}\left(\frac{Z_1}{Z_2}\right)-\frac{1}{128}\sqrt{\frac{Z_1}{Z_2}}\left(\frac{Z_1}{Z_2}\right)^2+\cdots\ldots\right)\right]$$

$$e^{\gamma}=1+\sqrt{\frac{Z_1}{Z_2}}+\frac{1}{2}\left(\sqrt{\frac{Z_1}{Z_2}}\right)^2+\left(\frac{1}{8}\left(\sqrt{\frac{Z_1}{Z_2}}\right)^3-\frac{1}{128}\left(\sqrt{\frac{Z_1}{Z_2}}\right)^5+\cdots\ldots\right)$$

When applied to incremental length of line Δ_x , then series impedance $Z_1 = Z\Delta x$ and the shunt impedance $Z_2 = \frac{1}{Y\Delta x}$ and propagation constant , $\gamma\Delta x$

$$e^{\gamma\Delta x} = 1 + \sqrt{Z\Delta x Y\Delta x} + \frac{1}{2}\left(\sqrt{Z\Delta x Y\Delta x}\right)^2 + \left(\frac{1}{8}\left(\sqrt{Z\Delta x Y\Delta x}\right)^3 - \frac{1}{128}\left(\sqrt{Z\Delta x Y\Delta x}\right)^5 + \cdots \ldots\right)$$

Series expansion for an exponential $e^{\gamma\Delta x}$ is

$$e^{\gamma\Delta x} = 1 + \Delta x\sqrt{ZY} + \frac{1}{2}\left(\Delta x\sqrt{ZY}\right)^2 + \left(\frac{1}{8}\left(\Delta x\sqrt{ZY}\right)^3 - \frac{1}{128}\left(\Delta x\sqrt{ZY}\right)^5 + \cdots \ldots\right) \text{-----------------------------} \ 4$$

$$e^{\gamma\Delta x} = 1 + \Delta x\gamma + \frac{1}{2!}(\Delta x\,\gamma)^2 + \left(\frac{1}{3!}(\Delta x\,\gamma)^3 + \cdots \ldots\right) \text{--------------} \ 5$$

Equating 4 and 5

$$1 + \Delta x\sqrt{ZY} + \frac{1}{2}\left(\Delta x\sqrt{ZY}\right)^2 + \left(\frac{1}{8}\left(\Delta x\sqrt{ZY}\right)^3 - \frac{1}{128}\left(\Delta x\sqrt{ZY}\right)^5 + \cdots \ldots\right)$$
$$= 1 + \Delta x\gamma + \frac{1}{2!}(\Delta x\,\gamma)^2 + \left(\frac{1}{3!}(\Delta x\,\gamma)^3 + \cdots \ldots\right)$$

$$\Delta x\sqrt{ZY} + \frac{1}{2}\left(\Delta x\sqrt{ZY}\right)^2 + \left(\frac{1}{8}\left(\Delta x\sqrt{ZY}\right)^3 - \frac{1}{128}\left(\Delta x\sqrt{ZY}\right)^5 + \cdots \ldots\right)$$
$$= \Delta x\gamma + \frac{1}{2!}(\Delta x\,\gamma)^2 + \left(\frac{1}{3!}(\Delta x\,\gamma)^3 + \cdots \ldots\right)$$

When Δx tends to zero then

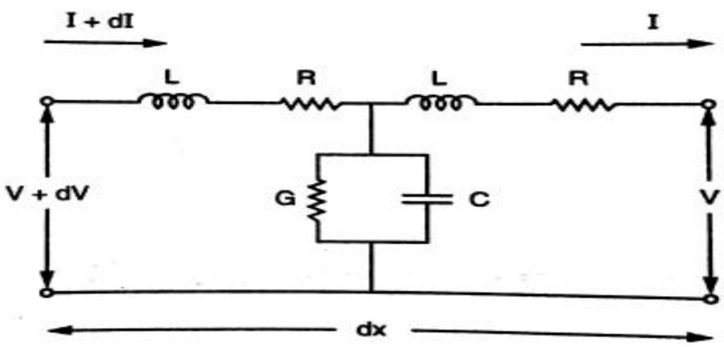
$$\boldsymbol{\gamma = \sqrt{ZY}}$$

This is the value of propagation constant in terms of Z and Y.

General solution - The infinite line

Transmission line is conductive method of guiding electrical energy from one place to another.

A uniform transmission line can be considered to be made up of an infinite number of T sections, each of infinitesimal size dx. The equivalent circuit of T section of transmission line is shown below



The parameters R, L, G and C are distributed throughout the transmission line. The constants of an incremental length dx of a line. The series impedance per unit length and shunt admittance per unit length are given by,

Z=R+jwL   Y=G+jwC

Consider a T section of transmission line of length dx. Let V+dV be the voltage and I+dI be the current at one end of T section .let V be the voltage and I be the current at the other end of this section.

The series impedance of a small section dx is (Z=R+jwL) dx

The shunt admittance of this section dx is (G+jwC) dx

The voltage drop across the series impedance of T sections i.e. the potential difference between the two ends of T section is

V+dV-V = I(R+jwL) dx -----------------1

$\frac{dv}{dx}$ = I(R+jwL),

$\frac{dv}{dx}$ = IZ-----------------------------------2

The current difference between the two ends of T section is due to the voltage drop across the shunt admittance

I+dI-I = V (G+jwC) dx

$\frac{di}{dx}$ = V (G+jwC)

$\frac{di}{dx}$ = VY -----------------------------3

Differentiating 2 w.r.to x

$\frac{d^2V}{dx^2}$ = (R+jwL) $\frac{di}{dx}$ = (R+jwL) (G+jwC) V ----------------------- 4

Differentiating 3 w.r.to x

$\frac{d^2I}{dx^2}$ = (G+jwC) $\frac{dv}{dx}$ = (R+jwL) (G+jwC) I ---------------------------- 5

But we know that propagation constant, $\gamma = \sqrt{(\mathrm{R+jwL})\ (\mathrm{G+jwC})} = \sqrt{ZY}$

Substituting the value of γ in 4 and 5, we get

$\frac{d^2V}{dx^2} = \gamma^2$V , $\frac{d^2I}{dx^2} = \gamma^2$I

The solutions of the above linear differential equations are

V = A$e^{\gamma x}$ + B$e^{-\gamma x}$ ------ 6

I = C $e^{\gamma x}$ + D$e^{-\gamma x}$ ------ 7

Where A, B, C, D are arbitrary constants

Differentiating 6 w.r.to x

$\frac{dv}{dx}$ = A$\gamma e^{\gamma x}$- B$\gamma e^{-\gamma x}$

Substitute From 2

IZ = A$\gamma e^{\gamma x}$- B$\gamma e^{-\gamma x}$

IZ = A$\sqrt{ZY}e^{\sqrt{ZY}x}$- B$\sqrt{ZY}e^{-\sqrt{ZY}x}$

I = A$\sqrt{Y/Z}e^{\sqrt{ZY}x}$- B$\sqrt{Y/Z}e^{-\sqrt{ZY}x}$ ------------------ 8

Similarly, differentiating the equation 7 w.r.to x

$\frac{di}{dx}$ = C$\gamma e^{\gamma x}$ - D$\gamma e^{-\gamma x}$

Substitute from 3

VY = C$\gamma e^{\gamma x}$ - D$\gamma e^{-\gamma x}$

= C$\sqrt{ZY}e^{\sqrt{ZY}x}$ - D$\sqrt{ZY}e^{-\sqrt{ZY}x}$

V = C$\sqrt{Z/Y}e^{\sqrt{ZY}x}$ - D$\sqrt{Z/Y}e^{-\sqrt{ZY}x}$

Since the distance x is measured from the receiving end of the transmission line,

X=0. I = I_R, V = V_R, $V_R = I_R Z_R$

Where I_R is the current in the receiving end of the line.

V_R is the voltage across the receiving end of the line.

Z_R is the impedance of receiving end.

Substituting this condition in equations 6, 7, 8, and 9

V_R = A+B ---10

I_R = C+D --- 11

$I_R = A\sqrt{Y/Z} - B\sqrt{Y/Z}$ ----------12

$V_R = C\sqrt{Z/Y} - D\sqrt{Z/Y}$ -------------13

To solve these equations let x = $\sqrt{Z/Y}$ and 1/x = $\sqrt{Y/Z}$

$I_R = A/x – B/x$ ----------14

$V_R = Cx - Dx$ -------------15

Comparing 10 and 15

A+B = Cx – Dx ----16

A – B = Cx+Dx ----17

Add equations 17 + 16, we get

A = Cx

Subtract equations 16 – 17 we get

B = Dx

Substituting the values of A & B in 10

V_R = Cx – Dx -----18

But from 11 we get

I_R = C+D

11 * x =>xI_R = Cx+Dx ----19

18 => V_R = Cx – Dx

Adding 19 and 18, we get

$2Cx = xI_R + V_R$

$C = I_R/2 + V_R/2x$

W.k.t $1/x = \sqrt{Y/Z}$

$C = I_R/2 + (V_R/2)\sqrt{Y/Z}$ --------20

Subtracting 19 – 18

$2Dx = xI_R - V_R$

$D = I_R/2 - V_R/2x$

$D = I_R/2 - (V_R/2)\sqrt{Y/Z}$ --------21

W.k.t A = Cx

Substitute in 20

$A = I_R x/2 + V_R/2$

$A = (I_R/2)\sqrt{Z/Y} + V_R/2$---------------- 22

W.k.t B = - Dx

Substitute in 20

$B = - I_R x/2 + V_R/2$

$B = -(I_R/2)\sqrt{Z/Y} + V_R/2$---------------- 23

The characteristic impedance is defines as

$Z_0 = \sqrt{Z/Y}$

22 => $A = (I_R/2) Z_0 + V_R/2 = \frac{V_r}{2} + \frac{V_r Z_0}{2 Z_r}$

$A = \frac{V_r}{2}\left[1 + \frac{Z_0}{Z_r}\right]$ ------------ 24

23 => $B = \frac{V_r}{2} - \frac{V_r Z_0}{2 Z_r}$

$B = \frac{V_r}{2}\left[1 - \frac{Z_0}{Z_r}\right]$ ------------ 25

20 -=> $C = \frac{I_r}{2} - \frac{I_r Z_R}{2 Z_0}$

$C = \frac{I_r}{2}\left[1 - \frac{Z_R}{Z_0}\right]$ -------------- 26

21 => $D = \frac{I_r}{2} + \frac{I_r Z_R}{2 Z_0}$

$D = \frac{I_r}{2}[1+\frac{Z_R}{Z_0}]$ -------------- 27

Substituting the values of A, B, C and D in 6 and 7

$V = \frac{V_r}{2}\left[1+\frac{Z_0}{Z_r}\right]e^{\sqrt{ZY}x} + \frac{V_r}{2}\left[1-\frac{Z_0}{Z_r}\right]e^{-\sqrt{ZY}x}$

$I = \frac{I_r}{2}[1+\frac{Z_R}{Z_0}]\, e^{\sqrt{ZY}x} + \frac{I_r}{2}\left[1-\frac{Z_R}{Z_0}\right]e^{-\sqrt{ZY}x}$

$V = \frac{V_r}{2}e^{\sqrt{ZY}x} + \frac{V_r}{2}\frac{Z_0}{Z_r}e^{\sqrt{ZY}x} + \frac{V_r}{2}e^{-\sqrt{ZY}x} - \frac{V_r}{2}\frac{Z_0}{Z_r}e^{-\sqrt{ZY}x}$

$V = \frac{V_r}{2}\left[e^{\sqrt{ZY}x} + e^{-\sqrt{ZY}x}\right] + \frac{V_r}{2}\frac{Z_0}{Z_r}\left[e^{\sqrt{ZY}x} + e^{-\sqrt{ZY}x}\right]$

Since $2\cosh ax = (e^{ax} + e^{-ax})$

$2\sinh ax = (e^{ax} - e^{-ax})$

$V = V_r \cosh\sqrt{ZY}x + I_r Z_r \frac{Z_0}{Z_r}\sinh\sqrt{ZY}x$

$V = V_r \cosh\sqrt{ZY}x + I_r Z_0 \sinh\sqrt{ZY}x$ ---------------------------- 28

$I = \frac{I_r}{2}e^{\sqrt{ZY}x} + \frac{I_r}{2}\frac{Z_R}{Z_0}e^{\sqrt{ZY}x} + = \frac{I_r}{2}e^{-\sqrt{ZY}x} - \frac{I_r}{2}\frac{Z_R}{Z_0}e^{-\sqrt{ZY}x}$

$I = \frac{I_r}{2}\left[e^{\sqrt{ZY}x} + e^{-\sqrt{ZY}x}\right] + \frac{I_r}{2}\frac{Z_R}{Z_0}\left[e^{\sqrt{ZY}x} + e^{-\sqrt{ZY}x}\right]$

$I = I_r \cosh\sqrt{ZY}x + \frac{V_r}{Z_0}\sinh\sqrt{ZY}x$ -- 29

The equations of voltage and current at the sending end of the transmission line of length l are given by

$V_s = V_r \cosh\sqrt{ZY}l + V_r Z_0 \sinh\sqrt{ZY}l$

$V_s = V_r[\cosh\sqrt{ZY}l + \frac{Z_0}{Z_R}\sinh\sqrt{ZY}l]$ ------------------------- 30

$I_S = I_r \cosh\sqrt{ZY}l + \frac{I_r Z_r}{Z_0}\sinh\sqrt{ZY}x$

$I_S = I_r[\cosh\sqrt{ZY}l + \frac{Z_r}{Z_0}$sin

h $\sqrt{ZY}x$] ------------------------------ 31

Physical significance of the equation – infinite line.

Input impedance:

The equations for voltage and current at the sending end of a transmission line of length l are given by

$$\boldsymbol{V_s = V_r\,[\cosh\sqrt{ZY}l + Z_r Z_0 \sinh\sqrt{ZY}l\,]}$$

$$\boldsymbol{I_S = I_r\,[\cosh\sqrt{ZY}l + \frac{Z_r}{Z_0}\sinh\sqrt{ZY}x\,]}$$

The input impedance of the transmission line is

$$Z_s = \frac{V_S}{I_R}$$

$$Z_s = \frac{\boldsymbol{V_r\,[\cosh\sqrt{ZY}l + \frac{Z_0}{Z_r}\sinh\sqrt{ZY}l\,]}}{\boldsymbol{I_r\,[\cosh\sqrt{ZY}l + \frac{Z_r}{Z_0}\sinh\sqrt{ZY}x\,]}}$$

$$= \frac{\boldsymbol{I_r Z_r\,[\cosh\sqrt{ZY}l + \frac{Z_0}{Z_r}\sinh\sqrt{ZY}l\,]}}{\boldsymbol{I_r\,[\cosh\sqrt{ZY}l + \frac{Z_r}{Z_0}\sinh\sqrt{ZY}x\,]}}$$

$$= \frac{\boldsymbol{I_r Z_r(Z_r\cosh\sqrt{ZY}l + Z_0\sinh\sqrt{ZY}l)}}{\boldsymbol{Z_r}} \Bigg/ \frac{\boldsymbol{I_r\,[Z_0\cosh\sqrt{ZY}l + Z_r\sinh\sqrt{ZY}l\,]}}{\boldsymbol{Z_0}}$$

$$\boldsymbol{Z_s = \frac{Z_0(Z_r\cosh\sqrt{ZY}l + Z_0\sinh\sqrt{ZY}l)}{Z_0\cosh\sqrt{ZY}l + Z_r\sinh\sqrt{ZY}l}}$$

$$\boldsymbol{Z_s = \frac{Z_0(Z_r + Z_0\tanh\sqrt{ZY}l)}{Z_0 + Z_r\tanh\sqrt{ZY}l}}$$

Reflection loss and Return loss

When the load impedance is not equal to the characteristic impedance of transmission line, reflection takes place.

The expression for voltage and current on the transmission line are,

$$\mathrm{V} = \frac{V_r(Z_r+Z_0)}{2Z_r}[e^{\gamma x} + \frac{Z_r - Z_0}{Z_r+Z_0}\, e^{-\gamma x}]$$

$$\mathrm{I} = \frac{I_r(Z_r+Z_0)}{2Z_0}\,[e^{\gamma x} - \left(\frac{Z_r - Z_0}{Z_r+ Z_0}\right) e^{-\gamma x}]$$

$$\gamma = \sqrt{ZY}$$

If the transmission line is not terminated with the characteristic impedance i.e. $Z_r \neq Z_0$ the expressions for voltage and current exist.

It consist of two waves i) incident wave ii) reflected wave

Incident wave move in the forward direction

Reflected wave move on the opposite direction

The term varying with $e^{\gamma x}$ represents a wave progressing from the sending end towards receiving end.

The term varying with $e^{-\gamma x}$ represents a wave progressing from the receiving end towards sending end.

If the transmission line is terminated with characteristic impedance (i.e.)

$Z_r = Z_0$

$V = V_r e^{\gamma x}$, $I = I_r e^{\gamma x}$

Only incident wave exist, no reflected wave.

Reflection coefficient:

It is defined as the ratio of the reflected voltage to the incident voltage at the receiving end of the line.

$$K = \frac{reflected\ voltage\ at\ load}{incident\ voltage\ at\ load} = \frac{V_r}{V_s}$$

w.k.t

$$V = \frac{V_r(Z_r+Z_0)}{2Z_r}[e^{\gamma x}+\frac{Z_r-Z_0}{Z_r+Z_0}e^{-\gamma x}]$$

$e^{\gamma x}$represents incident wave

$e^{-\gamma x}$represents reflected wave

$$K = \frac{\frac{V_r(Z_r-Z_0)}{2Z_r}}{\frac{V_r(Z_r+Z_0)}{2Z_r}}$$

$$\mathbf{K} = \frac{(\boldsymbol{Z_r}-\boldsymbol{Z_0})}{(\boldsymbol{Z_r}+\boldsymbol{Z_0})}$$

It is also defined in terms of current but it is negative

-K = $\boldsymbol{\frac{I_r}{I_s}}$

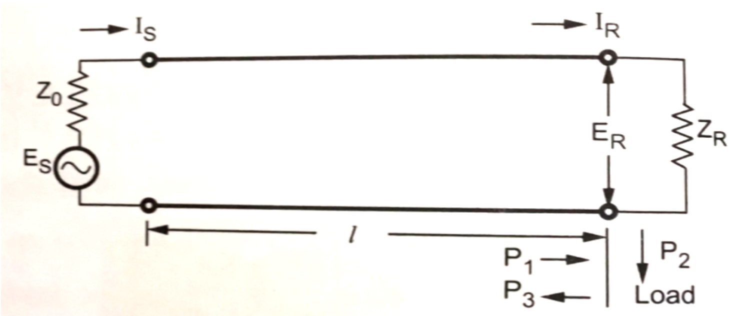
Reflection factor and reflection loss:

Consider a transmission line with a voltage source V_s and its impedance Z_s and load impedanceZ_r.

If $Z_r \neq Z_s$, reflection takes place.

The power delivered to the load is reduced

Reflection results in power loss. This loss is known as reflection loss



Impedance matching between Z_r *and* Z_s can be done by inserting an ideal transformer and a phase shifting network.

I_1 *and* I_2be the currents in primary and secondary of the transformer.

The current ratio of the transformer is given by

$$\frac{I_2}{I_1} = \sqrt{\frac{Z_s}{Z_r}}$$

$Z_r$may be adjusted to that of Z_s by choosing proper transformation ratio and phase angle.

$Z_r$is the image impedance of Z_s

The current through the source is

$$I_1 = \frac{V_s}{2Z_s}$$

The current flow in the secondary of the transformer under image impedance matching is

$$I_2' = I_1\sqrt{\frac{Z_s}{Z_r}} = \frac{V_s}{2Z_s}\sqrt{\frac{Z_s}{Z_r}} = \frac{V_s}{2\sqrt{Z_s Z_r}}$$

The current in the load impedance Z_2 without imager impedance matching.

$$|I_2| = \frac{|V_s|}{|Z_s + Z_r|}$$

The ratio of the current actually flowing in the load to that which might flow under matched condition is known as reflection factor.

$\left|\frac{I_2}{I_2{'}}\right| = \frac{\frac{|V_S|}{|Z_S+Z_r|}}{\left|\frac{V_S}{2\sqrt{Z_S Z_r}}\right|}$ k = $\frac{\left|2\sqrt{Z_S Z_r}\right|}{|Z_S+Z_r|}$

Reflection loss = ln 1/k = ln$\left|\frac{Z_S+Z_r}{2\sqrt{Z_S Z_r}}\right|$ dB

V = $\frac{V_r(Z_r+Z_0)}{2Z_r}[e^{\gamma x}+Ke^{-\gamma x}]$

I = $\frac{I_r(Z_r+Z_0)}{2Z_r}[\, e^{\gamma x}-Ke^{-\gamma x}]$

These 2 quantities comprise of incident and reflected wave.

The term involving $e^{\gamma x}$ is the incident wave and $e^{-\gamma x}$is the reflected wave

The reflected wave depends upon the reflection coefficients.

The voltage and current distributions for open circuit and short circuit are shown below.

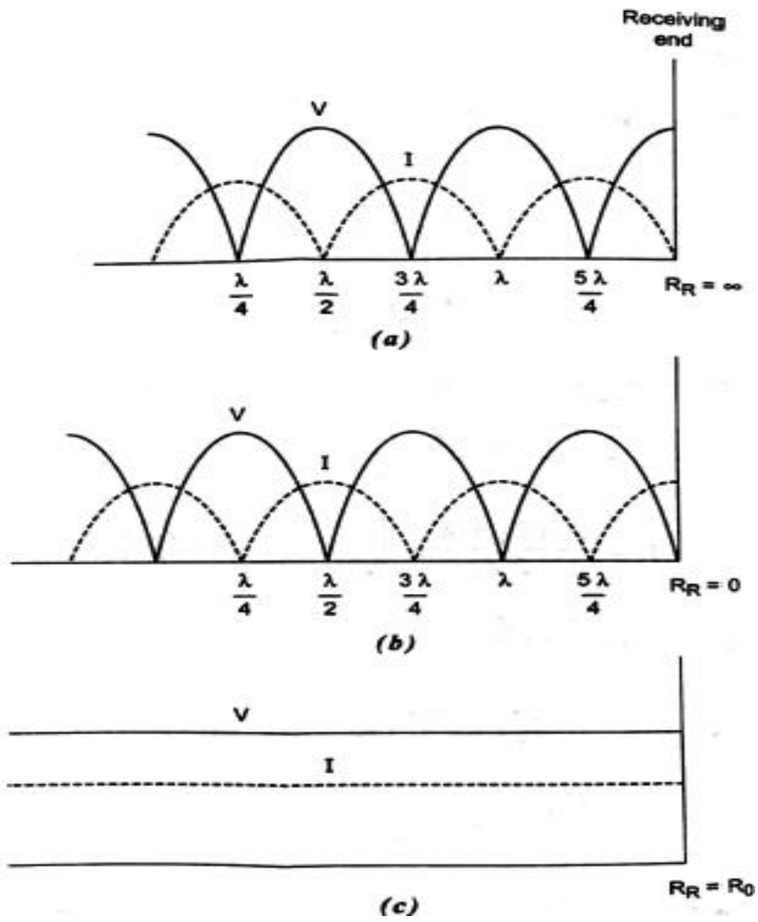


Fig.Voltages and Currents on dissipation less line

a.Open Circuit b.Short Circuit c.R_R=R_0

Return Loss

The return loss is defined as,

Return loss=10 log $\frac{p_1}{P_3}$dB

It indicates the ratio of the power at receiving end due to incident wave to the power reflected by the load.

Return loss=10 log$\frac{p_1}{(|k|^2 p_1)} = 10\,\frac{log 1}{(|k|^2}$.

Return loss=20log$\frac{Z_r+Z_0}{Z_r-Z_0}$ dB

Wave form distortion and also derive the condition for distortion less line:

Signal transmitted over transmission line is normally complex and consists of many frequencies components. Such voice voltage will not have all frequencies transmitted with equal attenuation and equal; time delay. The received wave form will not be identical with input waveform at the sending end. This variation is known as distortion.

There are two types of line distortion.

Frequency distortion

Delay distortion

*i)**Frequency distortion** :*

A complex voltage transmitted on a transmission line will not be attenuated equally and the received waveform will not be identical with the received waveform will not be identical with the input waveform at the transmitting end. This variation is known as frequency distortion.

The attenuation constant is given by

$$\alpha = \sqrt{\frac{\mathrm{RG} - \omega^2\mathrm{LC} \pm \sqrt{(\mathrm{RG} - \omega^2\mathrm{LC})^2 - 4 * \frac{\omega^2}{4}(\mathrm{RC} + \mathrm{LG})^2}}{2}}$$

$\alpha$is a function of frequency and therefore the line will introduce frequency distortion.

ii) Delay or phase distortion:

All the frequencies applied to a transmission line will not travel uniformly. Some of them may be delayed more than others. This phenomenon is known as delay or phase distortion.

The phase constant is

$$\beta = \sqrt{\frac{-(\mathrm{RG} - \omega^2\mathrm{LC}) \pm \sqrt{(\mathrm{RG} - \omega^2\mathrm{LC})^2 - 4 * \frac{\omega^2}{4}(\mathrm{RC} + \mathrm{LG})^2}}{2}}$$

Frequency distortion is reduced in the transmission line of high quality wire line by the use of equalizers at the line terminals.

Delay distortion is of relatively less importance to voice and music transmission. But causes problem in video transmission. This can be avoided by the use of co axial cables.

The distortion less line:

If a line is to have neither frequency nor delay distortion, then attenuation factor and the velocity of propagation V cannot be functions of frequency.

If V = $\frac{\omega}{\beta}$ in this equation β must be a direct fuction of frequency.

$$\beta = \sqrt{\frac{-(\mathrm{RG}-\omega^2\mathrm{LC}) \pm \sqrt{(\mathrm{RG}-\omega^2\mathrm{LC})^2-4*\frac{\omega^2}{4}(\mathrm{RC}+\mathrm{LG})^2}}{2}} \text{-----------------------1}$$

For β to be a direct function of frequency, the term $(\mathrm{RG}-\omega^2\mathrm{LC})^2-4*\frac{\omega^2}{4}(\mathrm{RC}+\mathrm{LG})^2$ must be equal to $(\mathrm{RG}+\omega^2\mathrm{LC})^2$

$(\mathrm{RG}-\omega^2\mathrm{LC})^2-4*\frac{\omega^2}{4}(\mathrm{RC}+\mathrm{LG})^2 = (\mathrm{RG}+\omega^2\mathrm{LC})^2$

$R^2G^2+\omega^4L^2C^2-2RG\omega^2LC + \omega^2L^2G^2+\omega^2C^2R^2+2\omega^2LGCR = R^2G^2+\omega^4L^2C^2+2RG\omega^2LC$

$\omega^2L^2G^2+\omega^2C^2R^2-2\omega^2LGCR = 0$

$$\omega^2(L^2G^2+C^2R^2-2LGCR) = 0$$

$(\mathrm{LG}-\mathrm{CR})^2 = 0$

LG-CR = 0

LG = CR ------------------------2

$$\frac{R}{L} = \frac{G}{C}$$

This is the condition for distortion less line.

Therefore

$$\beta = \sqrt{\frac{-(RG - \omega^2 LC) + \sqrt{(RG + \omega^2 LC)^2}}{2}}$$

$$\beta = \sqrt{\frac{-(RG - \omega^2 LC) + RG + \omega^2 LC}{2}}$$

$\beta = \sqrt{\frac{2\omega^2 LC}{2}} = \sqrt{\omega^2 LC}$

$\beta = \omega\sqrt{LC}$

Velocity, $V = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$

This is the same velocity for all frequencies, thus eliminating delay distortion.

Attenuation factor,

$$\alpha = \sqrt{\frac{RG - \omega^2 LC \pm \sqrt{(RG - \omega^2 LC)^2 - 4 * \frac{\omega^2}{4}(RC + LG)^2}}{2}}$$

To make α independent of frequency, the term $(RG - \omega^2 LC)^2 - 4 * \frac{\omega^2}{4}(RC + LG)^2$ must be equal to $(RG + \omega^2 LC)^2$

From 2 LG = CR, $\frac{L}{C} = \frac{R}{G}$

This will make α and v independent of frequency to achieves this condition, it requires a very large value of L and small of G.

$$\alpha = \sqrt{\frac{RG - \omega^2 LC \pm \sqrt{(RG + \omega^2 LC)^2}}{2}}$$

$\alpha = \sqrt{\frac{RG - \omega^2 LC + RG + \omega^2 LC}{2}} = \sqrt{RG}$

It is independent of frequency, thus eliminating frequency distortion on the line.

The characteristic impedance

$Z_O = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L(\frac{R}{L}+j\omega)}{C(\frac{G}{C}+j\omega)}}$

But $\frac{R}{L} = \frac{G}{C}$

$Z_O = \sqrt{\frac{L}{C}}$. It is purely real and is independent of frequency.

Telephone cable

In the ordinary telephone cable, the wires are insulated with paper and twisted in pairs. This construction results in negligible values of inductance and conductance so that reasonable assumptions in the audio range frequencies are that

$$Z = [R + j\omega L]_{L=0} = R$$

$$Y = [G + j\omega C]_{G=0} = j\omega C$$

$$\gamma = \sqrt{ZY} = \sqrt{j\omega CR} = \sqrt{j}\sqrt{\omega CR}$$

$$e^{j\frac{\pi}{2}} = \cos\frac{\pi}{2} + j\sin\frac{\pi}{2} = j$$

$$e^{j\frac{\pi}{4}} = \cos\frac{\pi}{4} + j\sin\frac{\pi}{4} = j^{\frac{1}{2}} = \sqrt{j} = \frac{(1+j)}{\sqrt{2}}$$

$$\gamma = \sqrt{ZY} = \sqrt{j\omega CR} = \sqrt{j}\sqrt{\omega CR} = (1+j)\sqrt{\frac{\omega CR}{2}} = \alpha + j\beta$$

$$\alpha = \sqrt{\frac{\omega CR}{2}}\ and\ \beta = \sqrt{\frac{\omega CR}{2}}$$

Velocity of propagation is

$$v = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\frac{\omega CR}{2}}} = \sqrt{\frac{2\omega}{CR}}$$

It should be observed that both and the velocity of propagation are functions of frequency, such that the higher frequencies are attenuated more and travel faster than the lower frequencies. Very considerable frequency and delay distortion is the result on a telephone cable.

Inductance loading of telephone cables

It is necessary to increase the L/C ratio to achieve distortionless conditions. Heaviside suggested that the inductance can be increased, and Pupin developed a theory that made possible this increase in the inductance by *lumped inductors* spaced at intervals along the line. This use of inductance is called ***loading*** the line. In some submarine cables, distributed or uniform loading is obtained by winding the cable with a high permeability steel tape such as permalloy. This method is employed because of the practical difficulties of designing lumped loading coils for such underwater circuits.

(i) Uniformly loaded cable

For a uniformly loaded cable, it may be assumed that G=0 and for which L has been increased so that is large with respect to R. Then

$$Z = R + j\omega L$$

$$Y = j\omega C$$

Since

$$Z = \sqrt{R^2 + \omega^2 L^2}\ \angle\left(\frac{\pi}{2} - \tan^{-1}\frac{R}{\omega L}\right)$$

$$\boxed{\tan^{-1} x = \frac{\pi}{2} - \tan^{-1}\frac{1}{x} \ \Rightarrow \tan^{-1}\frac{\omega L}{R} = \frac{\pi}{2} - \tan^{-1}\frac{R}{\omega L}}$$

and

$$Y = j\omega C = \omega C \angle \frac{\pi}{2}$$

$$\gamma = \sqrt{ZY} = \sqrt{\left\{\sqrt{R^2 + \omega^2 L^2}\ \angle\left(\frac{\pi}{2} - \tan^{-1}\frac{R}{\omega L}\right)\right\}\left\{\omega C \angle \frac{\pi}{2}\right\}}$$

$$\gamma = \sqrt{\omega C\sqrt{R^2+\omega^2 L^2}\angle\left(\pi - \tan^{-1}\frac{R}{\omega L}\right)}$$

$$\gamma = \sqrt{(\omega C)(\omega L)\sqrt{\left(\frac{R^2}{\omega^2 L^2}+1\right)}\angle\left(\frac{\pi}{2}-\frac{1}{2}\tan^{-1}\frac{R}{\omega L}\right)}$$

$$\gamma = \omega\sqrt{LC}\ \sqrt[4]{\left(\frac{R^2}{\omega^2 L^2}+1\right)}\angle\left(\frac{\pi}{2}-\frac{1}{2}\tan^{-1}\frac{R}{\omega L}\right)$$

In view of the fact that R is small with respect to ωL, the term $\frac{R^2}{\omega^2 L^2}$ may be dropped, and γ becomes

$$\gamma = \omega\sqrt{LC}\angle\left(\frac{\pi}{2}-\frac{1}{2}\tan^{-1}\frac{R}{\omega L}\right)$$

If $\theta = \left(\frac{\pi}{2}-\frac{1}{2}\tan^{-1}\frac{R}{\omega L}\right)$,

$$\cos\theta = cos\left(\frac{\pi}{2}-\frac{1}{2}\tan^{-1}\frac{R}{\omega L}\right) = \sin\left(\frac{1}{2}\tan^{-1}\frac{R}{\omega L}\right)$$

For small angle,

$$\sin\theta = \tan\theta = \theta \Rightarrow \cos\theta = \frac{R}{2\omega L}$$

Likewise

$$\sin\theta = \sin\left(\frac{\pi}{2}-\frac{1}{2}\tan^{-1}\frac{R}{\omega L}\right) = 1$$

$$\sin(A - B) = sinA\ cosB - cosA\ sinB$$

$$\sin\left(\frac{\pi}{2} - \frac{1}{2}\tan^{-1}\frac{R}{\omega L}\right) = sin\left(\frac{\pi}{2}\right) cos\left(\frac{1}{2}\tan^{-1}\frac{R}{\omega L}\right) - cos\left(\frac{\pi}{2}\right) sin\left(\frac{1}{2}\tan^{-1}\frac{R}{\omega L}\right) = 1$$

The value of γ is rewritten as

$$\gamma = \omega\sqrt{LC}(cos\theta + jsin\theta) = \omega\sqrt{LC}\left(\frac{R}{2\omega L} + j\right)$$

Therefore, for the uniformly loaded cable,

$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}}$$

$$\beta = \omega\sqrt{LC}$$

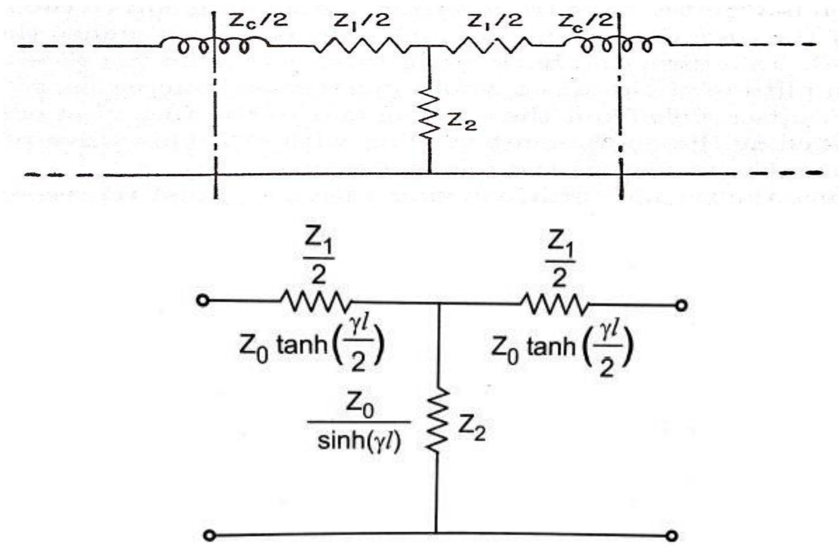
The velocity of propagation is

$$v = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

It is readily observed that, under the assumptions of G=0 and L large with respect R, attenuation and velocity are both independent of frequency and the loaded cable will be distortionless. This expression shows that may be reduced by increasing L, provided that R is not also increased too greatly. Continuous or uniform loading is expensive and achieves only a small increase in L per unit length.

(ii) Lumped loading

Lumped loading is preferred as a means for transmission improvement for cables. An analysis on the performance of a line loaded at uniform intervals can be obtained by considering a symmetrical section of line from the center of one loading coil to the center of the next, where the loading coil impedance is.



Equivalent T-section for part of a line between two lumped loading coils of impedance Z_C

$$\frac{Z_1}{2} = Z_0 \tan h \frac{N\gamma}{2}$$

Where N is the number of miles between the loading coils and $\gamma$is the propagation constant per mile. Upon including half a loading coil, the equivalent series arm of the loaded section becomes

$$\frac{Z_1{}'}{2} = \frac{Z_C}{2} + Z_0 \tan h \frac{N\gamma}{2}$$

The shunt Z_2 arm of the equivalent T section is

$$Z_2 = \frac{Z_0}{sinhN\gamma}$$

Also

$$coshN\gamma' = 1 + \frac{Z_1{}'}{2Z_2}$$

$$coshN\gamma' = 1 + \frac{\frac{Z_C}{2} + Z_0 \tan h \frac{N\gamma}{2}}{\frac{Z_0}{sinhN\gamma}} \qquad (13)$$

Equation (13) reduces to

$$coshN\gamma' = \frac{Z_C}{2Z_0} sinhN\gamma + coshN\gamma$$

This expression is known as Campbell's equation and permits the determination of a value for of a line section consisting partially of lumped and partially of distributed elements. Campbell's equation makes possible the calculation of the effects of the loading coils in reducing attenuation and distortion on lines.

A parallel wire transmission line is having the following line parameters at5KHz.Series resistance R=2.59×10^{-3}'Ω/m, series inductance=2µH/m shunt conductance G=0ʊ/m and capacitance between conductors C=5.56nF/m. Find the characteristic impedance, attenuation constant, phase shift constant, velocity of propagation and wavelength. NOV/DEC 2015(10)

The characteristic impedance is given by

$Z_0=\sqrt{\frac{R+jwL}{G+jwC}}$

$=\sqrt{\frac{2.59\times10^{-3}+j(2\pi\times5000)(2\times10^{-6})}{0+j(2\pi\times5000)(5.56\times10^{-9})}}$

$=\sqrt{\frac{2.59\times10^{-3}+j0.0628}{0+j1.7\times10^{-4})}} = \sqrt{\frac{0.0628\angle 87.64°}{1.7\times10^{-4}\angle 90°}}$

$=19.22\angle\frac{87.64°-90°}{2} = 19.22\angle -1.18°\Omega$

The propagation constant is given by

$\gamma\sqrt{(R+j\omega L)+(G+j\omega C)}$

$=\sqrt{(0.0628\angle 87.64°)(1.7\times10^{-4}}\angle 90°)$

$=3.267\times10^{-3}\angle\frac{87.64°+90°}{2} = 3.267\times10^{-3}\angle 88.82°$

$\alpha + j\beta = 6.728\times10^{-5} + j3.266\times10^{-3}$

Attenuation constant $\alpha = 6.728\times10^{-5} neper/m$

Phase shift constant$\beta = 3.266 \times 10^{-3} rad/m$

Velocity of propagation is given by

V=$\frac{\omega}{\beta} = \frac{2\pi f}{\beta}$=$\frac{2\pi\times 5000}{3.266\times 10^{-3}} = 1923.8km$

Wavelength is given by

λ=$\frac{2\pi}{\beta} = \frac{2\pi}{3.266\times 10^{-3}} = 1923.8km$

A meter long transmission line with characteristic impedance of 60+j40Ω is operating at ω=10^6rad/sec has attenuation constant of 0 rad/m. If the line is terminated by a load of 20+j50Ω, determine the input impedance of this line. NOV/DEC 2015(6)

Given that

Z_0=60+j40=72.111∠33.69°

Z_L=Z_R=20+j50=53.8516∠68.19°

L=2m,ω=10^6,α=0.921 and $\beta = 0$

The input impedance is given by

Z_s=$Z_o\left[\frac{ZRcosh\gamma l+Z0sinh\gamma l}{Zocosh\gamma l+ZRsinh\gamma l}\right]$

Coshγl=cosh(α+jβ)l=coshαl=cosh(0.921)(2)=3.2338

Sinhγl=sinh(α+jβ)l= sinhαl=sinh(0.921)(2)=3.07532

Therefore

Z_s=72.111∠33.69°$\left[\frac{(53.8516\angle 68.19°)(3.23)+(72.111\angle 33.69°)(3.075)}{(72.111\angle 33.69°)(3.23)+(53.85\angle 68.19°)(3.075)}\right]$

=72.111∠33.69° $\left[\frac{174.14\angle 68.19°)+(221.76\angle 33.69°)}{(233.19\angle 33.69°)+(165.61\angle 68.19°)}\right]$

Z_s=71.52∠34.58°Ω