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DEPARTMENT
OF
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LECTURE NOTES

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UNIT-2 CONDUCTORS AND DIELECTRICS

Boundary conditions for Electrostatic fields

In our discussions so far we have considered the existence of electric field in the homogeneous medium. Practical electromagnetic problems often involve media with different physical properties. Determination of electric field for such problems requires the knowledge of the relations of field quantities at an interface between two media. The conditions that the fields must satisfy at the interface of two different media are referred to as ***boundary conditions*** .

In order to discuss the boundary conditions, we first consider the field behavior in some common material media.

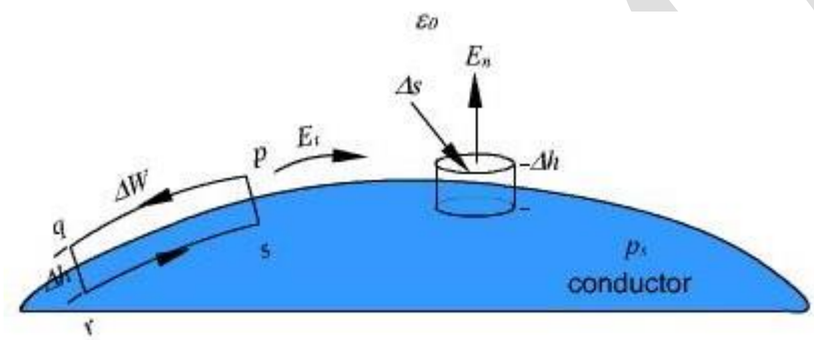


Fig 2.1: Boundary Conditions for at the surface of a Conductor

In general, based on the electric properties, materials can be classified into three categories: conductors, semiconductors and insulators (dielectrics). In *conductor* , electrons in the outermost shells of the atoms are very loosely held and they migrate easily from one atom to the other. Most metals belong to this group. The electrons in the atoms of *insulators* or *dielectrics* remain confined to their orbits and under normal circumstances they are not liberated under the influence of an externally applied field. The electrical properties of *semiconductors* fall between those of conductors and insulators since semiconductors have very few numbers of free charges.

The parameter *conductivity* is used characterizes the macroscopic electrical property of a material medium. The notion of conductivity is more important in dealing with the current flow and hence the same will be considered in detail later on.

If some free charge is introduced inside a conductor, the charges will experience a force due to mutual repulsion and owing to the fact that they are free to move, the charges will appear on the surface. The charges will redistribute themselves in such a manner that the field within the conductor is zero. Therefore, under steady condition, inside a conductor $\rho_v = 0$.

From Gauss's theorem it follows that

$\vec{E}=0$ ....*(2.51)*

The surface charge distribution on a conductor depends on the shape of the conductor. The charges on the surface of the conductor will not be in equilibrium if there is a tangential component of the electric field is present, which would produce movement of the charges. Hence under static field conditions, tangential component of the electric field on the conductor surface is zero. The electric field on the surface of the conductor is normal everywhere to the surface . Since the tangential component of electric field is zero, the conductor surface is an equipotential surface. As $\vec{E}=0$ inside the conductor, the conductor as a whole has the same potential. We may further note that charges require a finite time to redistribute in a conductor. However, this time is very small $\sim 10^{-19}$ sec for good conductor like copper.

Let us now consider an interface between a conductor and free space as shown in the figure *2*.1

Let us consider the closed path *pqrsp* for which we can write,

$$\oint \vec{E}\cdot d\vec{l} = 0 \quad \text{.............................}(2.52)$$

For $\Delta h \rightarrow 0$ and noting that $\vec{E}$ inside the conductor is zero, we can write

$$E_t \Delta w = 0 \text{.................................}(2.53)$$

Et is the tangential component of the field. Therefore we find that

$$E_t = 0 \text{........................} (2.54)$$

In order to determine the normal component *En*, the normal component of $\vec{E}$, at the surface of the conductor, we consider a small cylindrical Gaussian surface as shown in the Fig.12. Let Δs represent the area of the top and bottom faces and Δh represents the height of the cylinder. Once again, as $\Delta h \rightarrow 0$, we approach the surface of the conductor. Since $\vec{E}=0$ inside the conductor is zero,

$$\varepsilon_0 \oint_s \vec{E}\cdot d\vec{s} = \varepsilon_0 E_n \Delta s = \rho_s \Delta s \quad \text{............}(2.55)$$

$$E_n = \frac{\rho_s}{\varepsilon_0} \quad \text{..................}(2.56)$$

Therefore, we can summarize the boundary conditions at the surface of a conductor as:

$E_t = 0$

.......................(2.57)

$$E_n = \frac{\rho_s}{\varepsilon_0} \qquad ....(2.58)$$

Behavior of dielectrics in static electric field: Polarization of dielectric

Here we briefly describe the behavior of dielectrics or insulators when placed in static electric field. Ideal dielectrics do not contain free charges. As we know, all material media are composed of atoms where a positively charged nucleus (diameter ~ 10^{-15}m) is surrounded by negatively charged electrons (electron cloud has radius ~ 10^{-10}m) moving around the nucleus. Molecules of dielectrics are neutral macroscopically; an externally applied field causes small displacement of the charge particles creating small electric dipoles.These induced dipole moments modify electric fields both inside and outside dielectric material.

Molecules of some dielectric materials posses permanent dipole moments even in the absence of an external applied field. Usually such molecules consist of two or more dissimilar atoms and are called *polar* molecules. A common example of such molecule is water molecule H_2O. In polar molecules the atoms do not arrange themselves to make the net dipole moment zero. However, in the absence of an external field, the molecules arrange themselves in a random manner so that net dipole moment over a volume becomes zero.

Under the influence of an applied electric field, these dipoles tend to align themselves along the field as shown in figure 2.15. There are some materials that can exhibit net permanent dipole moment even in the absence of applied field. These materials are called *electrets* that made by heating certain waxes or plastics in the presence of electric field. The applied field aligns the polarized molecules when the material is in the heated state and they are frozen to their new position when after the temperature is brought down to its normal temperatures. Permanent polarization remains without an externally applied field.

As a measure of intensity of polarization, polarization vector $\vec{P}$ (in C/m^2) is defined as:

$$\vec{P} = \lim_{\Delta v \to 0} \frac{\sum_{i=1}^{n\Delta v} \vec{P_k}}{\Delta v} \qquad ....(2.59)$$

n being the number of molecules per unit volume i.e. $\vec{P}$ is the dipole moment per unit volume. Let us now consider a dielectric material having polarization $\vec{P}$ and compute the potential at an external point O due to an elementary dipole $\vec{P}dv'$.

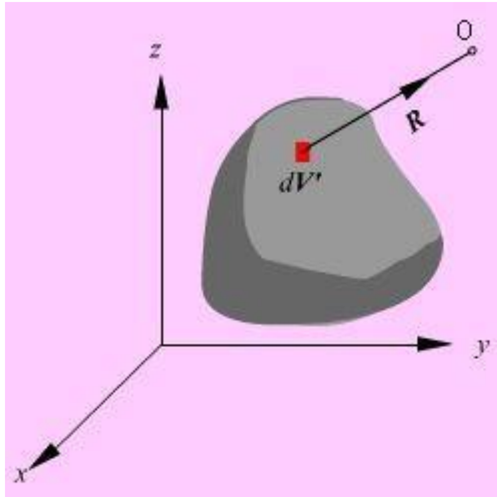


Fig 2.2: Potential at an External Point due to an Elementary Dipole $\vec{P}$ *dv'*.

With reference to the figure 2.16, we can write:

$$dV = \frac{\vec{P}dv' \cdot \hat{a}_R}{4\pi\varepsilon_0 R^2} \quad \text{...}(2.60)$$

Therefore,

$$R = \left\{(x-x')^2 + (y-y')^2 + (z-z')^2\right\}^{1/2} \quad \text{..................}(2.61)$$

where x,y,z represent the coordinates of the external point O and x',y',z' are the coordinates of the source point.

From the expression of *R*, we can verify that

$$\nabla'\left(\frac{1}{R}\right) = \frac{\hat{a}_R}{R^2} \quad \text{...}(2.63)$$

$$V = \frac{1}{4\pi\varepsilon_0}\int_{V'} \vec{P}\cdot\nabla'\left(\frac{1}{R}\right)dv' \quad \text{....................................}(2.64)$$

Using the vector identity, $\nabla'\cdot(f\vec{A}) = f\nabla'\cdot\vec{A} + \vec{A}\cdot\nabla' f$,where *f* is a scalar quantity , we have,

$$V = \frac{1}{4\pi\varepsilon_0}\left[\int_{V'}\nabla'\cdot\left(\frac{\vec{P}}{R}\right)dv' - \int_{V'}\frac{\nabla'\cdot\vec{P}}{R}dv'\right] \quad \text{.....................}(2.65)$$

Converting the first volume integral of the above expression to surface integral, we can write

$$V = \frac{1}{4\pi\varepsilon_0}\oint_{S'}\frac{\vec{P}\cdot\hat{a}'_n}{R}ds' + \frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{\left(-\nabla\cdot\vec{P}\right)}{R}dv' \quad \text{.................}(2.66)$$

where $\hat{a}'_n$ is the outward normal from the surface element *ds'* of the dielectric. From the above expression we find that the electric potential of a polarized dielectric may be found from the contribution of volume and surface charge distributions having densities

$$\rho_{ps} = \vec{P}\cdot\hat{a}_n \quad \text{..}(2.67)$$

$$\rho_{pv} = -\nabla\cdot\vec{P} \quad \text{......................}(2.68)$$

These are referred to as polarisation or bound charge densities. Therefore we may replace a polarized dielectric by an equivalent polarization surface charge density and a polarization volume charge density. We recall that bound charges are those charges that are not free to move within the dielectric material, such charges are result of displacement that occurs on a molecular scale during polarization. The total bound charge on the surface is

$$\oint_S \rho_{ps}ds = \oint_S \vec{P}\cdot d\vec{s} \quad \text{..................}(2.69)$$

The charge that remains inside the surface is

$$\int_V \rho_{pv}dv = \int_V -\nabla\cdot\vec{P}dv \quad \text{....................}(2.70)$$

The total charge in the dielectric material is zero as

$$\oint_S \rho_{ps}ds + \int_V \rho_{pv} = \oint_S \vec{P}\cdot d\vec{s} + \int_V -\nabla\cdot\vec{P}dv = \int_V \nabla\cdot\vec{P} - \int_V \nabla\cdot\vec{P} = 0 \quad \text{....................}(2.71)$$

If we now consider that the dielectric region containing charge density ρ_v the total volume charge density becomes

$$\rho_t = \rho_v + \rho_{pv} \quad \text{.................}(2.72)$$

Since we have taken into account the effect of the bound charge density, we can write

$$\nabla \cdot \vec{E} = \frac{(\rho_v + \rho_{pv})}{\varepsilon_0} \qquad \text{....................}(2.73)$$

Using the definition of ρ_{pv} we have

$$\nabla.(\varepsilon_0 \vec{E} + \vec{P}) = \rho_v \text{....................}(2.74)$$

Therefore the electric flux density $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

When the dielectric properties of the medium are linear and isotropic, polarisation is directly proportional to the applied field strength and

$$\vec{P} = \varepsilon_0 \chi_e \vec{E} \text{ ....}(2.75)$$

is the electric suJITptibility of the dielectric. Therefore,

$$\vec{D} = \varepsilon_0 (1 + \chi_e) \vec{E} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E} \text{.......................}(2.76)$$

$\varepsilon_r = 1 + \chi_e$ is called relative permeability or the dielectric constant of the medium.

$\varepsilon_0 \varepsilon_r$ is called the absolute permittivity.

A dielectric medium is said to be linear when is χ_e independent of and the $\vec{E}$ medium is homogeneous if χ_e is also independent of space coordinates. A linear homogeneous and isotropic medium is called a **simple medium** and for such medium the relative permittivity is a constant.

Dielectric constant ε_r may be a function of space coordinates. For anistropic materials, the dielectric constant is different in different directions of the electric field, D and E are related by a permittivity tensor which may be written as:

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \text{.......................}(2.77)$$

For crystals, the reference coordinates can be chosen along the principal axes, which make off diagonal elements of the permittivity matrix zero. Therefore, we have

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \quad \text{.......................(2.78)}$$

Media exhibiting such characteristics are called **biaxial**. Further, if $\varepsilon_1 = \varepsilon_2$ then the medium is called **uniaxial**. It may be noted that for isotropic media, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$.

Lossy dielectric materials are represented by a complex dielectric constant, the imaginary part of which provides the power loss in the medium and this is in general dependant on frequency.

Another phenomenon is of importance is **dielectric breakdown**. We observed that the applied electric field causes small displacement of bound charges in a dielectric material that results into polarization. Strong field can pull electrons completely out of the molecules. These electrons being accelerated under influence of electric field will collide with molecular lattice structure causing damage or distortion of material. For very strong fields, avalanche breakdown may also occur. The dielectric under such condition will become conducting.

The maximum electric field intensity a dielectric can withstand without breakdown is referred to as the **dielectric strength** of the material.

Method Of Images:

The replacement of the actual problem with boundaries by an enlarged region or with image charges but no boundaries is called the method of images.
Method of images is used in solving problems of one or more point charges in the presence of boundary surfaces.

Continuity of equation:

The relation between density and the volume charge density at a point called continuity of equation

$\nabla \bullet J = - \rho / t_v$

Boundary Conditions for perfect Electric Fields:

Let us consider the relationship among the field components that exist at the interface between two dielectrics as shown in the figure 2.17. The permittivity of the medium 1 and medium 2 are ε_1 and ε_2 respectively and the interface may also have a net charge density ρ_s Coulomb/m.

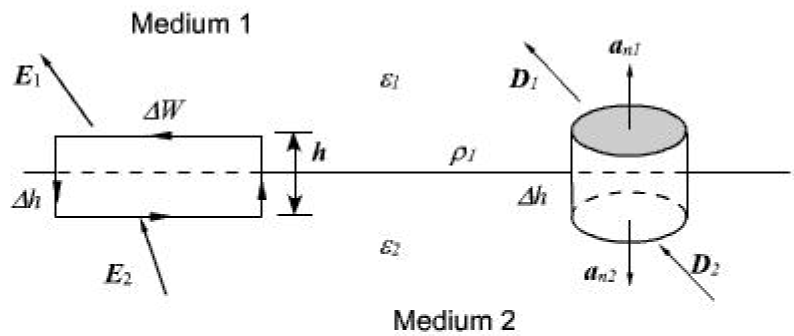


Fig *2.3*: Boundary Conditions at the interface between two dielectrics

We can express the electric field in terms of the tangential and normal components

$$\vec{E_1} = \vec{E_{1t}} + \vec{E_{1n}}$$
$$\vec{E_2} = \vec{E_{2t}} + \vec{E_{2n}} \quad \text{..........(2.79}$$

where Et and En are the tangential and normal components of the electric field respectively. Let us assume that the closed path is very small so that over the elemental path length the variation of E can be neglected. Moreover very near to the interface, $\Delta h \to 0$. Therefore

$$\oint \vec{E}.d\vec{l} = E_{1t}\Delta w - E_{2t}\Delta w + \frac{h}{2}\left(E_{1n} + E_{2n}\right) - \frac{h}{2}\left(E_{1n} + E_{2n}\right) = 0 \quad \text{.......................(2.80)}$$

Thus, we have,

$E_{1t} = E_{2t}$ or $\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$ i.e. the **tangential component of an electric field is continuous across the interface**.

For relating the flux density vectors on two sides of the interface we apply Gauss's law to a small pillbox volume as shown in the figure. Once again as $\Delta h \to 0$, we can write

$$\oint \vec{D} \cdot d\vec{s} = (\vec{D_1} \cdot \hat{a}_{n2} + \vec{D_2} \cdot \hat{a}_{n1})\Delta s = \rho_s \Delta s \quad \text{..................(2.81a)}$$

i.e.,

$$D_{1n} - D_{2n} = \rho_s \quad \text{..(2.81b)}$$

.e., $\varepsilon_1 E_{1n} - \varepsilon_2 E_{2n} = \rho_s$ .... (2.81c)

Thus we find that the **normal component of the flux density vector *D* is discontinuous across an interface by an amount of discontinuity equal to the surface charge density at the interface.**

Example

Two further illustrate these points; let us consider an example, which involves the refraction of D or E at a charge free dielectric interface as shown in the figure 2.18.

Using the relationships we have just derived, we can write

$$E_{1t} = E_1 \sin\theta_1 = \frac{D_1}{\varepsilon_1}\sin\theta_1 = E_{2t} = E_2\sin\theta_2 = \frac{D_2}{\varepsilon_2}\sin\theta_2 \quad \text{.......................(2.82a)}$$

$$D_{1n} = D_1\cos\theta_1 = D_{2n} = D_2 Cos\theta_2 \quad \text{.......................(2.82b)}$$

In terms of flux density vectors,

$$\frac{D_1}{\varepsilon_1}\sin\theta_1 = \frac{D_2}{\varepsilon_2}\sin\theta_2 \quad \text{.......................(2.83a)}$$

$$D_1\cos\theta_1 = D_2 Cos\theta_2 \quad \text{.....................}\textit{(2.83b)}$$

Therefore,

$$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}} \quad \text{.......................}\textit{(2.84)}$$

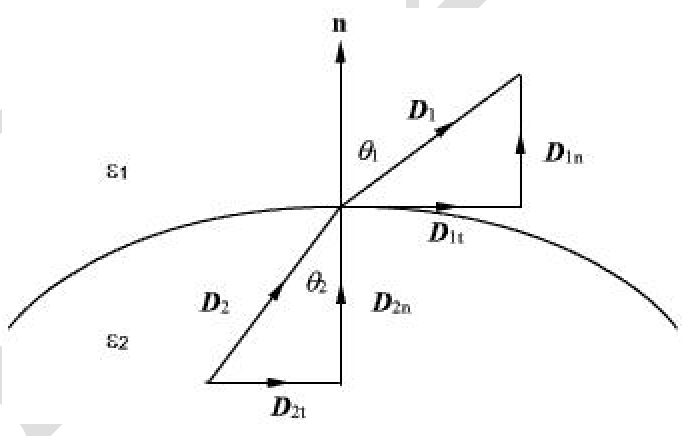


Fig *2*.4: Refraction of D or E at a Charge Free Dielectric Interface

Capacitance and Capacitors

We have already stated that a conductor in an electrostatic field is an Equipotential body and any charge given to such conductor will distribute themselves in such a manner that electric field inside the conductor vanishes. If an additional amount of charge is supplied to an isolated conductor at a given potential, this additional charge will increase the surface charge density ρ_s. Since the potential of the conductor is given by $V = \frac{1}{4\pi\varepsilon_0}\int_{s'}\frac{\rho_s ds'}{r}$, the potential

of the conductor will also increase maintaining the ratio $\frac{Q}{V}$ same. Thus we can write

$$C = \frac{Q}{V}$$

where the constant of proportionality C is called the capacitance of the isolated conductor. SI
unit of capacitance is Coulomb/ Volt also called Farad denoted by *F*. It can It can be seen that if V=1, $C = Q$. Thus capacity of an isolated conductor can also be defined as the amount of charge in Coulomb required to raise the potential of the conductor by 1 Volt.

Of considerable interest in practice is a capacitor that consists of two (or more) conductors carrying equal and opposite charges and separated by some dielectric media or free space. The conductors may have arbitrary shapes. A two-conductor capacitor is shown in figure 2.5.

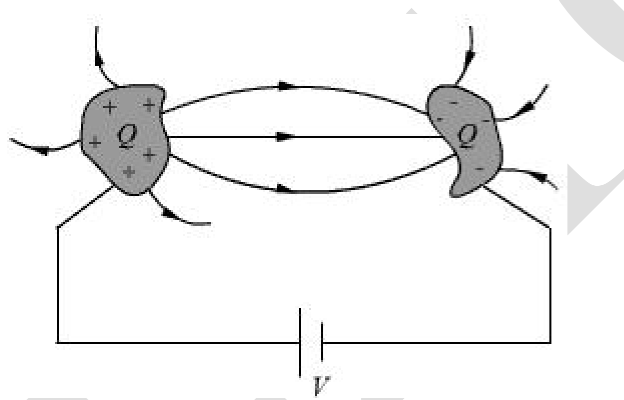


*Fig **2.5**:Capacitance and Capacitors*

When a d-c voltage source is connected between the conductors, a charge transfer occurs which results into a positive charge on one conductor and negative charge on the other conductor. The conductors are equipotential surfaces and the field lines are perpendicular to the conductor surface. If V is the mean potential difference between the conductors, the capacitance is given by $C = \frac{Q}{V}$. Capacitance of a capacitor depends on the geometry of the conductor and the permittivity of the medium between them and does not depend on the charge or potential difference between conductors. The capacitance can be computed by assuming Q(at the same time $-Q$ on the other conductor), first determining $\vec{E}$ using Gauss's theorem and then determining $V = -\int \vec{E}.d\vec{l}$. We illustrate this procedure by taking the example of a parallel plate capacitor.

Parallel plate capacitor

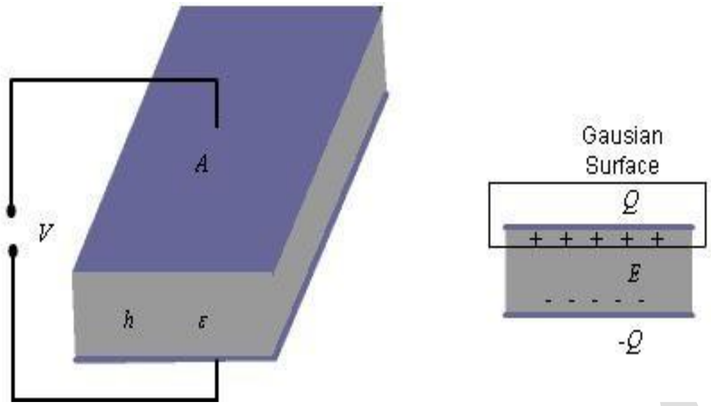


Fig 2.6: Parallel Plate Capacitor

For the parallel plate capacitor shown in the figure 2.20, let each plate has area A and a distance h separates the plates. A dielectric of permittivity ε fills the region between the plates. The electric field lines are confined between the plates. We ignore the flux fringing at the edges of the plates and charges are assumed to be uniformly distributed over the

ρ_s ρ_s $\rho_s = \frac{Q}{A}$

conducting plates with densities and , . $E = \frac{\rho_s}{\varepsilon} = \frac{Q}{A\varepsilon}$

By Gauss's theorem we can write, ...*(2.85)*

As we have assumed ρ_s to be uniform and fringing of field is neglected, we see that E is constant in the region between the plates and therefore, we can write .

Thus, $V = Eh = \frac{hQ}{\varepsilon A}$

for a parallel plate capacitor we have,. $C = \frac{Q}{V} = \varepsilon \frac{A}{h}$ ....*(2.86)*

Series and parallel Connection of capacitors

Capacitors are connected in various manners in electrical circuits; series and parallel connections are the two basic ways of connecting capacitors. We compute the equivalent capacitance for such connections.

Series Case: Series connection of two capacitors is shown in the figure 2.21. For this case we can write,

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{V}{Q} = \frac{1}{C_{eqs}} = \frac{1}{C_1} + \frac{1}{C_2} \quad ....(2.87)$$

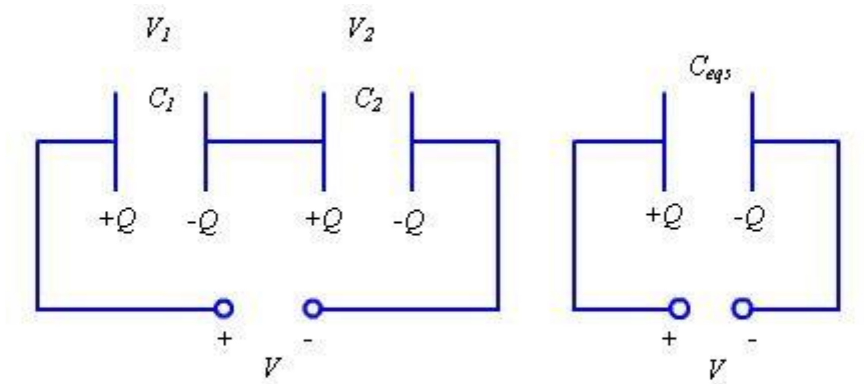
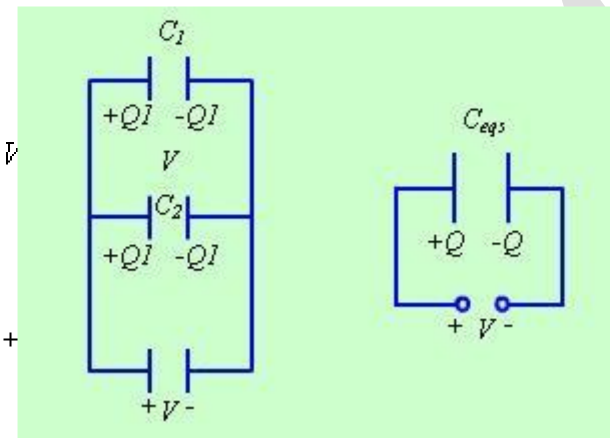


Fig 2.7: Series Connection of Capacitors

$W = (V_{1N}Q_1 + \ldots\ldots + V$ $\ldots_{-2)N}Q_{N-2}) + V_{(N-1)N}Q_{N-1}$



$2W = (V_{1N} + V_{1(N-1)} +$ $\ldots\ldots$

$\ldots\ldots V_{N2} + V_{N(N-1)})Q_N$

Fig2.7:Parallel Connection of Capacitors

The same approach may be extended to more than two capacitors connected in series. **Parallel Case:** For the parallel case, the voltages across the capacitors are the same. The total charge $Q = Q_1 + Q_2 = C_1V + C_2V$

Therefore,.. (2.88)

$$C_{eqp} = \frac{Q}{V} = C_1 + C_2$$

Poisson's and Laplace's Equations

For electrostatic field, we have seen that

$$\nabla \cdot \vec{D} = \rho_v$$
$$\vec{E} = -\nabla V$$
....(2.97)

Form the above two equations we can write

$$\nabla \cdot (\varepsilon \vec{E}) = \nabla \cdot (-\varepsilon \nabla V) = \rho_v$$ ....(2.98)

Using vector identity we can write, $\varepsilon \nabla \cdot \nabla V + \nabla V \cdot \nabla \varepsilon = -\rho_v$ ....(2.99)

For a simple homogeneous medium, is constant and $\nabla \varepsilon = 0$. Therefore,

$$\nabla \cdot \nabla V = \nabla^2 V = -\frac{\rho_v}{\varepsilon}$$ ....(2.100)

This equation is known as **Poisson's equation**. Here we have introduced a new operator, ∇^2 (del square), called the Laplacian operator. In Cartesian coordinates,

$$\nabla^2 V = \nabla \cdot \nabla V = \left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right) \cdot \left(\frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z\right)$$ ....(2.101)

Therefore, in Cartesian coordinates, Poisson equation can be written as:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial x^2} = -\frac{\rho_v}{\varepsilon}$$ ....(2.102)

In cylindrical coordinates,

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$ ....(2.103)

In spherical polar coordinate system,

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$ ....*(2.104)*

At points in simple media, where no free charge is present, Poisson's equation reduces to

$$\nabla^2 V = 0$$ ....*(2.105)*

which is known as Laplace's equation.

Application of poisons and Laplace's equations:

Laplace's and Poisson's equation are very useful for solving many practical electrostatic field problems where only the electrostatic conditions (potential and charge) at some boundaries are known and solution of electric field and potential is to be found throughout the volume. We shall consider such applications in the section where we deal with boundary value problems.

ASSIGNMENT PROBLEMS

1. A charged ring of radius d carrying a charge of ρ_L C/m lies in the x-y plane with its centre at the origin and a charge Q C is placed at the point $(0,0,2d)$. Determine ρ_S in terms of Q and d so that a test charge placed at $(0,0,2d)$ does not experience any force.

2. A semicircular ring of radius ε lies in the free space and carries a charge density ρ_s C/m. Find the electric field at the centre of the semicircle.
3. Consider a uniform sphere of charge with charge density ρ_0 and radius *b* , centered at the origin. Find the electric field at a distance *r* from the origin for the two cases: *r*<*b* and *r*>*b* . Sketch the strength of the electric filed as function of *r* .

4. A spherical charge distribution is given by

$$\rho_v = \begin{cases} \rho_0 (a^2 - r^2), & r \leq a \\ 0, & r > a \end{cases}$$

ε is the radius of the sphere. Find the following:

i. The total charge.
ii. $\vec{E}$ for $r \leq a$ and $r > a$.
iii The value of r where $\vec{E}$ becomes

5. With reference determine the potential and field at the point if the shaded $P(0,0,h)$ region contains uniform charge density ρ_s/m^2 .

6. A capacitor consists of two coaxial metallic cylinders of length L, radius of the inner conductor ε and that of outer conductor b . A dielectric material having dielectric $\varepsilon_r = 3 + 2/\rho$ constant , where ρ is the radius, fills the space between the conductors. Determine the capacitance of the capacitor.

7. Determine whether the functions given below satisfy Laplace 's equation

i) $V(x,y,z) = \dfrac{1}{\sqrt{x^2 + y^2 + z^2}}$

ii) $V(\rho,\phi,z) = \rho z \sin\phi + \rho^2$