EC8251 CIRCUIT ANALYSIS **L T P C**
4 0 0 4

OBJECTIVES:

To introduce the basic concepts of DC and AC circuits behavior

To study the transient and steady state response of the circuits subjected to step and sinusoidal excitations.

To introduce different methods of circuit analysis using Network theorems, duality and topology.

UNIT I BASIC CIRCUITS ANALYSIS AND NETWORK TOPOLOGY 12

Ohm‗s Law – Kirchhoff‗s laws – Mesh current and node voltage method of analysis for D.C and A.C. circuits - Network terminology - Graph of a network - Incidence and reduced incidence matrices – Trees –Cutsets - Fundamental cutsets - Cutset matrix – Tie sets - Link currents and Tie set schedules -Twig voltages and Cutset schedules, Duality and dual networks.

UNIT II NETWORK THEOREMS FOR DC AND AC CIRCUITS 12

Network theorems -Superposition theorem, Thevenin‗s theorem, Norton‗s theorem, Reciprocity theorem, Millman‗s theorem, and Maximum power transfer theorem ,application of Network theorems- Network reduction: voltage and current division, source transformation – star delta conversion.

UNIT III RESONANCE AND COUPLED CIRCUITS 12

Resonance - Series resonance - Parallel resonance - Variation of impedance with frequency - Variation in current through and voltage across L and C with frequency – Bandwidth - Q factor - Selectivity. Self inductance - Mutual inductance - Dot rule - Coefficient of coupling - Analysis of multiwinding coupled circuits - Series, Parallel connection of coupled inductors - Single tuned and double tuned coupled circuits.

UNITIV TRANSIENT ANALYSIS 12

Natural response-Forced response - Transient response of RC, RL and RLC circuits to excitation by Step Signal, Impulse Signal and exponential sources - Complete response of RC, RL and RLC Circuits to sinusoidal excitation.

UNIT V TWO PORT NETWORKS 12

Two port networks, Z parameters, Y parameters, Transmission (ABCD) parameters, Hybrid(H) Parameters, Interconnection of two port networks, Symmetrical properties of T and π networks. TOTAL : 60 PERIODS OUTCOMES: At the end of the course, the student should be able to: • Develop the capacity to analyze electrical circuits, apply the circuit theorems in real time • Design and understand and evaluate the AC and DC circuits.

TEXT BOOKS:

1. William H. Hayt, Jr. Jack E. Kemmerly and Steven M. Durbin, ―Engineering Circuit Analysis‖ , McGraw Hill Science Engineering, Eighth Edition, 11th Reprint 2016.

2. Joseph Edminister and Mahmood Nahvi, ―Electric Circuits‖, Schaum‗s Outline Series, Tata McGraw Hill Publishing Company, New Delhi, Fifth Edition Reprint 2016.

REFERENCES:

1. Charles K. Alexander, Mathew N.O. Sadiku, ―Fundamentals of Electric Circuits‖, Fifth Edition, McGraw Hill, 9th Reprint 2015.

2. A.Bruce Carlson, ―Cicuits: Engineering Concepts and Analysis of Linear Electric Circuits‖, Cengage Learning, India Edition 2nd Indian Reprint 2009.

3. Allan H.Robbins, Wilhelm C.Miller, ―Circuit Analysis Theory and Practice‖, Cengage Learning, Fifth Edition, 1st Indian Reprint 2013.

UNIT I

BASIC CIRCUITS ANALYSIS AND NETWORK TOPOLOGY

1.1 INTRODUCTION

Electric circuit is the interconnection of various electric elements in order to perform a desired function. The electric elements includes source of energy, resistors, capacitors, inductors, etc. The analysis of an electrical circuit is done to determine the quantities such as voltage, current and power associated with one or more elements in the circuit. To contribute to the solution of engineering problems one must acquire the basic knowledge of electric circuit analysis and laws. Many other systems, like mechanical, hydraulic, thermal, magnetic and power system are easy to analyze and model by a circuit. In order to analyze the models of any system, first one needs to learn the techniques of circuit analysis.

1.2 BASIC ELEMENTS & INTRODUCTORY CONCEPTS

Electrical Network:

A combination of various electric elements (Resistor, Inductor, Capacitor, Voltage source, Current source) connected in any manner what so ever is called an electrical network. The circuit elements are classified as

1. **Passive elements**

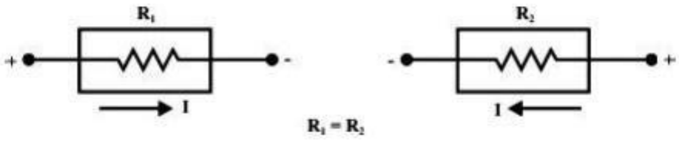
The element which receives or absorbs energy and then either converts it into heat or stored it in an electric (C) or magnetic (L) field is called passive element. Examples of passive elements are resistors, capacitors etc. Transformer is an example of passive element as it does not amplify the power level and power remains same both in primary and secondary sides.

2. **Active elements**

The elements that supply energy to the circuit is called active element. Examples of active elements include voltage and current sources, generators, and electronic devices that require power supplies. A transistor is an active circuit element, as it can amplify power of a signal.

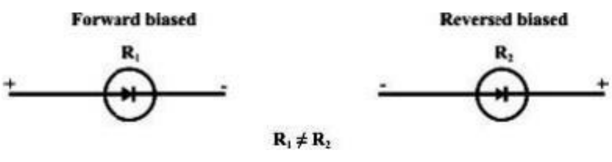
Bilateral Element:

Conduction of current in both directions in an element with same magnitude is termed as bilateral element.
Example: Resistance; Inductance; Capacitance



Unilateral Element:

Conduction of current in one direction is termed as unilateral element.
Example: Diode, Transistor



Response:

The behaviour of output signal with time when an input is given to the system is known as the response of the system.

Potential Energy Difference:

The amount of energy required to move a unit charge between the two points is known as the voltage or potential energy difference between two points in an electric circuit.

Ohm's Law:

Ohm's law states that the current through a conductor between two points is directly proportional to the potential difference or voltage across the two points, and inversely proportional to the resistance between them. The mathematical equation that describes this relationship is:

$$I = V / R$$

where I is the current through the resistance in units of amperes,

V is the potential difference measured across the resistance in units of volts, and

R is the resistance of the conductor in units of ohms.

More specifically, Ohm's law states that the R in this relation is constant, independent of the current.

1.3 KIRCHOFF'S LAW

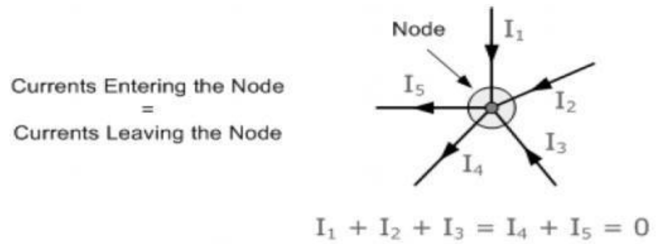
Kirchhoff's First Law - the Current Law, (KCL)

"The total current or charge entering a junction or node is exactly equal to the charge leaving the node as it has no other place to go except to leave, as no charge is lost within the node".

In other words the algebraic sum of ALL the currents entering and leaving a node must be equal to zero,

$$I(\text{exiting}) + I(\text{entering}) = 0.$$

This idea by Kirchoff is known as the Conservation of Charge**.**

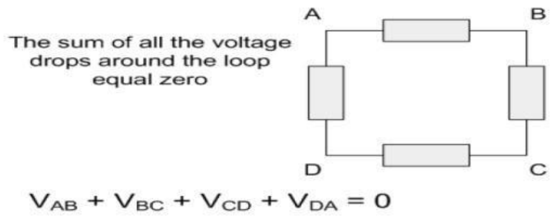


Here, the 3 currents entering the node, I_1, I_2, and I_3 are all positive in value and the 2 currents leaving the node, I_4 and I_5 are negative in value. Then the equation becomes $I_1 + I_2 + I_3 - I_4 - I_5 = 0$

Kirchhoff's Second Law - the Voltage Law, (KVL)

"In any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop" which is also equal to zero. In other words the algebraic sum of all voltages within the loop must be equal to zero.

Starting at any point in the loop continue in the same direction noting the direction of all the voltage drops, either positive or negative, and returning back to the same starting point. It is important to maintain the same direction either clockwise or anti-clockwise or the final voltage sum will not be equal to zero.



Problem 1:

A current of 0.5 A is flowing through the resistance of 10Ω.Find the potential difference between its ends.

Solution:
Current I = 0.5A.
Resistance R = 10Ω

Potential difference V =?

$$V = IR$$
$$= 0.5 \times 10$$
$$= 5V.$$

Problem: 2

A supply voltage of 220V is applied to a resistor100. Find Ω the current flowing through it.

Solution:

Voltage V = 220V Resistance R = 100Ω Current I = V/ R

= 2 2 0 /100
= 2.2 A.

Problem: 3

Calculate the resistance of the conductor if a current of 2A flows through it when the potential difference across its ends is 6V.

Solution:

Current I = 2A. Potential difference = V = 6. Resistance R = V/I

= 6 /2
= 3 ohm.

Problem: 4

Calculate the current and resistance of a 100 W, 200V electric bulb.

Solution:

Power, P = 100W
Voltage, V = 200V Power p = VI
Current I = P/V
= 100/200
= 0.5A
Resistance R = V /I
= 200/0.5
= 400W.

Problem: 5

Calculate the power rating of the heater coil when used on 220V supply taking 5 Amps.

Solution:

Voltage, V = 220V Current, I = 5A, Power, P = VI
= 220 × 5
= 1100W
= 1.1 KW.

Problem: 6

A circuit is made of 0.4wire,Ω a 150bulbΩ and a rheostat120connectedΩ in series. Determine the total resistance of the resistance of the circuit.

Solution:

Resistance of the wire = 0.4ResistanceΩ of bulb = 1 5 0 Ω Resistance of rheostat = 120Ω

In series,

Total resistance, R = 0.4 + 150 +120 = 270.4Ω

Problem: 7

Three resistances of values 2Ω,3Ωconnectedandin series5Ω across are 20 V,D.C supply. Calculate (a) equivalent resistance of the circuit (b) the total current of the circuit (c) the voltage drop across each resistor and (d) the power dissipated in each resistor.

Solution:

Total resistance R = R1 + R 2+ R3.

= 2 +3+5 = 10Ω

Voltage = 20V

Total current I = V/R = 20/10 = 2A.

Voltage drop across 2Ωresistor V1 = I R1

= 2× 2 = 4 volts.

Voltage drop across 3Ωresistor V 2 = IR2

= 2 × 3 = 6 volts.

Voltage drop across 5Ωresistor V3 = I R3

= 2 ×5 = 10 volts.

Power dissipated in 2Ωresistor is P1 = I2 R1

= 22 × 2 = 8 watts.

Power dissipated in 3 resistor is P2 = I2 R2.

= 22 × 3 = 12 watts.

Power dissipated in 5 resistor is P3 = I2 R3

= 22 × 5 = 20 watts.

Problem: 8

A lamp can work on 50 volt mains taking 2 amps. What value of the resistance must be connected in series with it so that it can be operated from 200 volt mains giving the same power?

Solution:

Lamp voltage, V = 50V Current, I = 2 amps.

Resistance of the lamp = V/I = 50/25 = 25 Ω

Resistance connected in series with lamp = r.

Supply voltage = 200 volt.

Circuit current I = 2A

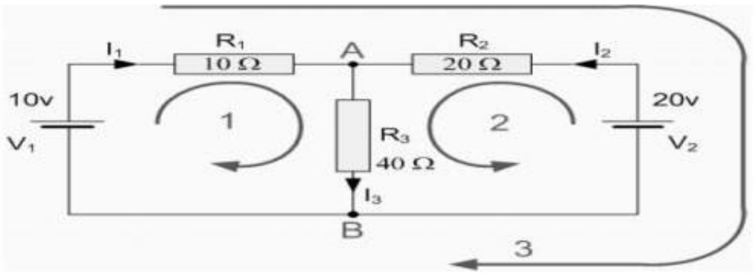
Total resistance Rt= V/I = 200/2 = 100Ω

Rt = R + r 100 = 25 + r

r = 75Ω

Problem: 9

Find the current flowing in the 40Ω Resistor,



Solution:
The circuit has 3 branches, 2 nodes (A and B) and 2 independent loops.

Using Kirchoff's Current Law, KCL the equations are given as;
At node A: I1 + I2 = I3
At node B: I3 = I1 + I2

Using Kirchoff's Voltage Law, KVL the equations are given as;
Loop 1 is given as: 10 = R1 x I1 + R3 x I3 = 10I1 + 40I3
Loop 2 is given as: 20 = R2 x I2 + R3 x I3 = 20I2 + 40I3
Loop 3 is given as: 10 - 20 = 10I1 - 20I2

As I3 is the sum of I1 + I2 we can rewrite the equations as;
Eq. No 1: 10 = 10I1 + 40(I1 + I2) = 50I1 + 40I2
Eq.No 2: 20 = 20I1 + 40(I1 + I2) = 40I1 + 60I2

We now have two "Simultaneous Equations" that can be reduced to give us the value of both I1 and I2

Substitution of I1 in terms of I2 gives us the value of I1 as -0.143 Amps

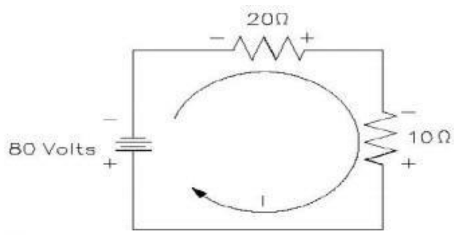
Substitution of I2 in terms of I1 gives us the value of I2 as +0.429 Amps

As: I3 = I1 + I2

The current flowing in resistor R3 is given as: -0.143 + 0.429 = 0.286 Amps and the voltage across the resistor R3 is given as: 0.286 x 40 = 11.44 volts

Problem: 10

Find the current in a circuit using Kirchhoff's voltage law



Solution:

80 = 20(I) + 10(I)

80 = 30(I)

I = 80/30 = 2.66 amperes

1.4 DC CIRCUITS:

A DC circuit (Direct Current circuit) is an electrical circuit that consists of any combination of constant voltage sources, constant current sources, and resistors. In this case, the circuit voltages and currents are constant, i.e., independent of time and a DC circuit has no memory. That is, a particular circuit voltage or current does not depend on the past value of any circuit voltage or current.

If a capacitor and/or inductor is added to a DC circuit, the resulting circuit is not a DC circuit. However, most such circuits have a DC solution. This solution gives the circuit voltages and currents when the circuit is in DC steady state. The solutions to these equations usually contain a time varying or transient part as well as constant or steady state part. There are some circuits that do not have a DC solution. Two simple examples are a constant current source connected to a capacitor and a constant voltage source connected to an inductor.

1.5 AC CIRCUITS:

Fundamentals of AC:

An alternating current (AC) is an electrical current, where the magnitude of the current varies in a cyclical form, as opposed to direct current, where the polarity of the current stays constant.

The usual waveform of an AC circuit is generally that of a sine wave, as these results in the most efficient transmission of energy. However in certain applications different waveforms are used, such as triangular or square waves.

Used generically, AC refers to the form in which electricity is delivered to businesses and residences. However, audio and radio signals carried on electrical wire are also examples of alternating current. In these applications, an important goal is often the recovery of information encoded (or modulated) onto the AC signal.

1.6 DIFFERENCE BETWEEN AC AND DC:

Current that flows continuously in one direction is called direct current. Alternating current (A.C) is the current that flows in one direction for a brief time then reverses and flows in opposite direction for a similar time. The source for alternating current is called AC generator or alternator.

Cycle:
One complete set of positive and negative values of an alternating quantity is called cycle.

Frequency:
The number of cycles made by an alternating quantity per second is called frequency. The unit of frequency is Hertz (Hz)

Amplitude or Peak value:
The maximum positive or negative value of an alternating quantity is called amplitude or peak value.

Average value:
This is the average of instantaneous values of an alternating quantity over one complete cycle of the wave.

Time period:
The time taken by the signal to complete one complete cycle.

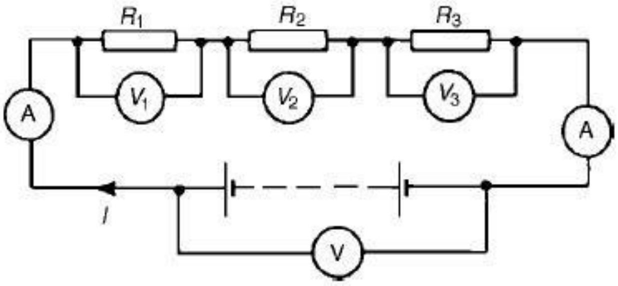
Average value derivation:

Let i = the instantaneous value of current and $i = I_m \sin\theta$ Where, I_m is the maximum value.

1.7 RESISTORS IN SERIES AND PARALLEL CIRCUITS

1.7.1 Series circuits

Figure shows three resistors R1, R2 and R3 connected end to end, i.e. in series, with a battery source of V volts. Since the circuit is closed a current I will flow and the potential difference across each resistor may be determined from the voltmeter readings V1, V2 and V3



In a series circuit

(a) The current I is the same in all parts of the circuit and hence the same reading is found on each of the two ammeters shown, and

(b) The sum of the voltages V1, V2 and V3 is equal to the total applied voltage, V, i.e.

$$V = V1 + V2 + V3$$

From Ohm's law,

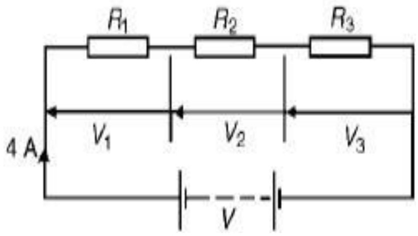
V1 = IR1,
V2 = IR2,
V3 = IR3 and V = IR
where R is the total circuit resistance.

Since V = V1 + V2 + V3,

then IR = IR1 + IR2 + IR3
Therefore, R = R1 + R2 + R3 (dividing throughout by I)

Thus for a series circuit, the total resistance is obtained by adding together the values of the separate resistances.

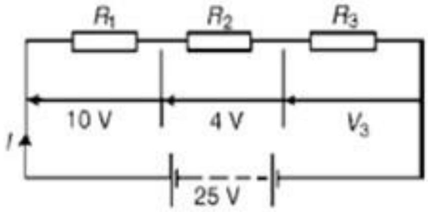
Problem 1: For the circuit shown in Figure, determine (a) the battery voltage V, (b) the total resistance of the circuit, and (c) the values of resistance of resistors R1, R2 and R3, given that the p.d.'sR1, R2acrossandR3are5V, 2V and 6V respectively.



(a) Battery voltage V = V1 + V2 + V3 = 5 + 2 + 6 = 13V
(b)Total circuit resistance R= V/ I
= 13/4 = 3.25Ω
(c) Resistance R1 = V1/ I
= 5/4
= 1.25 Ω Resistance R2 = V2/ I
= 2/4 = 0.5 Ω
Resistance R3 = V3/ I = 6/4 = 1.5 Ω

Problem2.

For the circuit shown in Figure determine the potential difference across resistor *R*3. If the total resistance of the circuit is 100Ω, determine the current flowing through resistor *R*1. Find also the value of resistor *R*2.



Potential difference across R3:

V3 =25 −10 −4=11V

Current I = V/ R

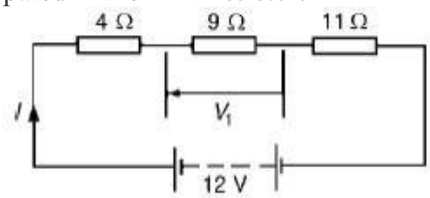
= 25/100 =0.25A, which is the current flowing in each resistor

Resistance R2 = V2/ I

= 4/0.25 =16 Ω

Problem 3:

A 12V battery is connected in a circuit having three series-connected resistors having resistances of 4 Ω, 9 Ω and 11 Ω. Determine the current flowing through, and the potential difference. across the 9 Ω resistors. Find also the power dissipated in the 11 Ω resistor.



Total resistance R=4 + 9 + 11=24 Ω

Current I = V/ R

= 12/24 =0.5A, which is the current in the 9 Ω resistor.

P.D. across the 9 Ω resistor, *V*1 = *I* × 9 = 0.5 × 9 = 4.5V

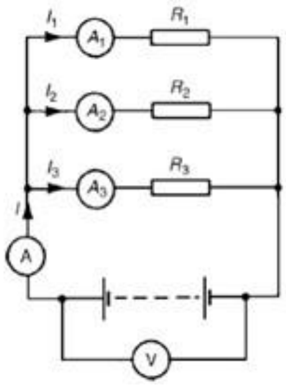
Power dissipated in the 11 Ω resistor, *P* = *I*2*R*=0.52(11)

= 0.25(11)

= 2.75W

1.7.2 Parallel Circuits

Figure shows three resistors, R1, R2 and R3 connected across each other, i.e. in parallel, across a battery source of V volts.



In a parallel circuit:

(a) The sum of the currents I1, I2 and I3 is equal to the total circuit current, I,
i.e. I =I1 +I2 +I3, and the source p.d., V volts, is the same across each of the

From Ohm's law:

I1 = V/R1
I2 = V/R2
I3 = V/R3 and I = V/R

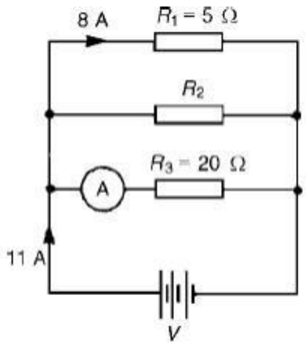
where R is the total circuit resistance.
Since I =I1 + I2 + I3, then

V/R= V/R1+ V/R2+ V/R3

Dividing throughout by V gives,

This equation can be used when finding the total resistance R of a parallel circuit. If two resistors are connected in parallel

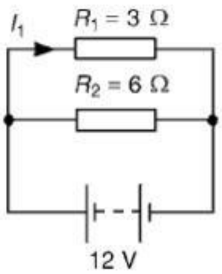
Problem 1: For the circuit shown in Figure, determine (a) the reading on the ammeter, and (b) the value of resistor R2.



P.D. across R1 is the same as the supply voltage V.
Hence supply voltage, V =8 × 5=40V

(a) Reading on ammeter, I = V R3= 40/20=2A

Current flowing through R2 =11−8−2=1A
Hence, R2 = V/I2= 40/1=40 Ω

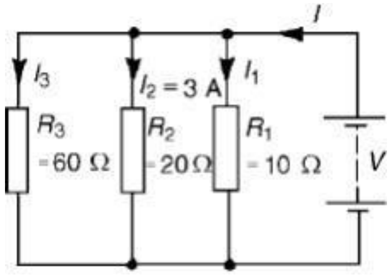


(a) The total circuit resistance R is given by 1/R= 1/R1+ 1/R2= 1/3+ 1/6

1/R= 2 + 1/6= 3/6 Hence, R = 6/3= 2 Ω

(b) Current in the 3 Ω resistance, I1 = V R1= 12/3= 4A

Problem 2: For the circuit shown in Figure find (a) the value of the supply voltage V and (b) the value of current I.



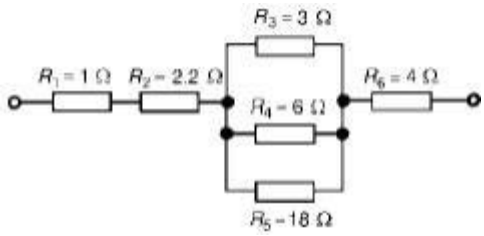
(a) P.D. across 20 Ω resistor = I_2R_2 = 3× 20 = 60V,
hence supply voltage V =60V since the circuit is connected in parallel.

(b) Current $I_1 = V/R_1$= 60/10= 6A;
I_2 = 3A
$I_3 = V/R_3$= 60/60= 1A

Current I =I1+I2+I3 and hence I =6+3+1=10A

Alternatively,
1/R= 1/60+ 1/20+ 1/10= 1 + 3 + 6/60= 10/60
Hence total resistance R= 60/10=6 Ω
Current I = V/R= 60/6=10A

Problem 4: Find the equivalent resistance for the circuit shown in Figure



R_3, R_4 andR_5 are connected in parallel and their equivalent resistance R is given by:
1/R= 1/3+ 1/6+ 1/18=6 + 3 + 1/18= 10/18

Hence R= 18/10=1.8 Ω

The circuit is now equivalent to four resistors in series and the equivalent circuit resistance =1+2.2+1.8+4=9 Ω

1.8 MESH ANALYSIS:

This is an alternative structured approach to solving the circuit and is based on calculating mesh currents. A similar approach to the node situation is used. A set of equations (based on KVL for each mesh) is formed and the equations are solved for unknown values. As many equations are needed as unknown mesh currents exist.

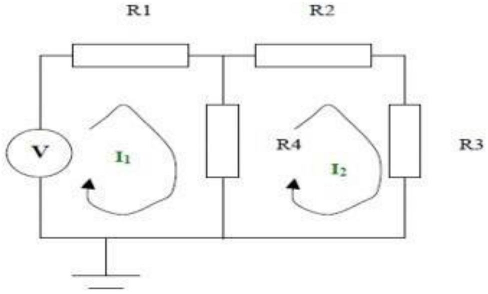
Step 1: Identify the mesh currents
Step 2: Determine which mesh currents are known
Step 3: Write equation for each mesh using KVL and that includes the mesh currents
Step 4: Solve the equations

Step 1:

The mesh currents are as shown in the figure given below

Step 2:

Neither of the mesh currents is known



Step 3:

KVL can be applied to the left hand side loop.
When writing down the voltages across each resistor equations are the mesh currents.

$I_1R_1 + (I_1 - I_2) R_4 - V = 0$ …..(1)

KVL can be applied to the right hand side loop.
When writing down the voltages across ea the equations are the mesh currents.

$I_2R_2 + I_2R_3 + (I_2 - I_1) R_4 = 0$ …..(2)

Step 4:

Solving the equations (1) and (2) we get

$$I_1 = V\frac{R_2 + R_3 + R_4}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_4 + R_3R_4}$$

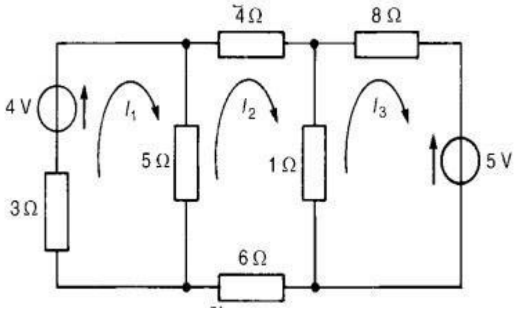
$$I_2 = V\frac{R_4}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_4 + R_3R_4}$$

The individual branch currents can be obtained from the these mesh currents and the node voltages can also be calculated using this information. For example:

$$V_C = I_2R_3 = V\frac{R_3R_4}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_4 + R_3R_4}$$

Problem 1:

Use mesh-current analysis to determine the current flowing in (a the 1Ω resistance of the DC circuit shown in



The mesh currents I1, I2 and I3 are shown in Figure
Using Kirchhoff's voltage law:

For loop 1, (3 + 5) I1 −I2 = 4...(1)
For loop 2, (4 + 1 + 6 + 5) I2 −(5) I1 −(1) I3 = 0…... (2)
For loop 3, (1 + 8) I3 −(1) I2 = 5−..(3)

Thus

8I1 −5I2 −4 =0
−5I1 + 16I2 −I3 =0
−I2 + 9I3 + 5 =0

$$\frac{I_1}{\begin{vmatrix} -5 & 0 & -4 \\ 16 & -1 & 0 \\ -1 & 9 & 5 \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} 8 & 0 & -4 \\ -5 & -1 & 0 \\ 0 & 9 & 5 \end{vmatrix}} = \frac{I_3}{\begin{vmatrix} 8 & -5 & -4 \\ -5 & 16 & 0 \\ 0 & -1 & 5 \end{vmatrix}} = \frac{-1}{\begin{vmatrix} 8 & -5 & 0 \\ -5 & 16 & -1 \\ 0 & -1 & 9 \end{vmatrix}}$$

Using determinants,

$$\frac{I_1}{-5\begin{vmatrix} -1 & 0 \\ 9 & 5 \end{vmatrix} - 4\begin{vmatrix} 16 & -1 \\ -1 & 9 \end{vmatrix}} = \frac{-I_2}{8\begin{vmatrix} -1 & 0 \\ 9 & 5 \end{vmatrix} - 4\begin{vmatrix} -5 & -1 \\ 0 & 9 \end{vmatrix}}$$

$$= \frac{I_3}{-4\begin{vmatrix} -5 & 16 \\ 0 & -1 \end{vmatrix} + 5\begin{vmatrix} 8 & -5 \\ -5 & 16 \end{vmatrix}}$$

$$= \frac{-1}{8\begin{vmatrix} 16 & -1 \\ -1 & 9 \end{vmatrix} + 5\begin{vmatrix} -5 & -1 \\ 0 & 9 \end{vmatrix}}$$

$$\frac{I_1}{-5(-5) - 4(143)} = \frac{-I_2}{8(-5) - 4(-45)}$$

$$= \frac{I_3}{-4(5) + 5(103)}$$

$$= \frac{I_3}{-4(5) + 5(103)}$$

$$= \frac{-1}{8(143) + 5(-45)}$$

$$\frac{I_1}{-547} = \frac{-I_2}{140} = \frac{I_3}{495} = \frac{-1}{919}$$

$$\text{Hence } I_1 = \frac{547}{919} = 0.595\text{ A,}$$

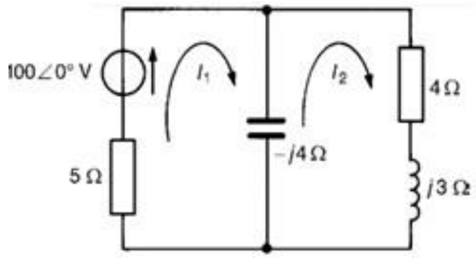
$$I_2 = \frac{140}{919} = 0.152\text{ A, and}$$

$$I_3 = \frac{-495}{919} = -0.539\text{ A}$$

(a) Current in the 5 Ω resistance = $I_1 - I_2$
= 0.595 −0.152
= 0.44A

(b) Current in the 1 Ω resistance = $I_2 - I_3$
= 0.152 − (−0.539)
= 0.69A

Problem 2: For the A.C. network shown in Figure determine, using mesh-current analysis, (a) the mesh currents I_1 and I_2 (b) the current flowing in the capacitor, and (c) the active power delivered by the $100\angle 0^\circ$ V voltage source.



(a) For the first loop

$(5-j4)\ I_1 - (-j4I_2) = 100\angle 0^\circ$ .... (1)

For the second loop

$(4+j3-j4)I_2 - (-j4I_1) = 0$ .... (2)

Rewriting equations (1) and (2) gives:

$(5-j4)I_1 + j4I_2 - 100 = 0$

$j4I_1 + (4-j)\ I_2 + 0 = 0$

Thus, using determinants I_1 and I_2 can be determined

$$\frac{I_1}{\begin{vmatrix} j4 & -100 \\ (4-j) & 0 \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} (5-j4) & -100 \\ j4 & 0 \end{vmatrix}} = \frac{1}{\begin{vmatrix} (5-j4) & j4 \\ j4 & (4-j) \end{vmatrix}}$$

(b) Current flowing

$= I_1 - I_2$

$= 10.77\angle 19.$

$= 4.44 + j12$

i.e. **the current**

(c) Source power P

$$\frac{I_1}{(400-j100)} = \frac{-I_2}{j400} = \frac{1}{(32-j21)}$$

Hence $I_1 = \dfrac{(400-j100)}{(32-j21)} = \dfrac{412.31\angle -14.04^\circ}{38.28\angle -33.27^\circ}$

$= 10.77\angle 19.23^\circ$ A $=$ **$10.8\angle 19.2^\circ$ A,** correct to one decimal place

(Check: power i

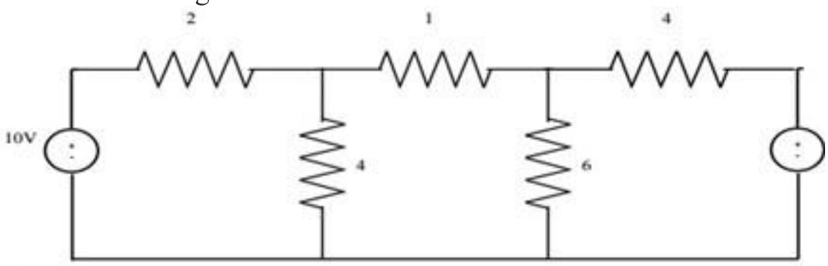
anc

$I_2 = \dfrac{400\angle -90^\circ}{38.28\angle -33.27^\circ} = 10.45\angle -56.73^\circ$ A

$=$ **$10.5\angle -56.7^\circ$ A,**

Thus total power dissipated = 579.97 + 436.81 = 1016.8W = 1020W

Problem 3: Calculate current through 6Ω resistance.



Solution:

Case(1): Consider loop ABGH ; Apply KVL .

$10 = 2I1+ 4(I_1-I_2)$

$10 = 6I_1-4I_2$ -------------------- (1)

Consider loop BCFG

$I_2+6(I_2+I_3) +4(I_2-I_1) = 0$

$1_1I_2+6I_3-4I_1 = 0$ ------------- (2)

Consider loop CDEF

$20 = 4I_3+6(I_2=I_3)$

$20 = 10I_3+6I_2$ ----------------- (3)

$$D = \begin{vmatrix} 6 & -4 & 0 \\ -4 & 11 & 6 \\ 0 & 6 & 10 \end{vmatrix}$$

$$= \begin{vmatrix} 10 \\ 0 \\ 20 \end{vmatrix}$$

$D = [\ 6\ (110 - 36) + 4(-40)] = 284.$

$$D_1 = \begin{vmatrix} 10 & -4 & 0 \\ 0 & 11 & 6 \\ 20 & 6 & 10 \end{vmatrix}$$

$D_1 = 10[110\text{-}36+ (-120)]$

$= 260$

$$D_2 = \begin{vmatrix} 6 & 10 & 0 \\ -4 & 0 & 6 \\ 0 & 20 & 10 \end{vmatrix}$$

$D_2 = 6(-120)-10(-40) = -320$

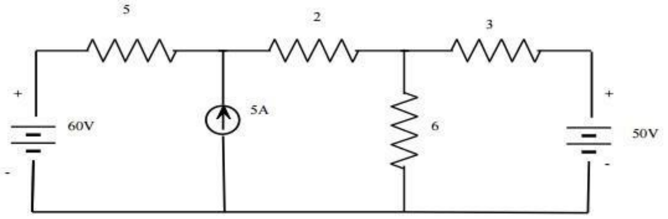
$$D_3 = \begin{vmatrix} 6 & -4 & 10 \\ -4 & 11 & 0 \\ 0 & 6 & 20 \end{vmatrix}$$

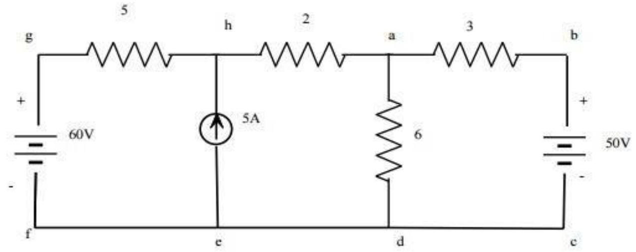
D3 = 6(220) +4(-80) +10(-24)
D3 = 760

I_1 = D1/D = 260/284 = 0.915A
I_2 = D2/D = -320/284 = -1.1267A
I_3 = D3/D = 760/284 = 2.676A

Current through 6Ω + Current through 3Ω = I
= -1.1267+2.676 = 1.55A

Problem 4: Find the current through branch a-b using mesh analysis.





Solution:

Consider loops

Loop HADE - > $5I_1+2I_2+6(I_2-I_3) = 60$

$5I_1+8I_2-6I_3 = 60$------------- (1)

Loop ABCDA - > $3I_3+6(I_3-I_2) = -50$

$3I_3+6I_3-6I_2 = -50$

$9I_3-6I_2 = -50$--------------- (2)

$I_2-I_1 = 5A$............................(3)

From (1), (2) & (3).

$$D = \begin{vmatrix} -1 & 1 & 0 \\ 5 & 8 & -6 \\ 0 & -6 & 9 \end{vmatrix}$$

$$= -1(72-36)-1(45)$$

$$D = -81.$$

$$D_3 = \begin{vmatrix} -1 & 1 & 5 \\ 5 & 8 & 60 \\ 0 & -6 & -50 \end{vmatrix}$$

$= -1(-400+360)-(-250) +5(-30)$

$= 40+250-150$

D3 = 140.

$I_3 = D3/D = 140/-81 = -1.7283$

The current through branch ab is 1.7283A which is flowing from b to a.

1.9 NODAL ANALYSIS

Nodal analysis involves determining all the node voltages in the circuit. The voltage at any given node of a circuit is the voltage drop between that node and a reference node which is usually ground. Once the node voltages are known any of the currents flowing in the circuit can be determined.

First all the nodes in the circuit are counted and identified. Then the nodes at which the voltage is already known are listed. A set of equations based on the node voltages are formed and these equations are solved for unknown quantities. The equations are obtained using KCL. Branch currents can then be found once the node voltages are known.

Step 1: Identify the nodes

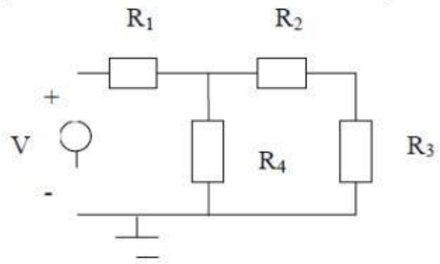
Step 2: Choose a reference node

Step 3: Identify which node voltages are known if any

Step 4: Identify the branch currents

Step 5: Use KCL to write an equation for each unknown node voltage
Step 6: Solve the equations

This is best illustrated with an example. Find all currents and voltages in the following circuit using the node method.



Step 1:

There are four nodes in the circuit. A, B, C and D

Step 2:

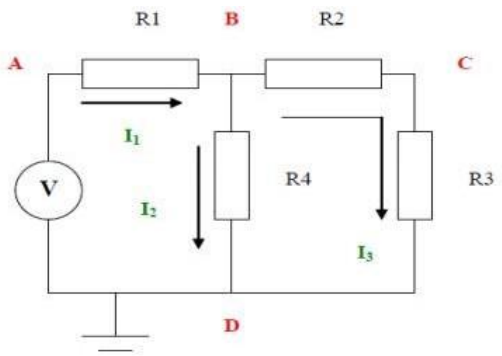
Node D is taken as the reference node.

Step 3:

Node voltage B and C are unknown. Voltage at A is V and at D is 0

Step 4:

The currents are as shown. There are 3 different currents



Step 5:

Apply KCL at node B and node C

Applying KCL for node B, the equation is as follows:

$$\frac{V - V_B}{R_1} + \frac{V_C - V_B}{R_2} + \frac{-V_B}{R_4} = 0 \qquad \text{.......(1)}$$

Applying KCL for node C, the equation is as follows:

$$\frac{V_C - V_B}{R_2} + \frac{-V_B}{R_3} = 0 \qquad \text{.......(2)}$$

Step 6:

Solving the equations (1) and (2) for V_B and V_C, we get

$$V_C = V\frac{R_3R_4}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_4 + R_3R_4}$$

$$V_B = V\frac{R_4(R_2 + R_3)}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_4 + R_3R_4}$$

The branch currents by a simple application of Ohm's Law:

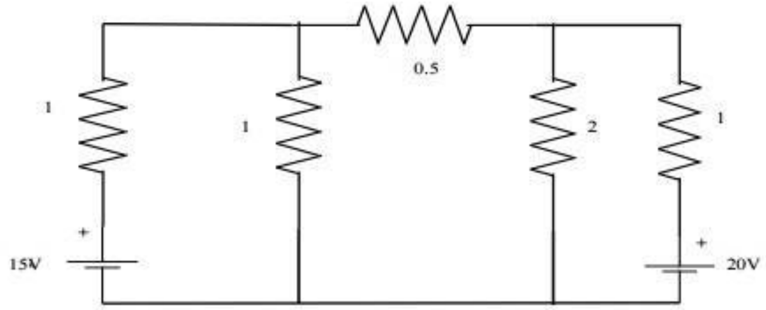
$$I_1 = (V - V_B) / R_1$$

$$I_2 = (V_B - V_C) / R_2$$

$$I_3 = (V_C) / R_3$$

$$I_4 = (V_B) / R_4$$

Problem 1: Find the current through each resistor of the circuit shown in fig, using nodal analysis



Solution:

At node1,

-I1-I2-I3 = 0 -[V1-15/1]-[V1/1][V1-V2/0.5] = 0
-V1+15-V1-2V1+2V2 = 0
4V1-2V2 = 15..............................(1)

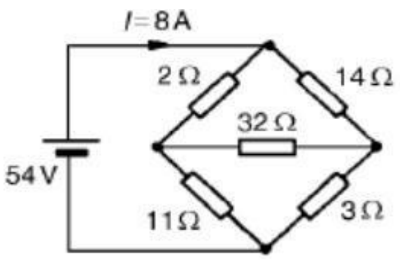
At node2,

I3-I4-I5 = 0
V1-V2/0.5 –V2/2 –V2-20/1 = 0
2V1-2V2-0.5V2-V2 + 20 = 0
2V1-3.5V2 = -20 --------------------(2)

Multiplying (2) by 2 & subtracting from (1)

5V2 = 55 V2 = 11V V = 9.25V
I1 = V1-5/1 = 9.25-15 = -5.75A = 5.75 I2 = V1/1 = 9.25A
I3 = V1-V2/0.5 = -3.5A = 3.5A □ I4 = V2/2 = 5.5A
I5 = V2-20/I = 11-20/1 = -9A=9A.

Problem 2: For the bridge network shown in Figure determine the currents in each of the resistors.



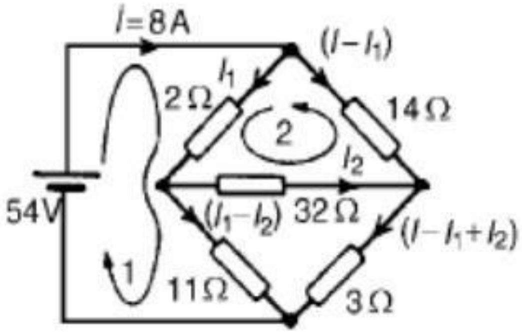
Let the current in the 2Ω resistor be I_1, By Kirchhoff's current law,

The current in the 14Ω resistor is $(I - I_1)$.

Let the current in the 32Ω resistor be I_2 as shown in Figure.

Then the current in the 11Ω resistor is $(I_1 - I_2)$ and that in the 3Ω resistor is $(I - I_1 + I_2)$.

Applying Kirchhoff's law and moving in a clockwise direction as shown in figure gives,



$54 = 2I_1 + 11(I_1 - I_2)$

i.e. $13I_1 - 11I_2 = 54$

Applying Kirchhoff's voltage law to loop 2 and direction as shown in Figure gives:

$0 = 2I_1 + 32I_2 - 14(I - I_1)$

However I = 8 A

Hence $0 = 2I_1 + 32I_2 - 14(8 - I_1)$ i.e. $16I_1 + 32I_2 = 112$

Equations (1) and (2) are simultaneous equations with two unknowns, I_1 and I_2.

16 * (1) gives: $208I_1 - 176I_2 = 864$

16 * (1) gives: $208I_1 - 176I_2 = 864$

13 * (2) gives: $208I_1 + 416I_2 = 1456$

(4) - (3) gives: $592I_2 = 592$, $I_2 = 1$ A

Substituting for I_2 in (1) gives:

$13I_1 - 11 = 54$

$I_1 = 65/13 = 5$ A

Hence,

the current flowing in the 2Ω resistor = $I_1 = 5$ A

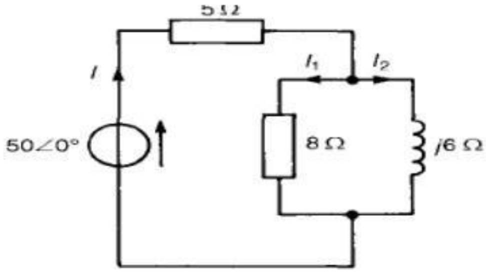
the current flowing in the 14Ω resistor = $I - I_1 = 8 - 5 = 3$ A

the current flowing in the 32Ω resistor = $I_2 = 1$ A

the current flowing in the 11Ω resistor = $I_1 - I_2 = 5 - 1 = 4$ A and

the current flowing in the 3Ω resistor = $I - I_1 + I_2 = 8 - 5 + 1 = 4$ A

Problem 3: Determine the values of currents I, I_1 and I_2 shown in the network of Figure



Total circuit impedance,

$Z_T = 5 + (8)(j6)/8 + j6$

$= 5 + (j48)(8 - j6)/82 + 62$

$= 5 + (j384 + 288)/100$

= (7.88 + j3.84) or 8.776 25.98° A Current I = V/ZT
= 50∟0°/8.77∟25.98°
= 5.7066 −25.98° A

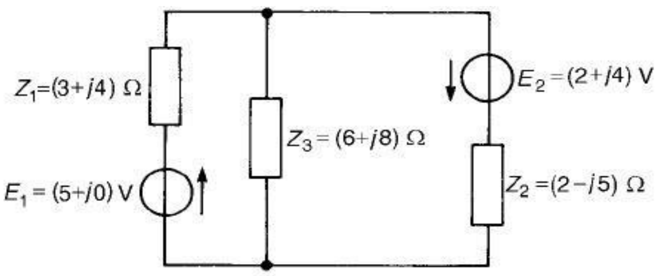
Current I_1 = I (j6/8 + j6)

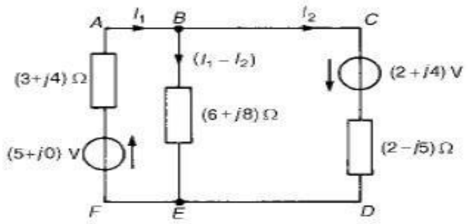
= (5.702∟5.98°) (6∟90°)/10∟36.87°)
= 3.426∟27.15° A

Current I_2 = I (8/ (8 + j6)
= (5-.2570∟.98°)*8∟0°/10∟36.87°
= 4.5666 −62.85° A

[Note: I = I_1 + I_2-62 = 85° 3.42 ∟27.15° + 4.56∟
= 3.043 + j1.561 + 2.081 - j4.058
= 5.124 - j2.497 A = 5.706 -25.98° A

Problem 4: For the AC network shown in figure, determine the current flowing in each branch using Kirchhoff's laws.





$$\text{from which, } \boldsymbol{I_1} = \frac{20 + j55}{64 + j27} = \frac{58.52\angle 70.02^\circ}{69.46\angle 22.87^\circ} = \mathbf{0.842\angle 47.15^\circ\ A}$$
$$= (0.573 + j0.617)\ \text{A}$$
$$= \mathbf{(0.57 + j0.62)\ A}, \text{ correct to two decimal places.}$$

From equation (1), $5 = (9 + j12)(0.573 + j0.617) - (6 + j8)\boldsymbol{I_2}$

$$5 = (-2.247 + j12.429) - (6 + j8)\boldsymbol{I_2}$$

$$\text{from which, } \boldsymbol{I_2} = \frac{-2.247 + j12.429 - 5}{6 + j8}$$
$$= \frac{14.39\angle 120.25^\circ}{10\angle 53.13^\circ}$$
$$= \mathbf{1.439\angle 67.12^\circ A} = (0.559 + j1.326)\ \text{A}$$
$$= \mathbf{(0.56 + j1.33)A}, \text{ correct to two decimal places.}$$

The current in the $(6 + j8)\Omega$ impedance,

$$\boldsymbol{I_1 - I_2} = (0.573 + j0.617) - (0.559 + j1.326)$$
$$= \mathbf{(0.014 - j0.709)A} \text{ or } \mathbf{0.709\angle -88.87^\circ\ A}$$

An alternative method of solving equations (1) and (2) is shown below, using determinants.

$$(9 + j12)I_1 - (6 + j8)I_2 - 5 = 0 \quad (1)$$
$$-(6 + j8)I_1 + (8 + j3)I_2 - (2 + j4) = 0 \quad (2)$$

$$\text{Thus}\quad \frac{I_1}{\begin{vmatrix} -(6+j8) & -5 \\ (8+j3) & -(2+j4) \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} (9+j12) & -5 \\ -(6+j8) & -(2+j4) \end{vmatrix}}$$

$$= \frac{1}{\begin{vmatrix} (9+j12) & -(6+j8) \\ -(6+j8) & (8+j3) \end{vmatrix}}$$

$$\frac{I_1}{(-20+j40)+(40+j15)} = \frac{-I_2}{(30-j60)-(30+j40)}$$

$$= \frac{1}{(36+j123)-(-28+j96)}$$

$$\frac{I_1}{20+j55} = \frac{-I_2}{-j100} = \frac{1}{64+j27}$$

$$\text{Hence}\quad \boldsymbol{I_1} = \frac{20+j55}{64+j27} = \frac{58.52\angle 70.02^\circ}{69.46\angle 22.87^\circ}$$

$$= \mathbf{0.842\angle 47.15^\circ\ A}$$

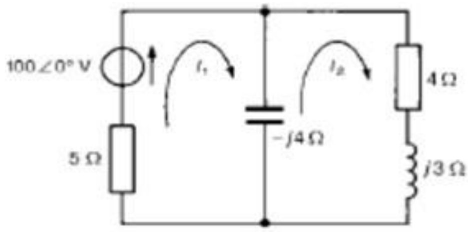
$$\text{and}\quad \boldsymbol{I_2} = \frac{100\angle 90^\circ}{69.46\angle 22.87^\circ} = \mathbf{1.440\angle 67.13^\circ\ A}$$

The current flowing in the $(6+j8)\ \Omega$ impedance is given by:

$$\boldsymbol{I_1} - \boldsymbol{I_2} = 0.842\angle 47.15^\circ - 1.440\angle 67.13^\circ A$$

$$= \mathbf{(0.013 - j0.709)\ A} \text{ or } \mathbf{0.709\angle{-88.95}^\circ\ A}$$

Problem 5: For the A.C. network shown in Figure determine, using mesh-current analysis, (a) the mesh currents I1 and I2 (b) the current flowing in the capacitor, and (c) the active power delivered by the 1006 0° V voltage source.



(a) For the first loop $(5 - j4)I_1 - (-j4I_2) = 100\angle 0^\circ$ (1)

For the second loop $(4 + j3 - j4)I_2 - (-j4I_1) = 0$ (2)

Rewriting equations (1) and (2) gives:

$$(5 - j4)I_1 + j4I_2 - 100 = 0 \quad (1')$$

$$j4I_1 + (4 - j)I_2 + 0 = 0 \quad (2')$$

Thus, using determinants,

$$\frac{I_1}{\begin{vmatrix} j4 & -100 \\ (4-j) & 0 \end{vmatrix}} = \frac{-I_2}{\begin{vmatrix} (5-j4) & -100 \\ j4 & 0 \end{vmatrix}} = \frac{1}{\begin{vmatrix} (5-j4) & j4 \\ j4 & (4-j) \end{vmatrix}}$$

$$\frac{I_1}{(400-j100)} = \frac{-I_2}{j400} = \frac{1}{(32-j21)}$$

$$\text{Hence}\quad \boldsymbol{I_1} = \frac{(400-j100)}{(32-j21)} = \frac{412.31\angle{-14.04}^\circ}{38.28\angle{-33.27}^\circ}$$

$$= 10.77\angle 19.23^\circ\ \text{A} = \mathbf{10.8\angle{-19.2}^\circ\ A},$$

correct to one decimal place

$$\boldsymbol{I_2} = \frac{400\angle{-90}^\circ}{38.28\angle{-33.27}^\circ} = 10.45\angle{-56.73}^\circ\ \text{A}$$

$$= \mathbf{10.5\angle{-56.7}^\circ\ A},$$

correct to one decimal place

(b) Current flowing in capacitor $= I_1 - I_2$

$$= 10.77\angle 19.23^\circ - 10.45\angle{-56.73^\circ}$$

$$= 4.44 + j12.28 = 13.1\angle 70.12^\circ \text{ A},$$

i.e., **the current in the capacitor is 13.1 A**

(c) Source power $P = VI\cos\phi = (100)(10.77)\cos 19.23^\circ$

$$= \mathbf{1016.9\ W} = 1020 \text{ W},$$

correct to three significant figures.

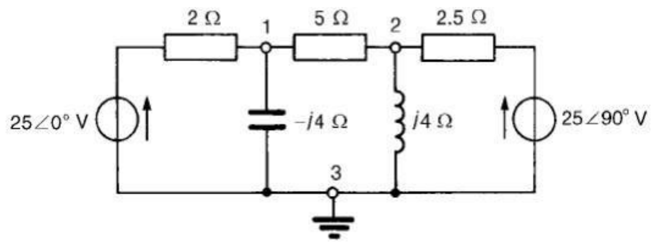
(Check: power in 5 Ω resistor $= I_1^2(5) = (10.77)^2(5) = 579.97$ W

and power in 4 Ω resistor $= I_2^2(4) = (10.45)^2(4) = 436.81$ W

Thus total power dissipated $= 579.97 + 436.81$

$= 1016.8$ W $= 1020$ W, correct to three significant figures.)

Problem 6: In the network of Figure use nodal analysis to determine (a) the voltage at nodes 1 and 2, (b) the current in the j4 Ω inductance, (c) of the active power dissipated-10) in the 2.5 Ω



(a) At node 1, $\dfrac{V_1 - 25\angle 0^\circ}{2} + \dfrac{V_1}{-j4} + \dfrac{V_1 - V_2}{5} = 0$

Rearranging gives:

$$\left(\frac{1}{2} + \frac{1}{-j4} + \frac{1}{5}\right)V_1 - \left(\frac{1}{5}\right)V_2 - \frac{25\angle 0^\circ}{2} = 0$$

i.e., $$(0.7 + j0.25)V_1 - 0.2V_2 - 12.5 = 0 \quad (1)$$

At node 2, $\dfrac{V_2 - 25\angle 90^\circ}{2.5} + \dfrac{V_2}{j4} + \dfrac{V_2 - V_1}{5} = 0$

Rearranging gives:

$$-\left(\frac{1}{5}\right)V_1 + \left(\frac{1}{2.5} + \frac{1}{j4} + \frac{1}{5}\right)V_2 - \frac{25\angle 90^\circ}{2.5} = 0$$

i.e., $$-0.2V_1 + (0.6 - j0.25)V_2 - j10 = 0 \quad (2)$$

Thus two simultaneous equations have been formed with two unknowns, V_1 and V_2. Using determinants, if

$$(0.7 + j0.25)V_1 - 0.2V_2 - 12.5 = 0 \quad (1)$$

and $-0.2V_1 + (0.6 - j0.25)V_2 - j10 = 0$ (2)

then $$\frac{V_1}{\begin{vmatrix} -0.2 & -12.5 \\ (0.6 - j0.25) & -j10 \end{vmatrix}} = \frac{-V_2}{\begin{vmatrix} (0.7 + j0.25) & -12.5 \\ -0.2 & -j10 \end{vmatrix}}$$

$$= \frac{1}{\begin{vmatrix} (0.7 + j0.25) & -0.2 \\ -0.2 & (0.6 - j0.25) \end{vmatrix}}$$

i.e.,
$$\frac{V_1}{(j2 + 7.5 - j3.125)} = \frac{-V_2}{(-j7 + 2.5 - 2.5)}$$

$$= \frac{1}{(0.42 - j0.175 + j0.15 + 0.0625 - 0.04)}$$

and $$\frac{V_1}{7.584\angle -8.53^\circ} = \frac{-V_2}{-7\angle 90^\circ} = \frac{1}{0.443\angle -3.23^\circ}$$

Thus **voltage,** $V_1 = \frac{7.584\angle -8.53^\circ}{0.443\angle -3.23^\circ} = 17.12\angle -5.30^\circ$ V

$= \mathbf{17.1\angle -5.3^\circ}$ **V**, correct to one decimal place,

and **voltage,** $V_2 = \frac{7\angle 90^\circ}{0.443\angle -3.23^\circ} = 15.80\angle 93.23^\circ$ V

$= \mathbf{15.8\angle 93.2^\circ}$ **V**, correct to one decimal place.

(b) The current in the $j4\ \Omega$ inductance is given by:

$\frac{V_2}{j4} = \frac{15.80\angle 93.23^\circ}{4\angle 90^\circ} = \mathbf{3.95\angle 3.23^\circ}$ **A** flowing away from node 2

(c) The current in the 5 Ω resistance is given by:

$$I_5 = \frac{V_1 - V_2}{5} = \frac{17.12\angle -5.30^\circ - 15.80\angle 93.23^\circ}{5}$$

i.e., $$I_5 = \frac{(17.05 - j1.58) - (-0.89 + j15.77)}{5}$$

$$= \frac{17.94 - j17.35}{5} = \frac{24.96\angle -44.04^\circ}{5}$$

$= \mathbf{4.99\angle -44.04^\circ}$ **A** flowing from node 1 to node 2

(d) The active power dissipated in the 2.5 Ω resistor is given by

$$P_{2.5} = (I_{2.5})^2(2.5) = \left(\frac{V_2 - 25\angle 90^\circ}{2.5}\right)^2 (2.5)$$

$$= \frac{(0.89 + j15.77 - j25)^2}{2.5} = \frac{(9.273\angle -95.51^\circ)^2}{2.5}$$

$= \frac{85.99\angle -191.02^\circ}{2.5}$ by de Moivre's theorem

$= 34.4\angle 169^\circ$ W

1.10 GRAPH

Network graph is simply called as **graph**. It consists of a set of nodes connected by branches. In graphs, a node is a common point of two or more branches. Sometimes, only a single branch may connect to the node. A branch is a line segment that connects two nodes.

Any electric circuit or network can be converted into its equivalent **graph** by replacing the passive elements and voltage sources with short circuits and the current sources with open circuits. That means, the line segments in the graph represent the branches corresponding to either passive elements or voltage sources of electric circuit.

1.10.1 Terms and definitions

The description of networks in terms of their geometry is referred to as network topology. The adequacy of a set of equations for analyzing a network is more easily determined topologically than algebraically.

Graph (or linear graph):

A network graph is a network in which all nodes and loops are retained but its branches are represented by lines. The voltage sources are replaced by short circuits and current sources are replaced by open circuits. (Sources without internal impedances or admittances can also be treated in the same way because they can be shifted to other branches by E-shift and/or I-shift operations.)

Branch:

A line segment replacing one or more network elements that are connected in series or parallel.

Node:

Interconnection of two or more branches. It is a terminal of a branch. Usually interconnections of three or more branches are nodes.

Path:

A set of branches that may be traversed in an order without passing through the same node more than once.

Loop: Any closed contour selected in a graph.

Mesh: A loop which does not contain any other loop within it.

Planar graph:

A graph which may be drawn on a plane surface in such a way that no branch passes over any other branch.

Non-planar graph: Any graph which is not planar.

Oriented graph:

When a direction to each branch of a graph is assigned, the resulting graph is called an oriented graph or a directed graph.

Connected graph: A graph is connected if and only if there is a path between every pair of nodes.

Sub graph: Any subset of branches of the graph.

Tree:

A connected sub-graph containing all nodes of a graph but no closed path. i.e. it is a set of branches of graph which contains no loop but connects every node to every other node not necessarily directly. A number of different trees can be drawn for a given graph.

Link:

A branch of the graph which does not belong to the particular tree under consideration. The links form a sub-graph not necessarily connected and is called the co-tree.

Tree compliment: Totality of links i.e. Co-tree.

Independent loop:

The addition of each link to a tree, one at a time, results one closed path called an independent loop. Such a loop contains only one link and other tree branches. Obviously, the number of such independent loops equals the number of links.

Tie set:

A set of branches contained in a loop such that each loop contains one link and the remainder are tree branches.

Tree branch voltages:

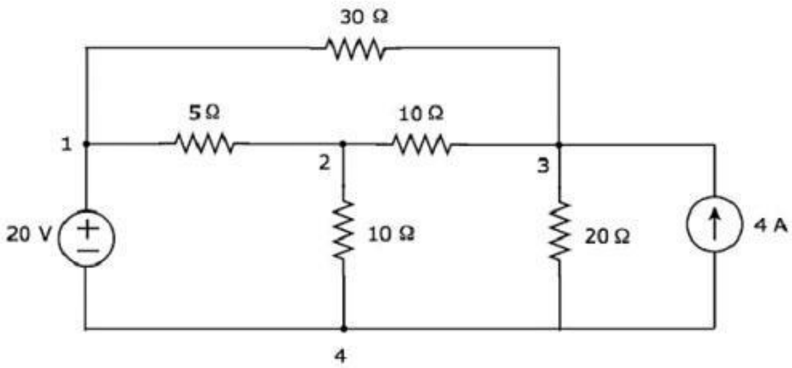
The branch voltages may be separated in to tree branch voltages and link voltages. The tree branches connect all the nodes. Therefore if the tree branch voltages are forced to be zero, then all the node potentials become coincident and hence all branch voltages are forced to be zero. As the act of setting only the tree branch voltages to zero forces all voltages in the network to be zero, it must be possible to express all the link voltages uniquely in terms of tree branch voltages. Thus tree branch form an independent set of equations.

Cut set:

A set of elements of the graph that dissociates it into two main portions of a network such that replacing any one element will destroy this property. It is a set of branches that if removed divides a connected graph in to two connected sub-graphs. Each cut set contains one tree branch and the remaining being links.

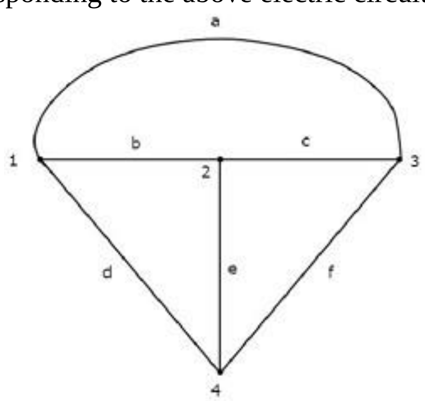
Example

Let us consider the following **electric circuit**.



In the above circuit, there are **four principal nodes** and those are labelled with 1, 2, 3, and 4. There are **seven branches** in the above circuit, among which one branch contains a 20 V voltage source, another branch contains a 4 A current source and the remaining five branches contain resistors having resistances of 30 Ω, 5 Ω, 10 Ω, 10 Ω and 20 Ω respectively.

An equivalent **graph** corresponding to the above electric circuit is shown in the following figure.



In the above graph, there are **four nodes** and those are labelled with 1, 2, 3 & 4 respectively. These are same as that of principal nodes in the electric circuit. There are **six branches** in the above graph and those are labelled with a, b, c, d, e & f respectively.

In this case, we got **one branch less** in the graph because the 4A current source is made as open circuit, while converting the electric circuit into its equivalent graph.

From this Example, we can conclude the following points −

- The **number of nodes** present in a graph will be equal to the number of principal nodes present in an electric circuit.
- The **number of branches** present in a graph will be less than or equal to the number of branches present in an electric circuit.

1.10.2 Types of Graphs

Following are the types of graphs −

- Connected Graph
- Unconnected Graph
- Directed Graph
- Undirected Graph

Connected Graph

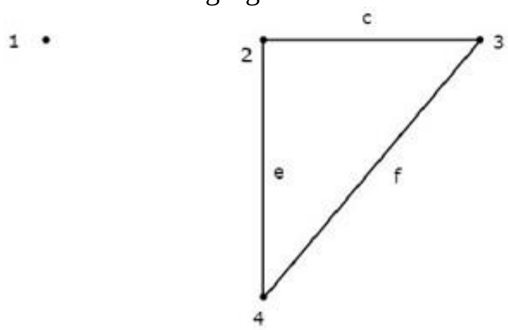
If there exists at least one branch between any of the two nodes of a graph, then it is called as a **connected graph**. That means, each node in the connected graph will be having one or more branches that are connected to it. So, no node will present as isolated or separated.

The graph shown in the previous Example is a **connected graph**. Here, all the nodes are connected by three branches.

Unconnected Graph

If there exists at least one node in the graph that remains unconnected by even single branch, then it is called as an **unconnected graph**. So, there will be one or more isolated nodes in an unconnected graph.

Consider the graph shown in the following figure.

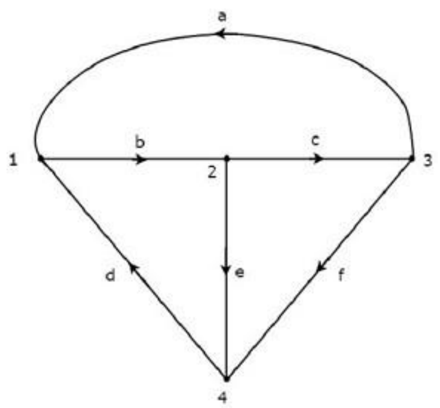


In this graph, the nodes 2, 3, and 4 are connected by two branches each. But, not even a single branch has been connected to the **node 1**. So, the node 1 becomes an **isolated node**. Hence, the above graph is an **unconnected graph**.

Directed Graph

If all the branches of a graph are represented with arrows, then that graph is called as a **directed graph**. These arrows indicate the direction of current flow in each branch. Hence, this graph is also called as **oriented graph**.

Consider the graph shown in the following figure.



In the above graph, the direction of current flow is represented with an arrow in each branch. Hence, it is a **directed graph**.

Undirected Graph

If the branches of a graph are not represented with arrows, then that graph is called as an **undirected graph**. Since, there are no directions of current flow; this graph is also called as an **unoriented graph**.

The graph that was shown in the first Example of this chapter is an unoriented graph, because there are no arrows on the branches of that graph.

Subgraph and its Types

A part of the graph is called as a **subgraph**. We get subgraphs by removing some nodes and/or branches of a given graph. So, the number of branches and/or nodes of a subgraph will be less than that of the original graph. Hence, we can conclude that a subgraph is a subset of a graph.

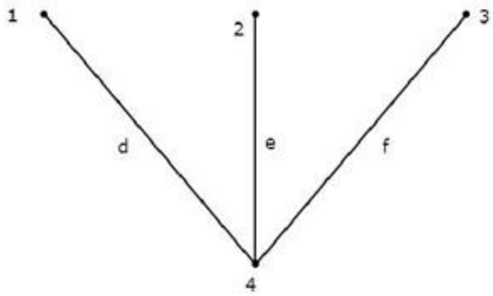
Following are the **two types** of subgraphs.

- Tree
- Co-Tree

Tree

Tree is a connected subgraph of a given graph, which contains all the nodes of a graph. But, there should not be any loop in that subgraph. The branches of a tree are called as **twigs**.

Consider the following **connected subgraph** of the graph, which is shown in the Example of the beginning of this chapter.

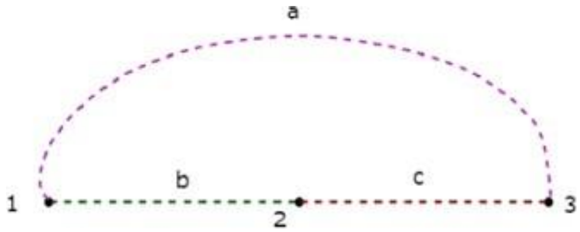


This connected subgraph contains all the four nodes of the given graph and there is no loop. Hence, it is a **Tree**. This Tree has only three branches out of six branches of given graph. Because, if we consider even single branch of the remaining branches of the graph, then there will be a loop in the above connected subgraph. Then, the resultant connected subgraph will not be a Tree.

From the above Tree, we can conclude that the **number of branches** that are present in a Tree should be equal to ***n* - 1** where ‘n’ is the number of nodes of the given graph.

Co-Tree

Co-Tree is a subgraph, which is formed with the branches that are removed while forming a Tree. Hence, it is called as **Complement** of a Tree. For every Tree, there will be a corresponding Co-Tree and its branches are called as **links** or chords. In general, the links are represented with dotted lines.
The **Co-Tree** corresponding to the above Tree is shown in the following figure.



This Co-Tree has only three nodes instead of four nodes of the given graph, because Node 4 is isolated from the above Co-Tree. Therefore, the Co-Tree need not be a connected subgraph. This Co-Tree has three branches and they form a loop.

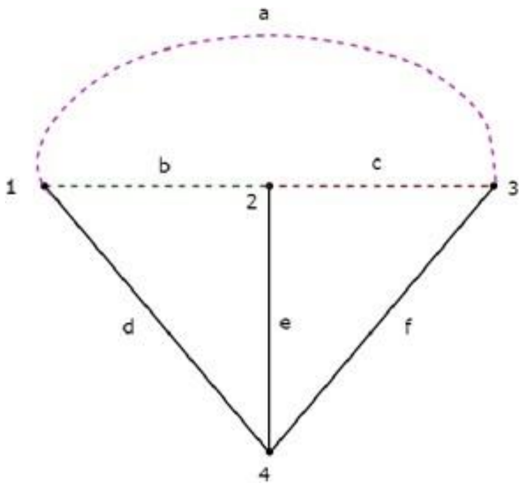
The **number of branches** that are present in a co-tree will be equal to the difference between the number of branches of a given graph and the number of twigs. Mathematically, it can be written as

$$l=b-(n-1)$$
$$l=b-n+1$$

Where,

- ***l*** is the number of links.
- ***b*** is the number of branches present in a given graph.
- ***n*** is the number of nodes present in a given graph.

If we combine a Tree and its corresponding Co-Tree, then we will get the **original graph** as shown below.



The Tree branches d, e & f are represented with solid lines. The Co-Tree branches a, b & c are represented with dashed lines.

1.10.3 Network Topology Matrices

Network Topology Matrices are useful for solving any electric circuit or network problem by using their equivalent graphs.

Matrices Associated with Network Graphs

Following are the three matrices that are used in Graph theory.

- Incidence Matrix
- Fundamental Loop Matrix
- Fundamental Cut set Matrix

Incidence Matrix

An Incidence Matrix represents the graph of a given electric circuit or network. Hence, it is possible to draw the graph of that same electric circuit or network from the **incidence matrix**. We know that graph consists of a set of nodes and those are connected by some branches. So, the connecting of branches to a node is called as incidence. Incidence matrix is represented with the letter A. It is also called as node to branch incidence matrix or **node incidence matrix**.

If there are 'n' nodes and 'b' branches are present in a **directed graph**, then the incidence matrix will have 'n' rows and 'b' columns. Here, rows and columns are corresponding to the nodes and branches of a directed graph. Hence, the **order** of incidence matrix will be **n × b**.

The **elements of incidence matrix** will be having one of these three values, +1, -1 and 0.

- If the branch current is leaving from a selected node, then the value of the element will be +1.
- If the branch current is entering towards a selected node, then the value of the element will be -1.
- If the branch current neither enters at a selected node nor leaves from a selected node, then the value of element will be 0.

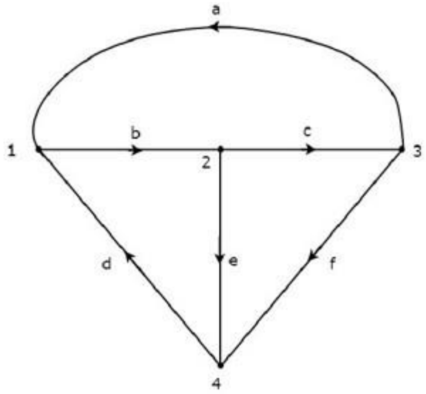
Procedure to find Incidence Matrix

Follow these steps in order to find the incidence matrix of directed graph.

- Select a node at a time of the given directed graph and fill the values of the elements of incidence matrix corresponding to that node in a row.
- Repeat the above step for all the nodes of the given directed graph.

Example

Consider the following **directed graph**.



The **incidence matrix** corresponding to the above directed graph will be

$$A = \begin{bmatrix} -1 & 1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{bmatrix}$$

The rows and columns of the above matrix represents the nodes and branches of given directed graph. The order of this incidence matrix is 4 × 6. By observing the above incidence matrix, we can conclude that the **summation** of column elements of incidence matrix is equal to zero. That means, a branch current leaves from one node and enters at another single node only.

Note − If the given graph is an un-directed type, then convert it into a directed graph by representing the arrows on each branch of it. We can consider the arbitrary direction of current flow in each branch.

Fundamental Loop Matrix

Fundamental loop or **f-loop** is a loop, which contains only one link and one or more twigs. So, the number of f-loops will be equal to the number of links. Fundamental loop matrix is represented with letter B. It is also called as **fundamental circuit matrix** and Tie-set matrix. This matrix gives the relation between branch currents and link currents.

If there are 'n' nodes and 'b' branches are present in a **directed graph**, then the number of links present in a co-tree, which is corresponding to the selected tree of given graph will be b-n+1.

So, the fundamental loop matrix will have 'b-n+1' rows and 'b' columns. Here, rows and columns are corresponding to the links of co-tree and branches of given graph. Hence, the order of fundamental loop matrix will be **(b - n + 1) × b**.

The **elements of fundamental loop matrix** will be having one of these three values, +1, -1 and 0.

- The value of element will be +1 for the link of selected f-loop.
- The value of elements will be 0 for the remaining links and twigs, which are not part of the selected f-loop.
- If the direction of twig current of selected f-loop is same as that of f-loop link current, then the value of element will be +1.
- If the direction of twig current of selected f-loop is opposite to that of f-loop link current, then the value of element will be -1.
-

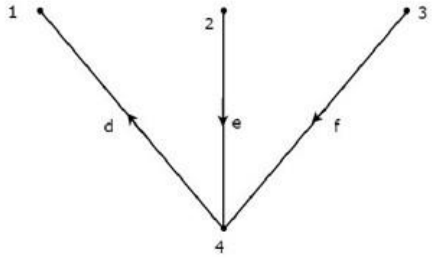
Procedure to find Fundamental Loop Matrix

Follow these steps in order to find the fundamental loop matrix of given directed graph.

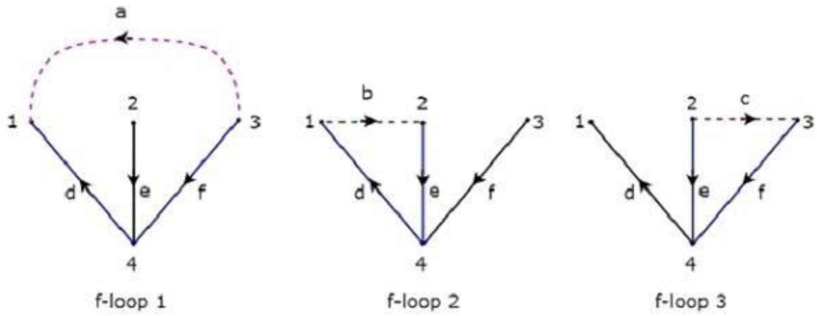
- Select a tree of given directed graph.
- By including one link at a time, we will get one f-loop. Fill the values of elements corresponding to this f-loop in a row of fundamental loop matrix.
- Repeat the above step for all links.

Example

Take a look at the following Tree of **directed graph**, which is considered for incidence matrix.



The above Tree contains three branches d, e & f. Hence, the branches a, b & c will be the links of the Co-Tree corresponding to the above Tree. By including one link at a time to the above Tree, we will get one **f-loop**. So, there will be three **f-loops**, since there are three links. These three f-loops are shown in the following figure.



In the above figure, the branches, which are represented with colored lines form f-loops. We will get the row wise element values of Tie-set matrix from each f-loop. So, the **Tieset matrix** of the above considered Tree will be

$$B = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{bmatrix}$$

The rows and columns of the above matrix represents the links and branches of given directed graph. The order of this incidence matrix is 3 × 6.

The **number of Fundamental loop matrices** of a directed graph will be equal to the number of Trees of that directed graph, because, every Tree will be having one Fundamental loop matrix.

Fundamental Cut-set Matrix

Fundamental cut set or **f-cut set** is the minimum number of branches that are removed from a graph in such a way that the original graph will become two isolated subgraphs. The f-cut set contains only **one twig** and one or more links. So, the number of f-cut sets will be equal to the number of twigs.
Fundamental cut set matrix is represented with letter C. This matrix gives the relation between branch voltages and twig voltages.

If there are 'n' nodes and 'b' branches are present in a **directed graph**, then the number of twigs present in a selected Tree of given graph will be n-1. So, the fundamental cut set matrix will have 'n-1' rows and 'b' columns. Here, rows and columns are corresponding to the twigs of selected tree and branches of given graph. Hence, the **order** of fundamental cut set matrix will be **(n-1) × b**.

The **elements of fundamental cut set matrix** will be having one of these three values, +1, -1 and 0.

- The value of element will be +1 for the twig of selected f-cutset.
- The value of elements will be 0 for the remaining twigs and links, which are not part of the selected f-cutset.
- If the direction of link current of selected f-cut set is same as that of f-cutset twig current, then the value of element will be +1.
- If the direction of link current of selected f-cut set is opposite to that of f-cutset twig current, then the value of element will be -1.

Procedure to find Fundamental Cut-set Matrix

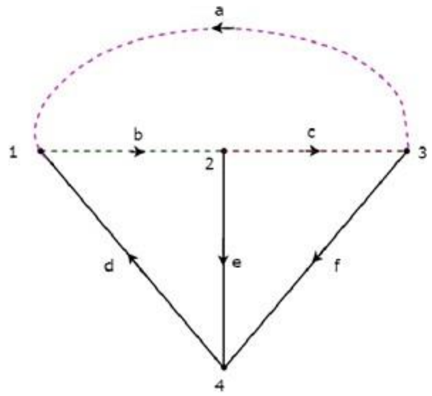
Follow these steps in order to find the fundamental cut set matrix of given directed graph.

- Select a Tree of given directed graph and represent the links with the dotted lines.
- By removing one twig and necessary links at a time, we will get one f-cut set. Fill the values of elements corresponding to this f-cut set in a row of fundamental cut set matrix.
- Repeat the above step for all twigs.

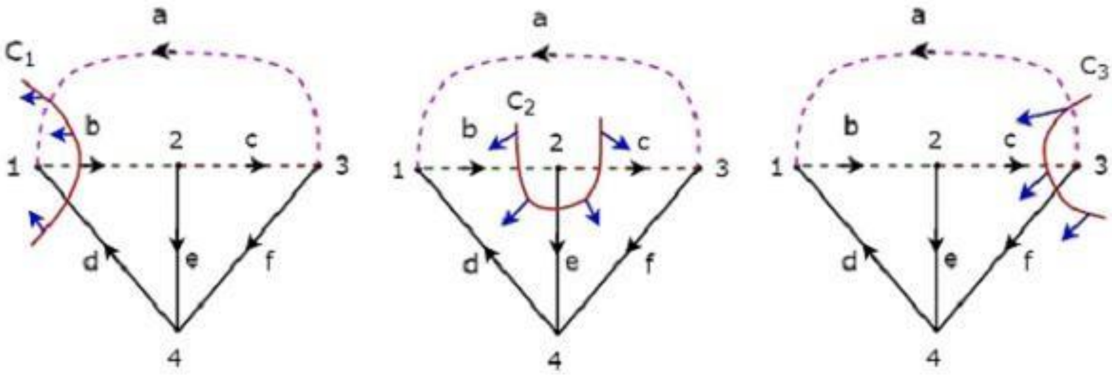
Example

Consider the same **directed graph**, which we discussed in the section of incidence matrix. Select the branches d, e & f of this directed graph as twigs. So, the remaining branches a, b & c of this directed graph will be the links.

The **twigs** d, e & f are represented with solid lines and **links** a, b & c are represented with dotted lines in the following figure.



By removing one twig and necessary links at a time, we will get one f-cut set. So, there will be three f-cut sets, since there are three twigs. These three **f-cut sets** are shown in the following figure.



We will be having three f-cut sets by removing a set of twig and links of C_1, C_2 and C_3. We will get the row wise element values of fundamental cut set matrix from each f-cut set. So, the **fundamental cut set matrix** of the above considered Tree will be

$$C = \begin{bmatrix} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

- The value of element will be +1 for the twig of selected f-cutset.
- The value of elements will be 0 for the remaining twigs and links, which are not part of the selected f-cutset.
- If the direction of link current of selected f-cut set is same as that of f-cutset twig current, then the value of element will be +1.
- If the direction of link current of selected f-cut set is opposite to that of f-cutset twig current, then the value of element will be -1.

The rows and columns of the above matrix represents the twigs and branches of given directed graph. The order of this fundamental cut set matrix is 3×6.

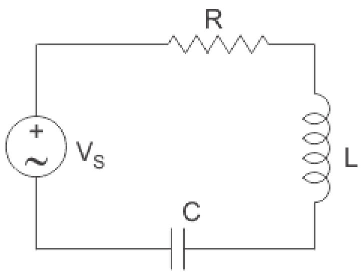
The **number of Fundamental cut set matrices** of a directed graph will be equal to the number of Trees of that directed graph. Because, every Tree will be having one Fundamental cut set matrix.

1.11 DUALITY AND DUAL NETWORKS

Duals: Two circuits are said to be dual of each other, if the mesh equations characterize one of them has the same mathematical form as the nodal equations that characterize the other.

Principle of Duality: Identical behavior patterns observed between voltages and currents between two independent circuits illustrate the principle of duality.

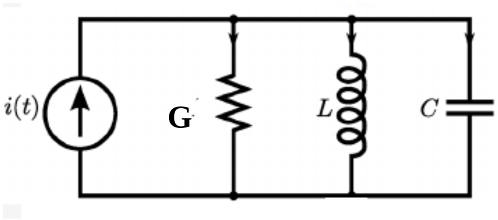
Ex: 1) series R-L-C circuit:



Mesh equation -

— – ∫

— – ∫



Some dual elements:

1) Voltage (V) ←→ Current (I)
2) Resistor (R)←→ Conductance (G)
3) Inductor (I) ←→ Capacitor (C)
4) KVL ←→ KCL
5) 5) V(t) ←→I(t)
6) Mesh ←→ nodal
7) Series ←→ parallel
8) Vsinωt←→Icosωt
9) Open circuit ←→ short circuit
10) Thevenin ←→ Norton
11) Link ←→ twig
12) Cut set ←→ tie set
13) Tree ←→ co-tree
14) Switch in series (getting closed) ←→ switching in parallel (getting opened) etc.

Requirements for a network to be dual:

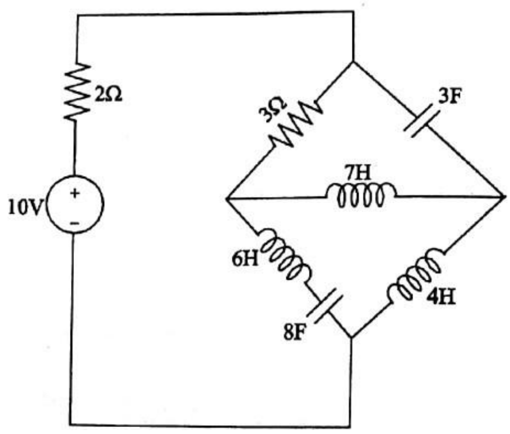
1. Number of meshes in original network should be equal to number of nodes in dual network.
2. Current equation of the original network should be of the same form as the voltage equation of the dual network.
3. Total Impedance of the original network should be equal to the total admittance of the dual network.
4. Sources or elements common between two loops in the original network should be represented common between two nodes in the dual network.
5. Magnitude of voltage sources in the original network should be same as that of the magnitude of the current sources in the dual network.

Procedure to Obtain a Dual Network:

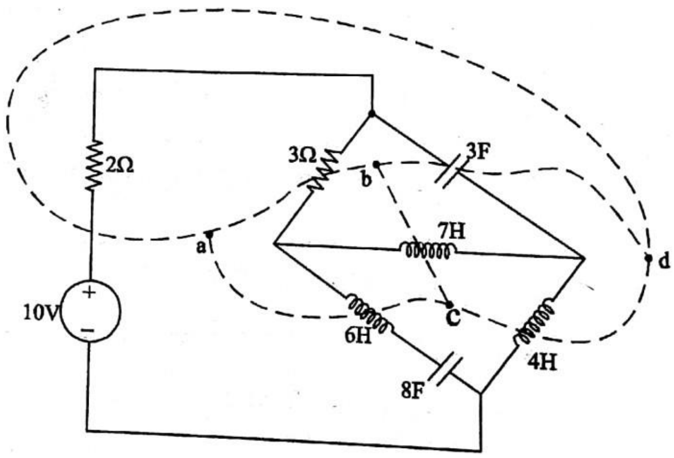
Steps for drawing the dual of a original network using graphical method:

1. Identify the number of loops in the network.
2. Place a node inside each of the available loops and name them.
3. Place a reference node outside the network.
4. Draw a dotted line from each node to the reference node, through all the available branches of the network.
5. No branch elements should be left and also one line should be drawn for one branch element.
6. Voltage rise in the original network should be represented with the current flow towards the node in the dual network.
7. Voltage drop in the original network should be represented with the current flow away from the node in the dual network.
8. For switches closed at t=0 in the original network should be represented as switches opened at t=0.

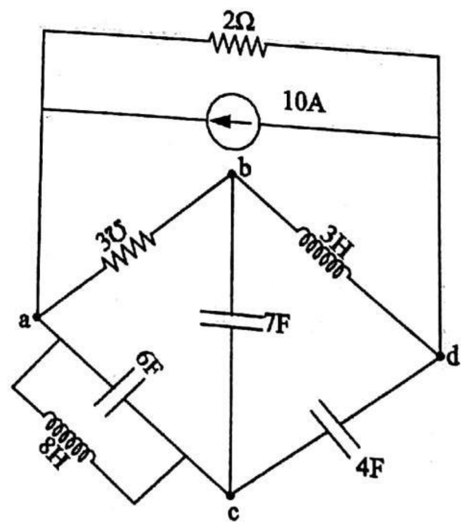
Example: 1



Original Network

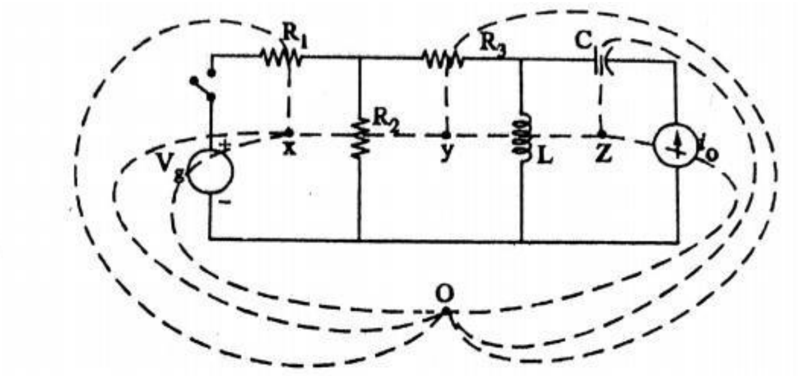


Step by step procedure to construct dual network

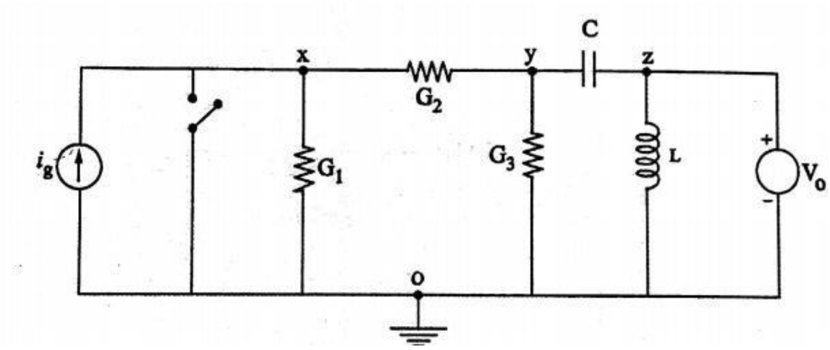


Dual of the original network

Example 2:

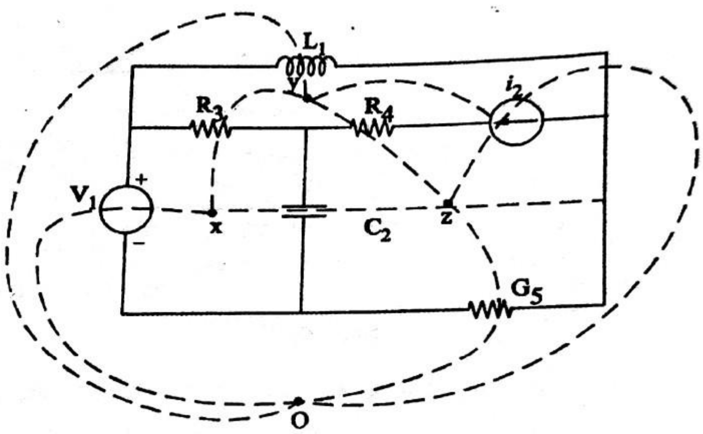


Original Network

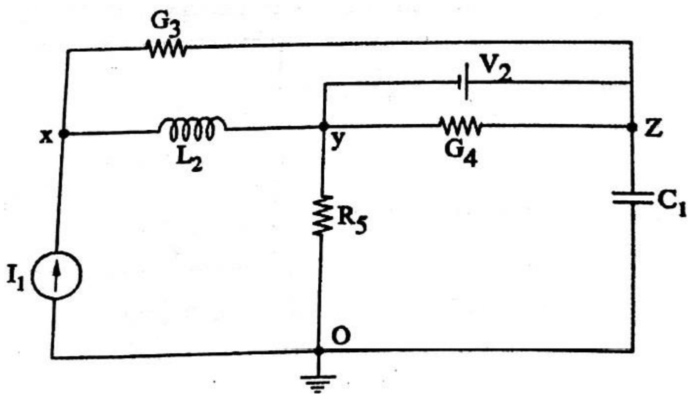


Dual of the original network

Example 3:



Original Network



Dual of the original network