UNIT 2 ANGLE MODULATION

Angle modulation is a class of analog modulation. These techniques are based on altering the angle (or phase) of a sinusoidal carrier wave to transmit data, as opposed to varying the amplitude, such as in AMtransmission.

Angle Modulation is modulation in which the angle of a sine-wave carrier is varied by a modulating wave. Frequency Modulation (FM) and Phase Modulation (PM) are two types of angle modulation. In frequency modulation the modulating signal causes the carrier frequency to vary. These variations are controlled by both the frequency and the amplitude of the modulating wave. In phase modulation the phase of the carrier is controlled by the modulatingwaveform.

The two main types of angle modulation are:

- Frequency modulation (FM), with its digital correspondence frequency-shift keying(FSK).
- Phase modulation (PM), with its digital correspondence phase-shift keying(PSK).

FREQUENCY MODULATION:

Frequency modulation (FM): the encoding of information in a carrier wave by varying the instantaneous frequency of thewave.

Besides using the amplitude of carrier to carrier information, one can also use the angle of a carrier to carrier information. This approach is called angle modulation, and includes frequency modulation (FM) and phase modulation (PM). The amplitude of the carrier is maintained constant. The major advantage of this approach is that it allows the trade-off between bandwidth and noise performance.

An angle modulated signal can be written as

$$st=A\cos\theta(t)$$

where$\theta(t)$ is usually of the form $\theta t= 2\pi fct + \emptyset(t)$ and fc is the carrier frequency. Thesignal $\emptyset(t)$ isderivedfromthemessagesignalm(t).If $\emptyset t=kpm(t)$ for some constant kp,the resulting modulation is called phase modulation. The parameter$kp$is called the phase SCE sensitivity.In telecommunications and signal processing, frequency modulation (FM) is the encoding of information in a carrier wave by varying the instantaneous frequency of the wave. (Compare with amplitude modulation, in which the amplitude of the carrier wave varies, while the frequency remains constant.) Frequency modulation is known as phase modulation when the carrier phase modulation is the time integral of the FM signal.

If the information to be transmitted (i.e., the baseband signal) is $x_m(t)$ and the sinusoidal carrier is $x_c(t) = A_c \cos(2\pi f_c t)$, where f_c is the carrier's base frequency, and A_c is the carrier's amplitude, the modulator combines the carrier with the baseband data signal to get the transmitted signal:

$$
\begin{aligned}
y(t) &= A_c \cos\left(2\pi \int_0^t f(\tau) d\tau\right) \\
&= A_c \cos\left(2\pi \int_0^t \left[f_c + f_\Delta x_m(\tau)\right] d\tau\right) \\
&= A_c \cos\left(2\pi f_c t + 2\pi f_\Delta \int_0^t x_m(\tau) d\tau\right)
\end{aligned}
$$

In this equation, $f(\tau)$ is the instantaneous frequency of the oscillator and f_Δ is the frequency deviation, which represents the maximum shift away from f_c in one direction, assuming $x_m(t)$ is limited to the range± 1.

While most of the energy of the signal is contained within $f_c \pm f_\Delta$, it can be shown by Fourier analysis that a wider range of frequencies is required to precisely represent an FM signal. The frequency spectrum of an actual FM signal has components extending infinitely, although their amplitude decreases and higher-order components are often neglected in practical design problems.

Sinusoidal baseband signal:

Mathematically, a baseband modulated signal may be approximated by a sinusoidal continuous wave signal with a frequency f_m.

The integral of such a signal is:

$$\int_0^t x_m(\tau) d\tau = \frac{A_m \cos(2\pi f_m t)}{2\pi f_m}$$

In this case, the expression for y(t) above simplifies to:

$$y(t) = A_c \cos\left(2\pi f_c t + \frac{f_\Delta}{f_m} \cos\left(2\pi f_m t\right)\right)$$

where the amplitude A_m of the modulating sinusoid is represented by the peakdeviation f_Δ

The harmonic distribution of a sine wave carrier modulated by such a sinusoidal signal can be represented with Bessel functions; this provides the basis for a mathematical understanding of frequency modulation in the frequency domain.

- ✓ **Modulationindex:**

As in other modulation systems, the value of the modulation index indicates by how much the modulated variable varies around its unmodulated level. It relates to variations in the carrier frequency:

$$h = \frac{\Delta f}{f_m} = \frac{f_\Delta |x_m(t)|}{f_m}$$

where f_m is the highest frequency component present in the modulating signal $x_m(t)$, and Δf is the peak frequency-deviation—i.e. the maximum deviation of the instantaneous frequency from the carrier frequency. For a sine wave modulation, the modulation index is seen to be the ratio of the amplitude of the modulating sine wave to the amplitude of the carrier wave (hereunity).

If $h \ll 1$, the modulation is called narrowband FM, and its bandwidth is approximately $2f_m$.

For digital modulation systems, for example Binary Frequency Shift Keying (BFSK), where a binary signal modulates the carrier, the modulation index is givenby:

$$h = \frac{\Delta f}{f_m} = \frac{\Delta f}{\frac{1}{2T_s}} = 2\Delta f T_s$$

where T_s is the symbol period, and $f_m = \frac{1}{2T_s}$ is used as the highest frequency of the modulating binary waveform by convention, even though it would be more accurate to say it isthe highest fundamental of the modulating binary waveform. In the case of digital modulation, the carrier f_c is never transmitted. Rather, one of two frequencies is transmitted,either $f_c + \Delta f$ or $f_c - \Delta f$, depending on the binary state 0 or 1 of the modulationsignal.

If $h \gg 1$, the modulation is called wideband FM and its bandwidth is approximately $2f_\Delta$. While wideband FM uses more bandwidth, it can improve the signal-to-noise ratiosignificantly; for example, doubling the value of Δf, while keeping f_m constant, results in an eight-fold improvement in the signal-to-noise ratio. (Compare this with Chirp spread spectrum, which uses extremely wide frequency deviations to achieve processing gains comparable to traditional, With a tone-modulated FM wave, if the modulation frequency is held constant and the modulation index is increased, the (non-negligible) bandwidth of the FM signal increases but the spacing between spectra remains the same; some spectral components decrease in strength as others increase. If the frequency deviation is held constant and the modulation frequency increased, the spacing between spectra increases.

Frequency modulation can be classified as narrowband if the change in the carrier frequency is about the same as the signal frequency, or as wideband if the change in the carrier frequency is much higher (modulation index >1) than the signal frequency. [6] For example, narrowband FM is used for two way radio systems such as Family Radio Service, in which the carrier is allowed to deviate only 2.5 kHz above and below the center frequency with speech signals of no more than 3.5 kHz bandwidth. Wideband FM is used for FM broadcasting, in which music and speech are transmitted with up to 75 kHz deviation from the center frequency and carry audio with up to a 20-kHz bandwidth.

Carson's rule:

$$BT = 2\Delta f + fm.$$

2.2 NARROW BAND FMMODULATION:

NarrowbandFM: If the modulation index of *FM* is kept under 1, then the *FM* produced is regarded as narrow band*FM*.

Thecasewhere$|\theta m(t)| \ll 1$foralltiscalled narrow band FM. Using the approximations cosx$\simeq$1andsinx $\simeq$x for$|x| \ll 1$,theFMsignalcanbeapproximatedas:

$$s(t) = Ac \cos[\omega ct + \theta m(t)]$$

$$= Ac \cos \omega ct \cos \theta m(t) - Ac \sin \omega ct \sin\theta m(t)$$

$$\simeq Ac \cos \omega ct - Ac\theta m(t) \sin \omega ct$$

or in complex notation

$$st= A_C RE\{e^{jwct} (1 + j\theta mt\}$$

ThisissimilartotheAMsignalexceptthatthediscretecarriercomponentAccoswc(t)is90°out of phase with the sinusoid Ac sinwc(t) multiplying the phase angle θm(t). The spectrum of narrow band FM is similar to that ofAM.

- ✓ **The Bandwidth of an FMSignal:**

The following formula, known as Carson‘s rule is often used as an estimate of the FM signal bandwidth: BT = 2(Δf + fm) Hz

whereΔf is the peak frequency deviation and fm is the maximum baseband message frequencycomponent.

- ✓ **FM Demodulation by a Frequency Discriminator:**

A frequency discriminator is a device that converts a received FM signal into a voltage that is proportional to the instantaneous frequency of its input without using a local oscillator and, consequently, in a non coherentmanner.

• When the instantaneous frequency changes slowly relative to the time-constantsofthe filter, a quasi-static analysis can beused.

• In quasi-static operation the filter output has the same instantaneous frequency as the input but with an envelope that varies according to the amplitude response of the filter at the instantaneousfrequency.

• Theamplitudevariationsarethendetectedwithanenvelopedetectorliketheonesused for AMdemodulation.

- ✓ **An FM Discriminator Using the Pre-Envelope:**

When θm(t) is small and band-limited so that cos θm(t) and sinθm(t) are essentially band-limited signals with cut off frequencies less than fc, the pre-envelope of the FM signal is

$$s+(t) = s(t) + j\hat{}s(t) = Ace^{j}(\omega ct+\theta m(t))$$ The

angle of the pre-envelope is φ'(t) = arctan[ˆs(t)/s(t)] = ωct + θm(t) The

derivative of the phase is =ωct+ kθm(t)

$$\frac{d\varphi t}{dt} = \frac{s t d}{dt^{st}} - \frac{s \frac{d}{dt} st}{s2t+s^2(t)} = \omega ct + k\omega m\ (t)$$

which is exactly the instantaneous frequency. This can be approximated in discrete-time by using FIR filters to form the derivatives and Hilbert transform. Notice that the denominator is the squared envelope of the FM signal.

This formula can also be derived by observing,

$$\frac{d}{dt} st = \frac{d}{dt} AC\cos\omega ct + \theta mt = -AC\omega ct + k\omega mt\sin[\omega ct + \theta mt]$$

$$\frac{d}{dt}\hat{s}t = \frac{d}{dt} AC\sin\omega ct + \theta mt = AC\omega ct + k\omega mt\cos[\omega ct + \theta mt]$$

So,

$$s t d \frac{}{dt}\hat{s}(t) - \frac{\hat{s}(t) d}{} = AC^2 \omega ct + k\omega mt * \cos 2[wct + \theta mt + \sin 2[wct + \theta mt\ dtst$$

The bandwidth of an FM discriminator must be at least as great as that of the received FM signal which is usually much greater than that of the baseband message. This limits the degree of noise reduction that can be achieved by preceding the discriminator by a bandpass receive filter.

- ✓ **Using a Phase-Locked Loop for FMDemodulation:**

A device called a phase-locked loop (PLL) can be used to demodulate an FM signal with better performance in a noisy environment than a frequency discriminator. The block diagram of a discrete-time version of a PLL as shown in figure,

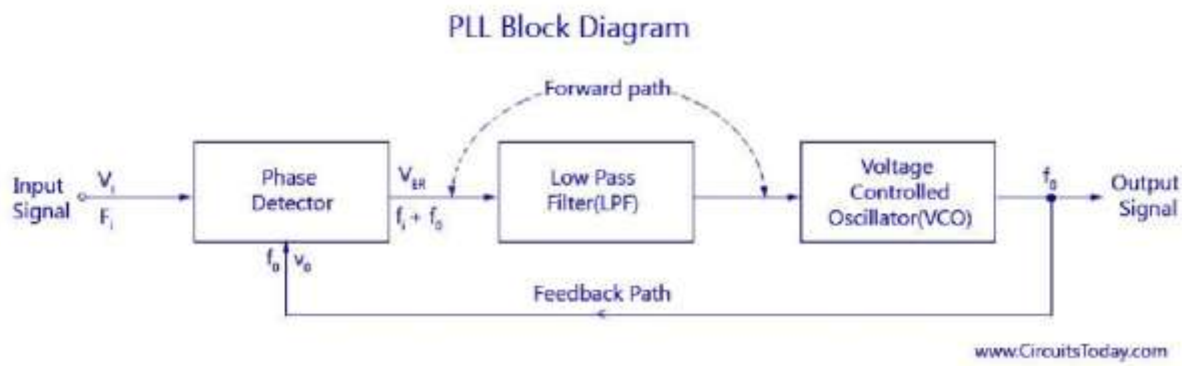


FIG 2.2 PLL Block diagram

The block diagram of a basic PLL is shown in the figure below. It is basically a flip flop consisting of a phase detector, a low pass filter (LPF),and a Voltage Controlled Oscillator (VCO) The input signal Vi with an input frequency fi is passed through a phase detector. A phase detector

basically a comparator which compares the input frequency fiwith the feedback frequency fo .The phase detector provides an output error voltage Ver (=fi+fo),which is a DC

voltage. This DC voltage is then passed on to an LPF. The LPF removes the high frequency noise and produces a steady DC level, Vf (=Fi-Fo). Vf also represents the dynamic characteristics of the PLL.

The DC level is then passed on to a VCO. The output frequency of the VCO (fo) is directly proportional to the input signal. Both the input frequency and output frequency are compared and adjusted through feedback loops until the output frequency equals the input frequency. Thus the

PLL works in these stages – free-running, capture and phase lock.

As the name suggests, the free running stage refer to the stage when there is no input voltage applied. As soon as the input frequency is applied the VCO starts to change and begin producing an output frequency for comparison this stage is called the capture stage. The frequency comparison stops as soon as the output frequency is adjusted to become equal to the input frequency. This stage is called the phase locked state.

- ✓ **PLLPerformance:**

• The frequency response of the linearized loop characteristics of a band-limiteddifferentiator.

• The loop parameters must be chosen to provide a loop bandwidth that passes the desired baseband message signal but is as small as possible to suppress out-of-bandnoise.

• The PLL performs better than a frequency discriminator when the FM signal is corrupted by additive noise. The reason is that the bandwidth of the frequency discriminator must be large enough to pass the modulated FM signal while the PLL bandwidth only has to be large enough to pass the baseband message. With wideband FM, the bandwidth of the modulated signal can be significantly larger than that of the basebandmessage.

- ✓ **Bandwidth of FM PLL vs. CostasLoop:**

The PLL described in this experiment is very similar to the Costas loop presented in coherent demodulation of DSBSC-AM. However, the bandwidth of the PLL used for FM demodulation must be large enough to pass the baseband message signal, while the Costas loop is used to generate a stable carrier reference signal so its bandwidth should be very small and just wide enough to track carrier drift and allow a reasonable acquisition time.

2.3 WIDE-BAND FM:

$$st= ACcos(2\pi fct + \varphi(t)$$

Finding its FT is not easy: $\phi(t)$ is inside the cosine.

To analyze the spectrum, we use complex envelope.

s(t) can be written as: Consider single tone FM: $s(t) = ACcos(2\pi fct + \beta sin2\pi fm(t))$

Wideband FM is defined as the situation where the modulation index is above 0.5. Under these circumstances the sidebands beyond the first two terms are not insignificant. Broadcast FM stations use wideband FM, and using this mode they are able to take advantage of the wide bandwidth available to transmit high quality audio as well as other services like a stereo channel, and possibly other services as well on a single carrier.

The bandwidth of the FM transmission is a means of categorising the basic attributes for the signal, and as a result these terms are often seen in the technical literature associatedwith frequency modulation, and products using FM. This is one area where the figure for modulation index is used.

- ✓ **GENERATION OF WIDEBAND FMSIGNALS:**

Indirect Method for Wideband FM Generation:

Consider the following blockdiagram

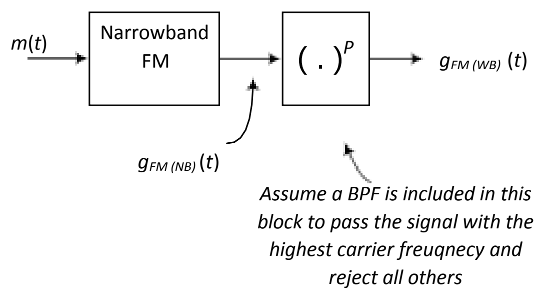


FIG 2.3 Block diagram of FM generation

A narrowband FM signal can be generated easily using the block diagram of the narrowband FM modulator that was described in a previous lecture. The narrowband FM modulator generates a narrowband FM signal using simple components such as an integrator (an OpAmp), oscillators, multipliers, and adders. The generated narrowband FM signal can be converted to a wideband FM signal by simply passing it through a non–linear device with power P. Both the carrier frequency and the frequency deviation Δf of the narrowband signal are increased by a factor P. Sometimes, the desired increase in the carrier frequency and the desired increase in Δf are different. In this case, we increase Δf to the desired value and use a frequency shifter (multiplication by a sinusoid followed by a BPF) to change the carrier frequency to the desired value.

- ✓ **System1:**

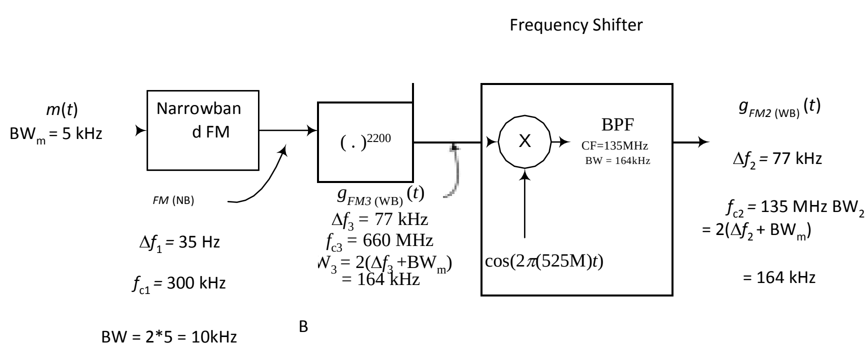


FIG 2.4 Block diagram of FM generation

In this system, we are using a single non–linear device with an order of 2200 or multiple devices withacombinedorderof2200.Itisclearthattheoutputofthenon–lineardevicehasthecorrect Δf but an incorrect carrier frequency which is corrected using a the frequency shifter with an oscillator that has a frequency equal to the difference between the frequency of its input signaland the desired carrier frequency. We could also have used an oscillator with a frequency that is the sum of the frequencies of the input signal and the desired carrier frequency. This system is characterized by having a frequency shifter with an oscillator frequency that is relatively large.

- ✓ **System2:**

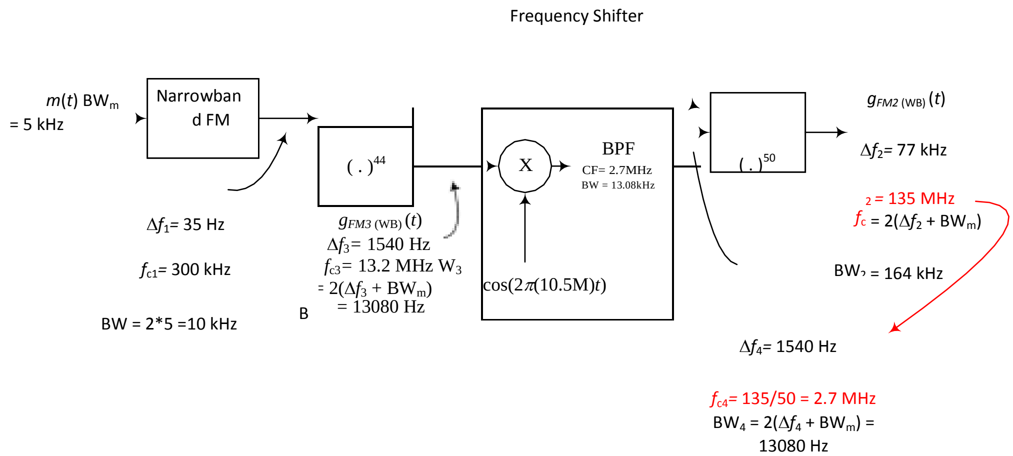


FIG 2.5 Block diagram of FM generation

In this system, we are using two non–linear devices (or two sets of non–linear devices) with orders 44 and 50 (44*50 = 2200). There are other possibilities for the factorizing 2200 such as 2*1100,4*550,8*275,10*220.. Depending on the available components, one of these factorizations may be better than the others. In fact, in this case, we could have used the same factorization but put 50 first followed by 44. We want the output signal of the overall system to be as shown in the block diagram above, so we have to insure that the input to the non–linear device with order 50 has the correct carrier frequency such that its output has a carrier frequency of 135 MHz. This is done by dividing the desired output carrier frequency by the non–linearity order of 50, which gives 2.7 Mhz. This allows us to figure out the frequency of the require oscillator which will be in this case either 13.2–2.7 = 10.5 MHz or 13.2+2.7 = 15.9 MHz. We are generally free to choose which ever we like unless the available components dictate the use of one of them and not the other. Comparing this system with System 1 shows that the frequency of the oscillator that is required here is significantly lower (10.5 MHz compared to 525 MHz), which is generally an advantage.

2.4 TRANSMISSIONBANDWIDTH:

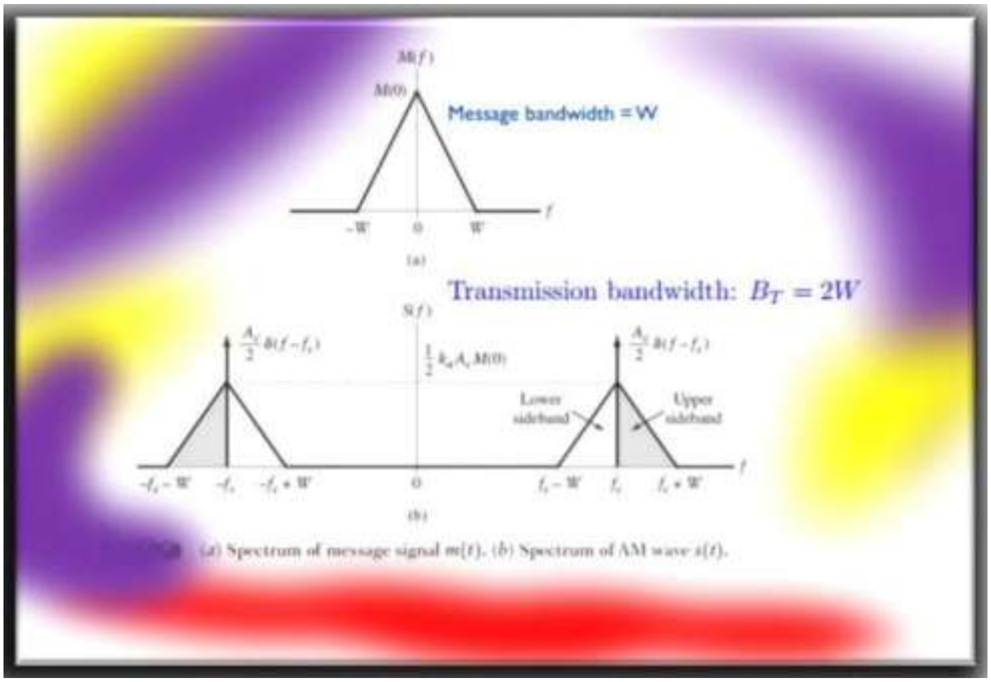


FIG 2.6 Spectrum of FM Bandwidth

2.5 FMTRANSMITTER

- ✓ **Indirect method (phase shift) of modulation**

The part of the Armstrong FM transmitter (Armstrong phase modulator) which is expressed in dotted lines describes the principle of operation of an Armstrong phase modulator. It should be noted, first that the output signal from the carrier oscillator is supplied to circuits that perform the task of modulating the carrier signal. The oscillator does not change frequency, as is the case of direct FM. These points out the major advantage of phase modulation (PM), or indirect FM, over direct FM. That is the phase modulator is crystal controlled for frequency.

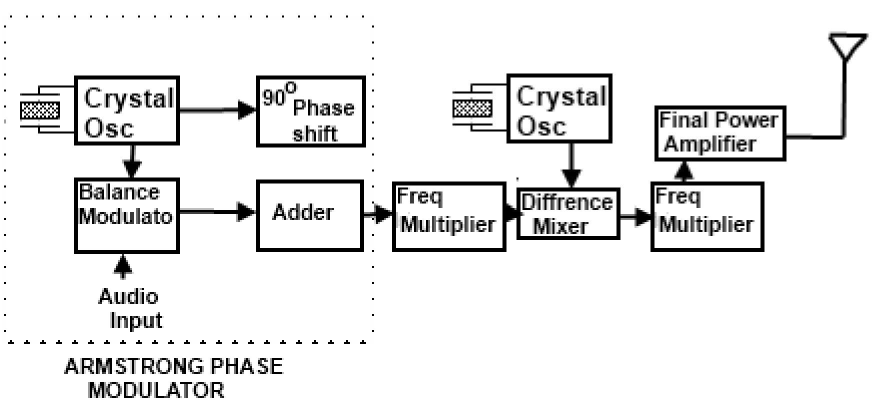


FIG 2.7 Armstrong Modulator

The crystal-controlled carrier oscillator signal is directed to two circuits in parallel. This signal (usually a sine wave) is established as the reference past carrier signal and is assigned a value 0°.The balanced modulator is an amplitude modulator used to form an envelope of double side-bands and to suppress the carrier signal (DSSC). This requires two input signals, the carrier signal and the modulating message signal. The output of the modulator is connected to the adder circuit; here the 90° phase-delayed carriers signal will be added back to replace the suppressed carrier. The act of delaying the carrier phase by 90° does not change the carrier frequency or its wave-shape. This signal identified as the 90° carriersignal.

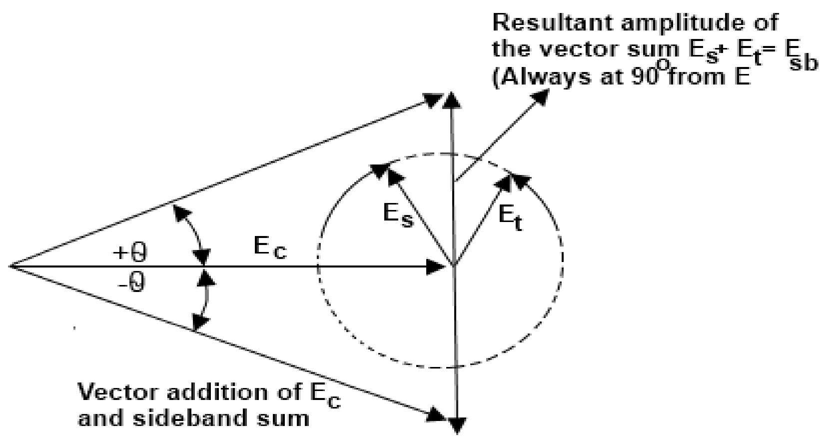


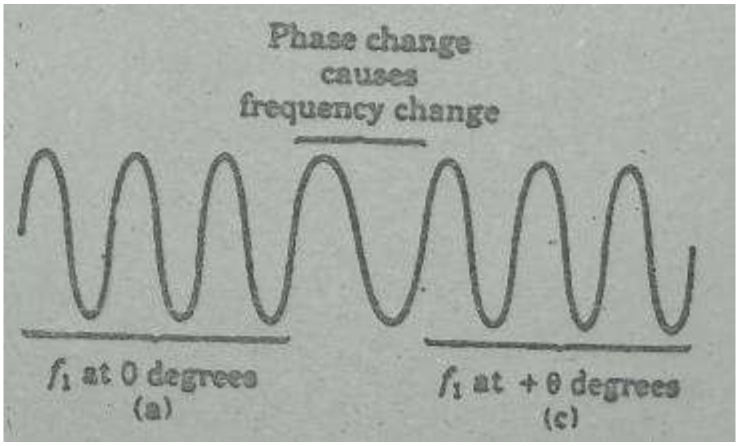
FIG 2.8 Phasor diagram of Armstrong Modulator

$$\% \text{ of modulation} = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} \times 100$$

The carrier frequency change at the adder output is a function of the output phase shift and is found by. fc = $\Delta\theta f_s$ (in hertz)

When θ is the phase change in radians and f_s is the lowest audio modulating frequency. In most FM radio bands, the lowest audio frequency is 50Hz. Therefore, the carrier frequency change at the adder output is 0.6125 x 50Hz = ± 30Hz since 10% AM represents the upper limit of carrier voltage change, then ± 30Hz is the maximum deviation from the modulator for PM.

The 90° phase shift network does not change the signal frequency because the components and resulting phase change are constant with time. However, the phase of the adder output voltage is in a continual state of change brought about by the cyclical variations of the message signal, and during the time of a phase change, there will also be a frequencychange.



In figure. (c). during time (a), the signal has a frequency f_1, and is at the zero reference phase. During time (c), the signal has a frequency f_1 but has changed phase to θ. During time (b) when the phase is in the process of changing, from 0 to θ. the frequency is less than f_1.

- ✓ **Using Reactance modulator directmethod**

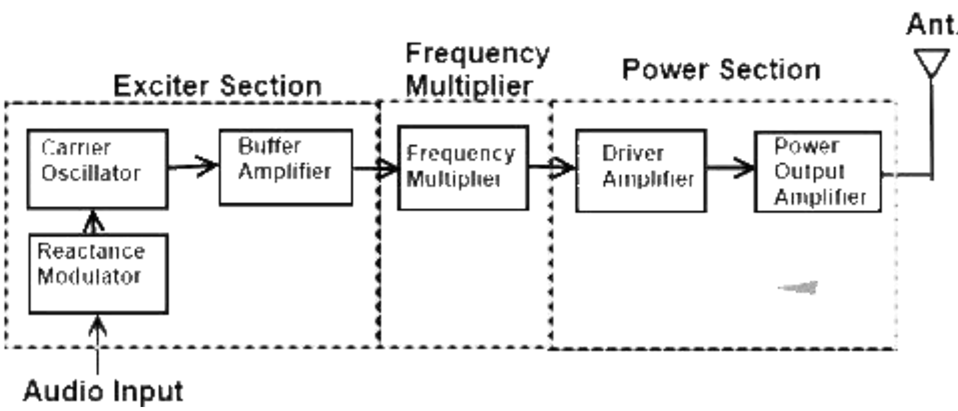


FIG 2.9 Reactance Modulator

The FM transmitter has three basic sections.

1. The exciter section contains the carrier oscillator, reactance modulator and thebuffer amplifier.
2. The frequency multiplier section, which features several frequencymultipliers.
3. The poweroutputection, which includes alow-
   level power amplifier, the final power amplifier, and the impedance matching network to properly load the power section with the antenna impedance.

The essential function of each circuit in the FM transmitter may be described as follows.

- ✓ **TheExciter**
  1. The function of the carrier oscillator is to generate a stable sine wave signal at the rest frequency, when no modulation is applied. It must be able to linearly change frequency when fully modulated, with no measurable change inamplitude.
  2. The buffer amplifier acts as a constant high-impedance load on the oscillator to help stabilize the oscillator frequency. The buffer amplifier may have a smallgain.
  3. The modulator acts to change the carrier oscillator frequency by application of the message signal. The positive peak of the message signal generally lowers the oscillator's frequency to a point below the rest frequency, and the negativemessage peak raises the oscillator frequency to a value above the rest frequency. Thegreater the peak-to-peak message signal, the larger the oscillatordeviation.
- ✓ Frequency multipliers are tuned-input, tuned-output RF amplifiers in which the output resonant circuit is tuned to a multiple of the input frequency. Common frequency multipliers are 2x, 3x and 4x multiplication. A 5x Frequency multiplier is sometimes seen, but its extreme low efficiency forbids widespread usage. Note that multiplicationis by whole numbers only. There can not a 1.5x multiplier, forinstance.
- ✓ The final power section develops the carrier power, to be transmitted and often has a low-power amplifier driven the final power amplifier. The impedance matchingnetwork is the same as for the AM transmitter and matches the antenna impedance to the correct load on the final overamplifier.
- ✓ **FrequencyMultiplier**

A special form of class C amplifier is the frequency. multiplier. Any class C amplifier is capable of performing frequency multiplidàtion if the tuned circuit in the collector resonates at some integer multiple of the inputfrequency.

For example a frequency doubler can be constructed by simply connecting a parallel tuned circuit in the collector of a class C amplifier that resonates at twice the input frequency. When the collector current pulse occurs, it excites or rings the tuned circuit at twice the input frequency. A current pulse flows for every other cycle of the input.

A Tripler circuit is constructed in the same way except that the tuned circuit resonates at 3 times the input - frequency. In this way, the tuned circuit receives one input pulse for every three cycles of oscillation it produces Multipliers can be constructed to increase the input frequency by any integer factor up to approximately 10. As' the multiplication factor gets higher, the power output of the multiplier decreases. For most practical applications, the best result is obtained with multipliers of 2 and 3.

Another way to look the operation of class C multipliers is .to .remember that the non-sinusoidal current pulse is rich in harmonics. Each time the pulse occurs, the second, third, fourth, fifth, and higher harmonics are generated. The purpose of the tuned circuit in the collector is to act as a filter to select the desired harmonics.

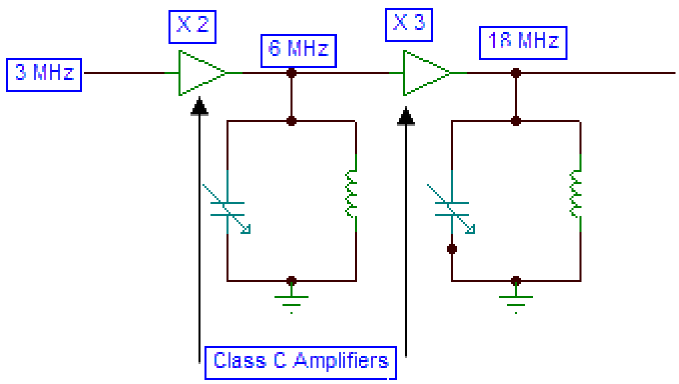


FIG 2.10 Block Diagram of Frequency Multiplier -1

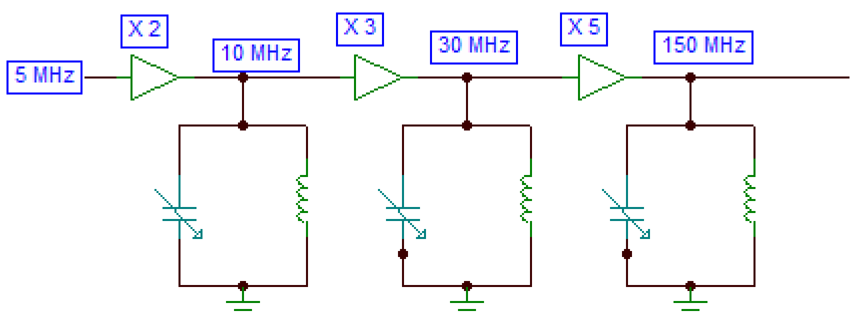


FIG 2.10 Block Diagram of Frequency Multiplier -2

In many applications a multiplication factor greater than that achievable with a single multiplier stage is required. In such cases two or more multipliers are cascaded to produce an overall multiplication of 6. In the second example, three multipliers provide an overall multiplication of 30. The total multiplication factor is simply the product of individual stage multiplicationfactors.

- ✓ **ReactanceModulator**

The reactance modulator takes its name from the fact that the impedance of the circuit acts as a reactance (capacitive or inductive) that is connected in parallel with the resonant circuit of the Oscillator. The varicap can only appear as a capacitance that becomes part of the frequency determining branch of the oscillator circuit. However, other discrete devices can appear as a capacitor or as an inductor to the oscillator, depending on how the circuit is arranged. A colpitts oscillator uses a capacitive voltage divider as the phase-reversing feedback path and would most likely tapped coil as the phase-reversing element in the feedback loop and most commonly uses a modulator that appears inductive.

2.6 COMPARISION OF VARIOUSMODULATIONS:

- ✓ **Comparisons of Various Modulations:**

Amplitude modulation	Frequency modulation	Phase modulation
1. Amplitude of the carrier wave is varied in accordance with the message signal.	1. Frequency of thecarrier wave is varied in accordance with the message signal.	1. Phase of the carrier wave is varied in accordance with the message signal.
2.Much affected by noise.	2.More immune to the noise.	2. Noise voltage is constant.
3.System fidelity is poor.	3.Improved system fidelity.	Improved system fidelity.
4.Linear modulation	4.Non Linear modulation	4.Non Linear modulation

- ✓ **Comparisons of Narrowband and WidebandFM:**

Narrowband FM	Wideband FM
Modulation index > 1.	Modulation index < 1.
Occupies more bandwidth.	Occupies less bandwidth.
Used in entertainment broadcastings	Used in FM Mobile communication services.

2.7 APPLICATION & ITS USES:

- Magnetic TapeStorage.
- Sound
- Noise FmReduction
- Frequency Modulation (FM) stereo decoders, FM Demodulation networks for FM operation.
- Frequency synthesis that provides multiple of a reference signalfrequency.
- Used in motor speed controls, trackingfilters.

1. Frequency modulation(FM), with its digital correspondence frequency-shift

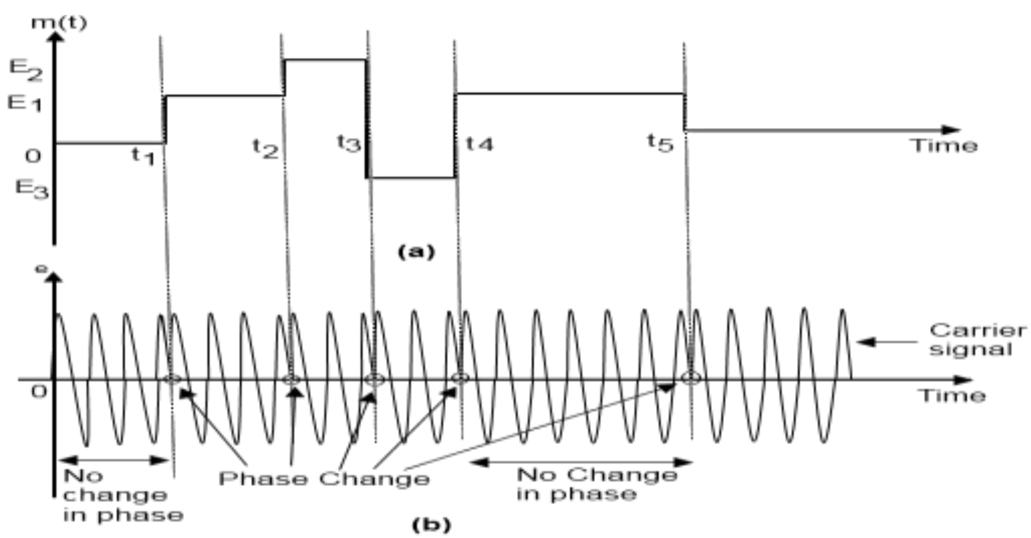
keying(FSK).

2. Phase modulation (PM), with its digital correspondence phase-shift keying(PSK).
3. In PM, the total phase of the modulated carrier changes due to the changes in the instantaneous phase of the carrier keeping the frequency of the carrier signalconstant.
4. A device called a phase-locked loop (PLL) can be used to demodulate an FM signal with better performance in a noisy environment than a frequencydiscriminator.
5. As in other modulation systems, the value of the modulation index indicates by how much the modulated variable varies around its unmodulatedlevel.
6. Amplitude Limiters, are used to keep the output constant despite changes in the input signal to removedistortion.

2.8 PHASEMODULATION:

Phase Modulation (PM) is another form of angle modulation. PM and FM are closely related to each other. In both the cases, the total phase angle θ of the modulated signal varies. In an FM wave, the total phase changes due to the change in the frequency of the carrier corresponding to the changes in the modulating amplitude.

In PM, the total phase of the modulated carrier changes due to the changes in the instantaneous phase of the carrier keeping the frequency of the carrier signal constant. These two types of modulation schemes come under the category of angle modulation. However, PM is not as extensively used as FM.



Phase Modulation Waveforms

(a) Non-Sinusoidal Modulating Signal,m(t). (b) Phase-Modulated Carrier Signal e

At time t1, the amplitude of m(t) increases from zero to E1. Therefore, at t1, the phase modulated carrier also changes corresponding to E1, as shown in Figure (a). This phase remains to this attained value until time t2, as between t1 and t2, the amplitude of m(t) remains constant at El. At t2, the amplitude of m(t) shoots up to E2, and therefore the phase of the carrier again increases corresponding to the increase in m(t). This new value of the phase attained at time t2remains constant up to time t3. At time t3, m(t) goes negative and its amplitude becomes E3. Consequently, the phase of the carrier also changes and it decreases from the previous value attained at t2.

The decrease in phase corresponds to the decrease in amplitude of m(t). The phase of the carrier remains constant during the time interval between t3 and t4. At t4, m(t) goes positive to reach the amplitude El resulting in a corresponding increase in the phase of modulated carrier at time t4. Between t4 and t5, the phase remains constant. At t5 it decreases to the phase of the unmodulated carrier, as the amplitude of m(t) is zero beyondt5.

- ✓ **Equation of a PM Wave:**

To derive the equation of a PM wave, it is convenient to consider the modulating signal as a pure sinusoidal wave. The carrier signal is always a high frequency sinusoidal wave. Consider the modulating signal, em and the carrier signal ec, as given by, equation 1 and 2, respectively.

$$e_m = E_m \cos \omega_m t \text{------------}(1)$$

$$e_c = E_c \sin \omega_c t \text{---------------}(2)$$

The initial phases of the modulating signal and the carrier signal are ignored in Equations (1) and (2) because they do not contribute to the modulation process due to their constant values. After PM, the phase of the carrier will not remain constant. It will vary according to the modulating signal em maintaining the amplitude and frequency as constants. Suppose, after PM, the equation of the carrier is represented as:

$$e = E_c \text{ Sin } \theta \text{ ------------------}(3)$$

Where θ, is the instantaneous phase of the modulated carrier, and sinusoid ally varies in proportion to the modulating signal. Therefore, after PM, the instantaneous phase of the modulated carrier can be written as:

$$\theta = \omega_c t + K_p e_m \text{-------------------}(4)$$

Where, kp is the constant of proportionality for phase modulation. Substituting Equation (1) in Equation (4), yonget:

$$\theta = \omega_c t + K_p E_m \text{ Cos } \omega_m t \text{ ---------------------}(5)$$

In Equation (5), the factor, kpEm is defined as the modulation index, and is given as:

$$m_p = K_p E_m \text{ ------------------------(6)}$$

where, the subscript p signifies; that mp is the modulation index of the PM wave. Therefore, equation (5) becomes

$$\theta = \omega_c t + m_p \cos \omega_m t \text{ ---------------------(7)}$$

Substituting Equation (7) and (3), you get:

$$e = E_c \sin (\omega_c t + m_p \cos \omega_m t) \text{ --------------------(8)}$$