JEPPIAAR INSTITUTE OF TECHNOLOGY
"Self-Belief| Self Discipline | Self Respect"

DEPARTMENT
OF
ELECTRICAL AND ELECTRONICS ENGINEERING

LECTURE NOTES

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Prepared by
Dr. Prajith Prabhakar
Assistant Professor / EEE

UNIT IV MAGNETIC FORCES AND MATERIALS

: Force On A Moving Charge:

In electric field, force on a charged particle is

F=QE

Force is in the same direction as the electric field intensity
(positive charge)

A charged particle in motion in a magnetic field force magnitude is proportional to the product of magnitudes of the charge Q, its velocity V and the flux density B and to the sine of the angle between the vectors V and B.

The direction of force is perpendicular to both V and B and is given by a unit vector in the direction of V x B.

The force may therefore be expressed as

F=QV x B

Force on a moving particle due to combined electric and magnetic fields is obtained by superposition.

$$F=Q\,(E + V \times B)$$

This equation is known as Lorentz force equation.

Force On A Differential Current Element:

The force on a charged particle moving through a steady magnetic field may be written as the differential; force exerted on a differential element of charge.

$dF \quad dQ$

Convection current density in terms of the velocity of the volume charge density

Differential element of charge may also be expressed in terms of volume charge density.

$$dQ \quad {}_{v}\, dv$$

Thus,

$$dF \quad {}_{v}\, dvVxB$$

$$dF$$
$$JxBdv$$

JdV is the differential current element

$$Jdv \quad Kds \quad IdL$$

Lorentz force equation may be applied to surface current density.

$$dF$$
$$KxBds$$

Differential current element

$$dF \quad IdLxB$$

Integrating the above equations over a volume, surface open or closed

$$F \quad JxBdv$$
$$vol$$

$$F$$
$$KxBds$$
$$s$$

To a straight conductor in a uniform magnetic field

F= $IdLxB$

$I\ BxdL$

$$F \quad ILxB$$

The magnitude of the force is given by the familiar equation

F=BILsin

Force on a current-carrying conductor

Charges confined to wires can also experience a force in a magnetic field. A current (I) in a magnetic field (**B**) experiences a force (**F**) given by the equation $\mathbf{F} = I\,\mathbf{l} \times \mathbf{B}$ or $F = IlB \sin\theta$, where **l** is the length of the wire, represented by a vector pointing in the direction of the current. The direction of the force may be found by a right-hand rule similar to the one shown in Figure . In this case, point your thumb in the direction of the current—the direction of motion of positive charges. The current will experience no force if it is parallel to the magnetic field.

Force and Torque on a current loop

A loop of current in a magnetic field can experience a torque if it is free to turn. Figure (a) depicts a square loop of wire in a magnetic field directed to the right. Imagine in Figure (b) that the axis of the wire is turned to an angle (θ) with the magnetic field and that the view is looking down on the top of the loop. The *x* in a circle depicts the current traveling into the page away from the viewer, and the dot in a circle depicts the current out of the page toward the viewer.

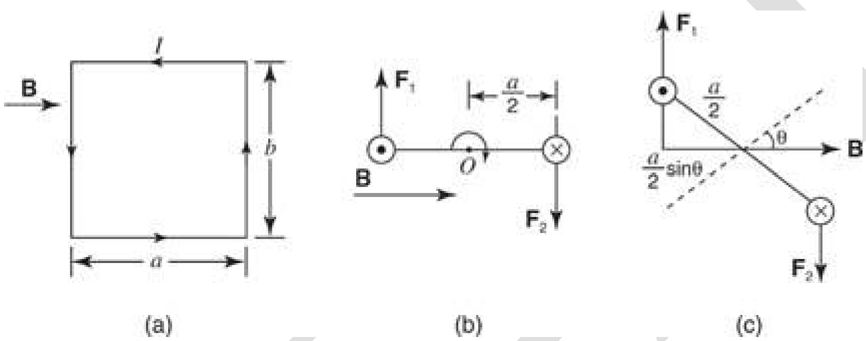

Figure 4.1
(a) Square current loop in a magnetic field **B**. (b) View from the top of the current loop. (c) If the loop is tilted with respect to **B**, a torque results.

MAGNETIC MATERIALS:

All material shows some magnetic effects. In many substances the effects are so weak that the materials are often considered to be non magnetic.

A vacuum is the truly nonmagnetic medium.

Material can be classified according to their magnetic behavior into

- Diamagnetic
- Paramagnetic
- Ferromagnetic

DIAMAGNETIC:

In diamagnetic materials magnetic effects are weak. Atoms in which the small magnetic fields produced by the motion of the electrons in their orbit and those produced by the electron spin combine to produce a net field of zero.

The fields produced by the electron motion itself in the absence of any external magnetic field.

This material as one in which the permanent magnetic moment m_0 of each atom is zero. Such a material is termed diamagnetic.

PARAMAGNETIC:

In paramagnetic materials the magnetic moments of adjacent atoms align in opposite directions so that the net magnetic moment of a specimen is nil even in the presence of applied field.

FERROMAGNETIC:

In ferromagnetic substance the magnetic moments of adjacent atoms are also aligned opposite, but the moments are not equal, so there is a net magnetic moment.

It is less than in ferromagnetic materials.

The ferrites have a low electrical conductivity, which makes them useful in the cores of ac inductors and transformers.

Since induced currents are less and ohmic losses are reduced.

BOUNDARY CONDITIONS:

A boundary between two isotropic homogeneous linear materials with permeability μ_1 and μ_2.

The boundary condition on the normal components is determined by allowing the surface to cut a small cylindrical gaussian surface.

Applying gauss' s law for the magnetic field.

$$\oint_s B.ds = 0$$

We find that

$$B_{N1}\,\Delta S - B_{N2}\,\Delta S = 0$$

$$B_{N1} = B_{N2}$$

$$\frac{\mu_1}{\mu_2} H_{N1} = H_{N2}$$

The normal component of B is continuous, but the normal component of H is discontinuous by the ratio $\frac{1}{2}$.

The relationship between the normal components of M,is fixed once the relationship between the normal components of H is known .

For linear magnetic materials, the result is written simply as

M_{N2}

Next, Ampere' s circuital law

$$H.dL \quad I$$

Is applied about a small closed path in a plane normal to the boundary surface.

Taking trip around the path, we find that

$$H_{t1} \quad L \quad H_{t2} \quad L \quad K \; L$$

Boundary may carry a surface current K whose component normal to the plane of the closed path is K.Thus

$$H_{t1} \quad H_{t2} \qquad K$$

The direction are specified more exactly by using the cross product to identify the tangential components,

$$(H_1 \quad H_2) x a_{N12} \qquad K$$

Where a_{N12} is the normal at the boundary directed from region 1 to region 2.

An equivalent formulation in term of the vector tangential components may be more convenient for H:

$$H_{t1} \quad H_{t2} \qquad K \text{ x } a_{N12}$$

For tangential B, we have

$$\frac{B_{t1}}{1} \quad \frac{B_{t2}}{2} \qquad K$$

INDUCTANCE:

self inductance and mutual inductance:

Like capacitance, inductance L is a property of a physical arrangement of conduc- tors. It is a measure of magnetic flux which links the circuit when a current I flows in the circuit. It is also a measure of how much energy is stored in the magnetic field of an inductor, such as a coil, solenoid, etc.

The definition of inductance rests on the concept of flux linkage. It is not a very precise concept unless one is willing to introduce a complicated topological description. For our purposes it will be sufficient to define *flux linkage* A as the flux that links all the circuit, multiplied by the number of turns N. For example, in the case of the solenoid shown in Fig. , flux linkage will be given by

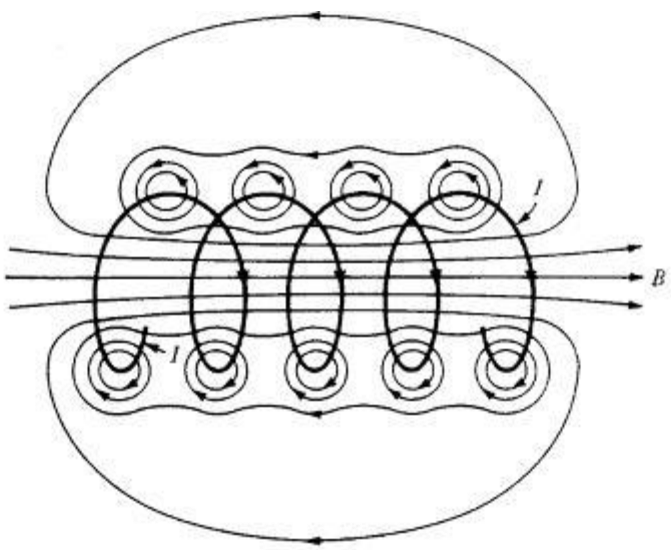

Fig 4.2 solenoid flux linkage

A = N¢ = N JJ B· **dA ~** *NBA* Wb

that is, only the flux that goes through the inside of the solenoid and therefore links all turns is used. The small flux loops about each turn are ignored in a first-order analysis because they link only one or two turns and flow through a small area. The area *A* is that area through which the flux that links all turns flows. For the solenoid of Fig. 7.7, a good approximation to *A* is the cross section of the solenoid

The unit of inductance is the henry (H). Inductors for filter applications in power supplies are usually wire-wound solenoids on an iron core with inductances in the range from 1 to 10 H. Inductors found in high-frequency circuits are air-core solenoids with values in the millihenry (mH) range. The definition for inductance, (7.28), even though it is derived for steady currents, is valid up to very high frequencies.
Let us calculate *L* for some useful geometries.

Solenoid

A good approximation of the *B* field in a solenoid that links all turns is the *B* field at the center of the solenoid; that is, *B* = *J10 Ni]l* from (7.22) or (6.40). There is some leakage at the ends of the solenoid (recall that the value of the *B* field drops to one-half at the ends), which we will ignore because it occurs mainly at the ends. The inductance *L* of a solenoid is therefore

L= A = *NBA* = *J1oN2A*

where *I* is the length and *A* is the cross section of the solenoid.
If we have a short solenoid of N turns, that is, one where the length *I* is smaller

TOROID

For example, the inductance of a 2000-turn toroid having a cross-sectional area of 1 em? and mean radius of 5 em is

L = *(4n* x 10-^{7} H/m)(2000)2(10-4 *m2)/2n(O.05* m) = 1.6 mH

If the toroid were filled with iron instead of air, the inductance could be increased many thousand fold.

Note that we have neglected the variation of *B* across the cross section of the toroid. By using an average *r,* as, for example, *r = (a + b)/2,* we have in effect used an average value of *B* in the calculation of inductance. **I**f this is not sufficiently accurate, the variation of *B* should be considered by integrating (7.25) between *a* and *b.*

Coaxial Transmission Line

The student usually does not have any difficulty in grasping the concept ofinduc- tance as long as the geometries involve windings (such as in coils and toroids). In the following examples flux linkage is used in a broader sense and should clarify that concept further. Figure 7.8 shows a longitudinal and transverse cross section of a coaxial line (already considered in Sec. 5.5 when the capacitance' per unit length was calculated). The current ***I*** flows in the center conductor and returns

The inductance per unit length *L/Z* is

L A *11 0* *b*
\- = - = -In - *Him* *(7.34) Z* *IZ* *2n* *a*

For an air-filled coaxial line the above expression can be written as *LIZ* = 0.2 In *(bla)* microhenrys per meter *(I1H/m).*

We have ignored the contribution of the magnetic field inside the inner con- ductor for several reasons. **t** First, as shown in Fig. 7.4, the magnetic flux within the inner conductor (assuming the current ***I*** is distributed uniformly throughout the cross section of the inner conductor, which is a valid assumption for direct current and for current at low frequencies) links only a fraction of that con- ductor; that fraction is proportional to *(rlaf* because *Iatr* = *(rla)2I.* Second, at the higher frequencies the current is effectively confined to a thin layer (skin depth) at *r* = *a* for the inner conductor and at *r* = *b* for the outer. Third, most practical transmission lines use a small inner conductor and a thin-walled outer conductor. Hence the flux linkages within the conductors can be neglected, and (7.34) is an accurate expression for inductance per unit length.

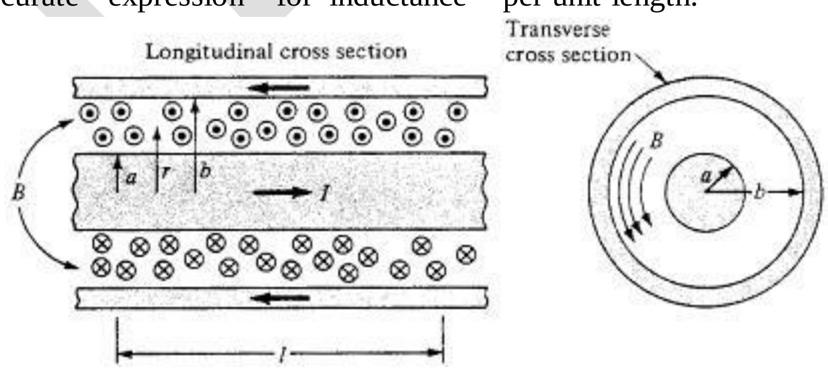

Fig: 4.3 coxial transmission-cross section

We have approximated the upper limit *d* - *a* by *d* because for practical transmis- sion lines *d* ~ *a*. This approximation also accounts for the flux from the lower conductor which partly links the current inside the upper wire. As a matter of fact it can be shown that the replacement of *d* - *a* by *a* gives an exact result for the flux linkages. t The inductance per unit length *LII.*
which is the desired result and gives the total stored magnetic energy in an inductance *L* carrying current *I*. For example, a solenoid with an inductance of
8 H and a current of -!- A has an energy stored of *W* = *-!-LI2* = 1 J.

Just as a capacitor stores energy in its electric field, so does an inductor in its magnetic field. A measure of the effectiveness of energy storage in the electric field is the capacitance ($W = \frac{1}{2}CV^2$); in the magnetic field, it is the inductance ($W = \frac{1}{2}LI^2$).

To derive the expression for the storage of energy in the magnetic field of an Inductor

$$V = RI + L\frac{dI}{dt} + \frac{1}{C}\int I\,dt$$

Since the instantaneous power is $P = VI = dW/dt$, we can obtain the energy in the inductor L by integrating power $P_L = V_L I = LI\,dI/dt$. We obtain

$$W = \int_0^t P_L\,dt = L\int_0^t I\frac{dI}{dt}\,dt = L\int_0^I I\,dI = \tfrac{1}{2}LI^2$$

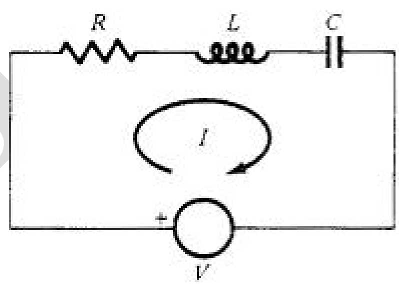

Fig:4.4 An *RLC* series circuit. Over a large range of current

which is the desired result and gives the total stored magnetic energy in an inductance L carrying current I. For example, a solenoid with an inductance of 8 H and a current of $\frac{1}{2}$ A has an energy stored of $W = \frac{1}{2}LI^2 = 1$ J.

4.8 ENERGY STORED INA MAGNETIC FIELD:

the charge on the capacitor plates or the electric field between the plates. We determined that the energy per unit volume in the electric E field is $w = \frac{1}{2}\varepsilon E^2$. Similarly, energy is stored in a magnetic field; the energy density will turn out to be $\frac{1}{2}\mu H^2$. To show this in the simplest way, we look for an inductor with a uniform field which is well confined. A long solenoid is suitable as is a toroid which has the magnetic field confined entirely to the region within the windings.

A toroid with a diameter that is large compared with that of its cross section

in the magnetic field is then

$$\boxed{w = \frac{W}{v} = \frac{\frac{1}{2}LI^2}{A2\pi r} = \frac{1}{2}\mu_0\left(\frac{NI}{2\pi r}\right)^2 = \frac{B^2}{2\mu_0} = \frac{1}{2}\mu_0 H^2}$$

where A = cross section

$2\pi r$ = mean circumference

v = volume, $v = A2\pi r$

L = inductance, $L = \mu_0 N^2 A/2\pi r$

B = magnetic field of the toroid, $B = \mu_0 H = \mu_0 NI/2\pi r$

We state now without proof that (7.45) expresses the energy density in any magnetic field. The total magnetic energy stored in the field of an inductor can therefore be obtained by integrating the magnetic energy density:

$$W = \iiint w\, dv$$

The integration is over the entire volume in which the field exists and must, of course, be equal to $\frac{1}{2}LI^2$.

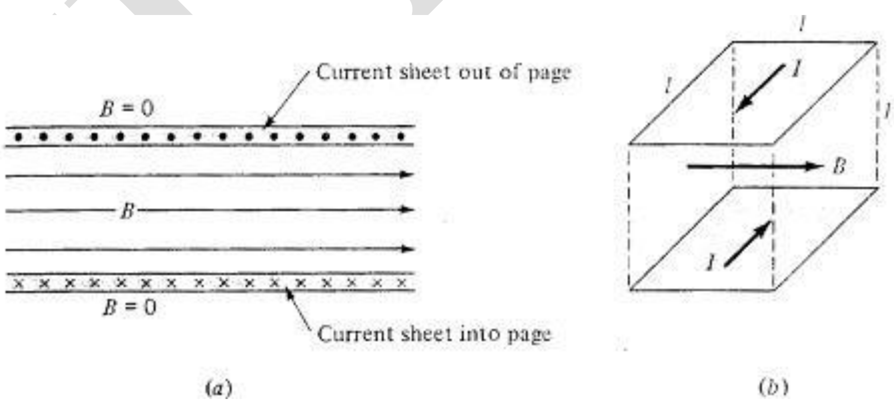

Fig:4.5 (a) The cross section of two infinite parallel current sheets; (b) a field cell is a cubical cut from the infinite sheets.

UNIT V TIME VARYING FIELDS AND MAXWELL'S EQUATIONS

Introduction:

In our study of static fields so far, we have observed that static electric fields are produced by electric charges, static magnetic fields are produced by charges in motion or by steady current. Further, static electric field is a conservative field and has no curl, the static magnetic field is continuous and its divergence is zero. The fundamental relationships for static electric fields among the field quantities can be summarized as:

$$\nabla \times \vec{E} = 0 \qquad (5.1a)$$

$$\nabla \cdot \vec{D} = \rho_v \qquad (5.1b)$$

For a linear and isotropic medium,

$$\vec{D} = \varepsilon \vec{E} \qquad (5.1c)$$

Similarly for the magnetostatic case

$$\nabla . \vec{B} = 0 \qquad (5.2a)$$

$$\nabla \times \vec{H} = \vec{J} \qquad (5.2b)$$

$$\vec{B} = \mu \vec{H} \qquad (5.2c)$$

It can be seen that for static case, the electric field vectors $\vec{E}$ and $\vec{D}$ and magnetic field vectors $\vec{B}$ and $\vec{H}$ form separate pairs.

In this chapter we will consider the time varying JITnario. In the time varying case we will observe that a changing magnetic field will produce a changing electric field and vice versa.

We begin our discussion with Faraday's Law of electromagnetic induction and then present the Maxwell's equations which form the foundation for the electromagnetic theory.

Faraday's Law of electromagnetic Induction

Michael Faraday, in 1831 discovered experimentally that a current was induced in a conducting loop when the magnetic flux linking the loop changed. In terms of fields, we can say that a time varying magnetic field produces an electromotive force (emf) which causes a current in a closed circuit. The quantitative relation between the induced emf (the voltage that arises from conductors moving in a magnetic field or from changing magnetic fields) and the rate of change of flux linkage developed based on experimental observation is known as Faraday's law. Mathematically, the induced emf can be written as

$$\text{Emf} = -\frac{d\phi}{dt} \quad \text{Volts} \qquad (5.3)$$

where ϕ is the flux linkage over the closed path.

A non zero $\frac{d\phi}{dt}$ may result due to any of the following: (a) time changing flux linkage a stationary closed path.

(b) relative motion between a steady flux a closed path.

(c) a combination of the above two cases.

The negative sign in equation (5.3) was introduced by Lenz in order to comply with the polarity of the induced emf. The negative sign implies that the induced emf will cause a current flow in the closed loop in such a direction so as to oppose the change in the linking magnetic flux which produces it. (It may be noted that as far as the induced emf is concerned, the closed path forming a loop does not necessarily have to be conductive).

If the closed path is in the form of N tightly wound turns of a coil, the change in the magnetic flux linking the coil induces an emf in each turn of the coil and total emf is the sum of the induced emfs of the individual turns, i.e.,

$$Emf = -N\frac{d\phi}{dt} \quad Volts \qquad (5.4)$$

By defining the total flux linkage as

$$\lambda = N\phi \qquad (5.5)$$

The emf can be written as

$$Emf = -\frac{d\lambda}{dt} \qquad (5.6)$$

Continuing with equation (5.3), over a closed contour '*C*' we can write

$$Emf = \oint_C \vec{E}.d\vec{l} \qquad (5.7)$$

where $\vec{E}$ is the induced electric field on the conductor to sustain the current. Further, total flux enclosed by the contour 'C' is given by

$$\phi = \int_S \vec{B}.d\vec{s} \qquad (5.8)$$

Where S is the surface for which 'C' is the contour.

From (5.7) and using (5.8) in (5.3) we can write

$$\oint_C \vec{E}.d\vec{l} = -\frac{\partial}{\partial t}\oint_S \vec{B}.d\vec{s} \qquad (5.9)$$

By applying stokes theorem

$$\int_S \nabla\times\vec{E}.d\vec{s} = -\int_S \frac{\partial\vec{B}}{\partial t}.d\vec{s} \qquad (5.10)$$

Therefore, we can write

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad (5.11)$$

which is the Faraday's law in the point form

We have said that non zero $\frac{d\phi}{dt}$ can be produced in a several ways. One particular case is when a time varying flux linking a stationary closed path induces an emf. The emf induced in a stationary closed path by a time varying magnetic field is called a transformer emf **.**

Ideal transformers

As shown in figure 5.1, a transformer consists of two or more numbers of coils coupled magnetically through a common core. Let us consider an ideal transformer whose winding has zero resistance, the core having infinite permittivity and magnetic losses are zero.

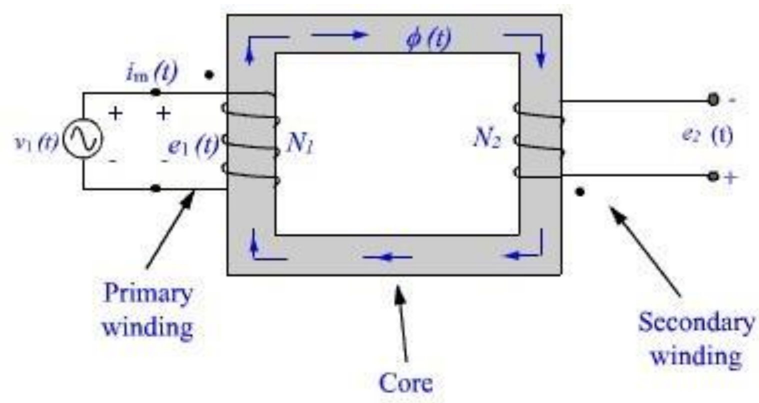

Fig 5.1: Transformer with secondary open

These assumptions ensure that the magnetization current under no load condition is vanishingly small and can be ignored. Further, all time varying flux produced by the primary winding will follow the magnetic path inside the core and link to the secondary coil without any leakage. If N1 and N2 are the number of turns in the primary and the secondary windings respectively, the induced emfs are

$$e_1 = N_1 \frac{d\phi}{dt} \qquad (5.12a)$$

$$e_2 = N_2 \frac{d\phi}{dt} \qquad (5.12b)$$

(The polarities are marked, hence negative sign is omitted. The induced emf is +ve at the dotted end of the winding.)

$$\therefore \frac{e_1}{e_2} = \frac{N_1}{N_2} \qquad (5.13)$$

i.e., the ratio of the induced emfs in primary and secondary is equal to the ratio of their turns. Under ideal condition, the induced emf in either winding is equal to their voltage rating.

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = a \qquad (5.14)$$

where 'a' is the transformation ratio. When the secondary winding is connected to a load, the current flows in the secondary, which produces a flux opposing the original flux. The net flux in the core decreases and induced emf will tend to decrease from the no load value. This causes the primary current to increase to nullify the decrease in the flux and induced emf.
The current continues to increase till the flux in the core and the induced emfs are restored to the no load values. Thus the source supplies power to the primary winding and the secondary winding delivers the power to the load. Equating the powers

$$i_1 v_1 = i_2 v_2 \qquad (5.15)$$

$$\frac{i_2}{i_1} = \frac{v_1}{v_2} = \frac{e_1}{e_2} = \frac{N_1}{N_2} \qquad (5.16)$$

Further,

$$i_2 N_2 - i_1 N_1 = 0 \qquad (5.17)$$

i.e., the net magnetomotive force (mmf) needed to excite the transformer is zero under ideal condition.

Motional EMF:

Let us consider a conductor moving in a steady magnetic field as shown in the fig 5.2.

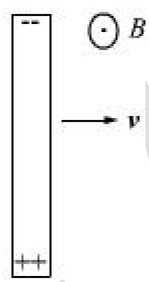

Fig 5.2

If a charge Q moves in a magnetic field $\vec{B}$, it experiences a force

$$\vec{F} = Q\vec{v} \times \vec{B} \qquad (5.18)$$

This force will cause the electrons in the conductor to drift towards one end and leave the other end positively charged, thus creating a field and charge separation continuous until electric and magnetic forces balance and an equilibrium is reached very quickly, the net force on the moving conductor is zero.

$\frac{\vec{F}}{Q} = \vec{v} \times \vec{B}$ can be interpreted as an induced electric field which is called the motional electric field

$$\vec{E}_m = \vec{v} \times \vec{B} \qquad (5.19)$$

If the moving conductor is a part of the closed circuit C, the generated emf around the circuit is $\oint_c \vec{v}\times\vec{B}.d\vec{l}$. This emf is called the motional emf.

Maxwell's Equation

Equation (5.1) and (5.2) gives the relationship among the field quantities in the static field. For time varying case, the relationship among the field vectors written as

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \qquad (5.20a)$$

$$\nabla\times\vec{H} = \vec{J} \qquad (5.20b)$$

$$\nabla.\vec{D} = \rho \qquad (5.20c)$$

$$\nabla.\vec{B} = 0 \qquad (5.20d)$$

In addition, from the principle of conservation of charges we get the equation of continuity

$$\nabla.\vec{J} = -\frac{\partial\rho}{\partial t} \qquad (5.21)$$

The equation 5.20 (a) - (d) must be consistent with equation

(5.21). We observe that

$$\nabla.\nabla\times\vec{H} = 0 = \nabla.\vec{J} \qquad (5.22)$$

Since $\nabla.\nabla\times\vec{A}$ is zero for any vector $\vec{A}$.

Thus $\nabla \times \vec{H} = \vec{J}$ applies only for the static case i.e., for the JITnario when
. A classic example for this is given below .

$$\frac{\partial \rho}{\partial t} = 0$$

Suppose we are in the process of charging up a capacitor as shown in fig 5.3.

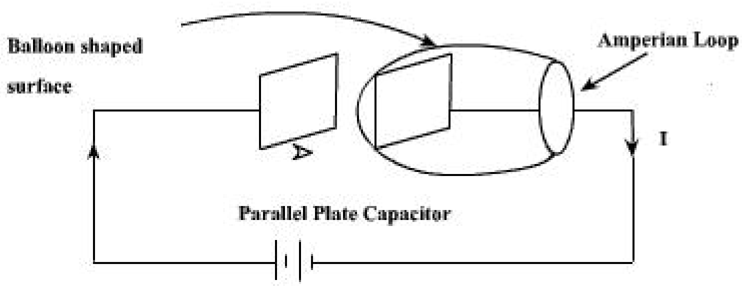

Fig 5.3 process of charging up a capacitor

Let us apply the Ampere's Law for the Amperian loop shown in fig 5.3. *Ienc* = *I* is the total current passing through the loop. But if we draw a baloon shaped surface as in fig 5.3, no current passes through this surface and hence *Ienc* = 0. But for non steady currents such as this one, the concept of current enclosed by a loop is ill-defined since it depends on what surface you use. In fact Ampere's Law should also hold true for time varying case as well, then comes the idea of displacement current which will be introduced in the next few slides.

We can write for time varying case,

$$\begin{aligned} \nabla . \left(\nabla \times \vec{H}\right) = 0 &= \nabla . \vec{J} + \frac{\partial \rho}{\partial t} \\ &= \nabla . \vec{J} + \frac{\partial}{\partial t} \nabla . \vec{D} \\ &= \nabla . \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right) \end{aligned} \tag{5.23}$$

$$\therefore \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{5.24}$$

The equation (5.24) is valid for static as well as for time varying case.

$\vec{J}$

Equation (5.24) indicates that a time varying electric field will give rise to a magnetic field even in the absence of . The term $\frac{\partial \vec{D}}{\partial t}$ has a dimension of current densities (A/m^2) and is called the displacement current density.

Introduction of $\frac{\partial \vec{D}}{\partial t}$ in $\nabla \times \vec{H}$ equation is one of the major contributions of Jame's Clerk Maxwell. The modified set of equations

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad (5.25b)$$

$$\nabla . \vec{D} = \rho \qquad (5.25c)$$

$$\nabla . \vec{B} = 0 \qquad (5.25d)$$

is known as the Maxwell's equation and this set of equations apply in the time varying JITnario, static fields are being a particular case $\left(\frac{\partial}{\partial t} = 0\right)$.

In the integral form

$$\oint_c \vec{E}.d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t}.d\vec{S} \qquad (5.26a)$$

$$\oint_c \vec{H}.d\vec{l} = \int_s \left(J + \frac{\partial D}{\partial t}\right).d\vec{S} = I + \int_s \frac{\partial \vec{D}}{\partial t}.d\vec{S} \qquad (5.26b)$$

$$\int_V \nabla.\vec{D}dv = \oint_S \vec{D}.d\vec{S} = \int_V \rho dv \qquad (5.26c)$$

$$\oint \vec{B}.d\vec{S} = 0 \qquad (5.26d)$$

The modification of Ampere's law by Maxwell has led to the development of a unified electromagnetic field theory. By introducing the displacement current term, Maxwell could predict the propagation of EM waves. Existence of EM waves was later demonstrated by Hertz experimentally which led to the new era of radio communication.

Boundary Conditions for Electromagnetic fields

The differential forms of Maxwell's equations are used to solve for the field v ectors provided the field quantities are single valued, bounded and continuous. At the media boundaries, the field vectors are discontinuous and their behaviors across the boundaries are governed by boundary conditions.
The integral equations(eqn 5.26) are assumed to hold for regions containing discontinuous media.Boundary conditions can be derived by applying the Maxwell's equations in the integral form to small regions at the interface of the two media. The procedure is similar to those used for obtaining boundary conditions for static electric fields (chapter 2) and static magnetic fields (chapter 4). The boundary conditions are summarized as follows

With reference to fig 5.3

$$\widehat{a_n} \times \left(\vec{E_1} - \vec{E_2}\right) = 0 \qquad 5.27(a)$$

$$\widehat{a_n} \cdot \left(\vec{D_1} - \vec{D_2}\right) = \rho_S \qquad 5.27(b)$$

$$\widehat{a_n} \times \left(\vec{H_1} - \vec{H_2}\right) = \vec{J_S} \qquad 5.27(c)$$

$$\widehat{a_n} \cdot \left(\vec{B_1} - \vec{B_2}\right) = 0 \qquad 5.27(d)$$

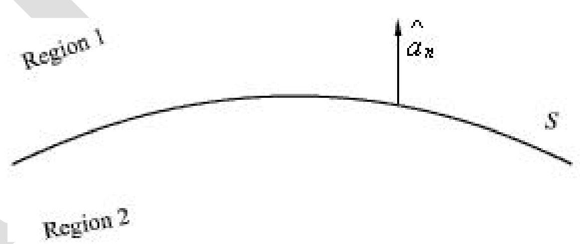

Fig 5.4

Equation 5.27 (a) says that tangential component of electric field is continuous across the interface while from 5.27 (c) we note that tangential component of the magnetic field is discontinuous by an amount equal to the surface current density. Similarly 5.27 (b) states that normal component of electric flux density vector $\vec{D}$ is discontinuous across the interface by an amount equal to the surface current density while normal component of the magnetic flux density is continuous.