JEPPIAAR INSTITUTE OF TECHNOLOGY

QUESTION BANK

II YEAR – 04TH SEMESTER

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

TABLE OF CONTENT

	Syllabus
I	Probability and Random Variable
II	Two Dimensional Random Variables
III	Classification of Random Processes
IV	Correlation And Spectral Densities
V	Linear Systems With Random Inputs

REGULATION :2017 **ACADEMIC YEAR : 2020-2021**

MA8451 **PROBABILITY AND RANDOM PROCESSES** **L T P C**

4 0 0 4

OBJECTIVES :

- To provide necessary basic concepts in probability and random processes for applications such as random signals, linear systems in communication engineering.
- To understand the basic concepts of probability, one and two dimensional random variables and to introduce some standard distributions applicable to engineering which can describe real life phenomenon.
- To understand the basic concepts of random processes which are widely used in IT fields.
- To understand the concept of correlation and spectral densities.
- To understand the significance of linear systems with random inputs.

UNIT I PROBABILITY AND RANDOM VARIABLES **12**

Probability – Axioms of probability – Conditional probability – Baye‘s theorem - Discrete and continuous random variables – Moments – Moment generating functions – Binomial, Poisson, Geometric, Uniform, Exponential and Normal distributions.

UNIT II TWO - DIMENSIONAL RANDOM VARIABLES **12**

Joint distributions – Marginal and conditional distributions – Covariance – Correlation and linear regression – Transformation of random variables – Central limit theorem (for independent and identically distributed random variables).

UNIT III RANDOM PROCESSES **12**

Classification – Stationary process – Markov process - Markov chain - Poisson process – Random telegraph process.

UNIT IV CORRELATION AND SPECTRAL DENSITIES **12**

Auto correlation functions – Cross correlation functions – Properties – Power spectral density – Cross spectral density – Properties.

UNIT V LINEAR SYSTEMS WITH RANDOM INPUTS **12**

Linear time invariant system – System transfer function – Linear systems with random inputs – Auto correlation and cross correlation functions of input and output.

TOTAL :60 PERIODS

OUTCOMES:

Upon successful completion of the course, students should be able to:

- Understand the fundamental knowledge of the concepts of probability and have knowledge of standard distributions which can describe real life phenomenon.
- Understand the basic concepts of one and two dimensional random variables and apply in engineering applications.

- Apply the concept random processes in engineering disciplines.
- Understand and apply the concept of correlation and spectral densities.
- The students will have an exposure of various distribution functions and help in acquiring skills in handling situations involving more than one variable. Able to analyze the response of random inputs to linear time invariant systems.

TEXT BOOKS:

1. Ibe, O.C.," Fundamentals of Applied Probability and Random Processes ", 1st Indian Reprint, Elsevier, 2007.

2. Peebles, P.Z., "Probability, Random Variables and Random Signal Principles ", Tata McGraw Hill, 4th Edition, New Delhi, 2002.

REFERENCES:

1. Cooper. G.R., McGillem. C.D., "Probabilistic Methods of Signal and System Analysis", Oxford University Press, New Delhi, 3rd Indian Edition, 2012.

2. Hwei Hsu, "Schaum's Outline of Theory and Problems of Probability, Random Variables and Random Processes ", Tata McGraw Hill Edition, New Delhi, 2004.

3. Miller. S.L. and Childers. D.G., ―Probability and Random Processes with Applications to Signal Processing and Communications ", Academic Press, 2004.

4. Stark. H. and Woods. J.W., ―Probability and Random Processes with Applications to Signal Processing ", Pearson Education, Asia, 3rd Edition, 2002.

5. Yates. R.D. and Goodman. D.J., ―Probability and Stochastic Processes", Wiley India Pvt. Ltd., Bangalore, 2nd Edition, 2012.

REGULATION :2017 **ACADEMIC YEAR : 2020-2021**

Subject Code: MA8451 **Year/Semester: II /04**

Subject Name: Probability & Random Processes **Subject Handler: Mr.C.Senthilkumar**

UNIT I –PROBABILITY & RANDOM VARIABLES

Probability – Axioms of probability – Conditional probability – Baye's theorem - Discrete and continuous random variables – Moments – Moment generating functions – Binomial, Poisson, Geometric, Uniform, Exponential and Normal distributions.

PART *A

Q.No.	Questions
1.	**Find the probability of a card drawn at random form an ordinary pack, is a diamond.** BTL2 Total number of ways of getting 1 card = 52 Number of ways of getting 1 diamond card is 13 $\text{Probability} = \frac{\text{Number of favourable events}}{\text{Number of exhaustive events}}$ $= \frac{13}{52} = \frac{1}{4}$
2	**A bag contains 7 white, 6 red and 5 black balls. Two balls are drawn at random. Find the probability that they both will be white.** BTL2 Total balls = 18 From these 18 balls 2 balls can be drawn in $18C_2$ ways Total number of ways of drawing 2 balls = 153 ---------------(1) 2 White balls can be drawn from 7 white balls in $7C_2$ ways. Therefore number of favourable cases = 21 Probability of drawing white balls $= \frac{\text{No., of favourable events}}{\text{Total no., of cases}}$ $= \frac{21}{153} = \frac{7}{51}$
3	**Write the axioms of probability**. BTL1 Let S be a sample space. To each event A, there is a real number P(A) satisfying the following axioms. (i) For any event A, $P(A) \geq 0$ (ii) P(S) = 1 (iii) If $A_1, A_2, ..., A_n$ are finite number of disjoint events of S then $P(A_1 \cup A_2 \cup A_3 \cup ...) = P(A_1) + P(A_2) + P(A_3) + ...$

4	**A and B are events such that** $P(A\cup B)=\frac{3}{4}$; $P(A\cap B)=\frac{1}{4}, P(\overline{A})=\frac{2}{3}$, *Find* $P(\overline{A}/B)$**.(Nov/Dec-2019)** BTL2 $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $\frac{3}{4} \quad =\frac{1}{3}+P(B)-\frac{1}{4}$ $P(B)\ =\frac{2}{3}$ $P(\overline{A}/B)=\frac{P(\overline{A}\cap B)}{P(B)}=\frac{P(B)-P(A\cap B)}{P(B)}=\frac{\frac{2}{3}-\frac{1}{4}}{\frac{2}{3}}=\frac{5}{8}$
5	**Define Baye's theorem.** BTL1 Let $A_1, A_2, ..., A_n$ be 'n' mutually exclusive and exhaustive events with $P(A_i)\neq 0$ for I = 1,2,…n. Let 'B' be an event such that $B\subset\bigcup_{I=1}^{N} A_i$, $P(B)\neq 0$ then $P(A_i/B)=\frac{P(A_i).P(B/A_i)}{\sum_{i=1}^{n} P(A_i).P(B/A_i)}$
6	**Define Random variable. (Nov/Dec2013, Apr/May 2017)** BTL1 A random variable is a function that assigns a real number X(S) to every element $s\in S$ where 'S' is the sample space corresponding to a random experiment E.
7	**Prove that the function P(x) is a legitimate probability mass function of a discrete random variable X, where** $p(x)=\begin{cases}\frac{2}{3}\left(\frac{1}{3}\right)^x, & x=0,1,2...\\ 0 \quad , & otherwise\end{cases}$ **(Apr/May 2017)** BTL5 $\sum p(x)=\sum_{x=0}^{\infty}\frac{2}{3}\left(\frac{1}{3}\right)^x=\frac{2}{3}\left(\frac{1}{3}\right)^0+\frac{2}{3}\left(\frac{1}{3}\right)^1+\frac{2}{3}\left(\frac{1}{3}\right)^2+...$ $=\frac{2}{3}\left[1+\frac{1}{3}+\left(\frac{1}{3}\right)^2+....\right]$ $=\frac{2}{3}\left[1-\frac{1}{3}\right]^{-1}=\frac{2}{3}\left[\frac{2}{3}\right]^{-1}$ $=\frac{2}{3}\left[\frac{3}{2}\right] \quad =1$ Since $\sum p(x)=1$, the given function P(x) is a legitimate probability mass function of a discrete random variable

'X'.

8

A random variable X has the following probability function.

X=x	0	1	2	3	4	5	6	7	8
P(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

Find the value of 'a'. BTL5

$$\sum P(x)=1$$

$$a+3a+5a+7a+9a+11a+13a+15a+17a=1$$

$$81a=1$$

$$a=\frac{1}{81}$$

9

If the random variable X takes the values 1, 2, 3 and 4 such that 2P[X=1] = 3P[X=2] = P[X=3] = 5P[X=4]. Find the probability distribution (Nov/Dec 2016) BTL3

Let P[X=3] = k

2P[X=1] = k $\Rightarrow$ $p[X=1]=\frac{k}{2}$

3P[X=2] = k $\Rightarrow$ $p[X=2]=\frac{k}{3}$

5P[X=4] = k $\Rightarrow$ $p[X=4]=\frac{k}{5}$

We know that $\sum P(x)=1$

$$\frac{k}{2}+\frac{k}{3}+k+\frac{k}{5}=1\Rightarrow\frac{61}{30}k=1\Rightarrow k=\frac{30}{61}$$

The probability distribution of X is given by

X	1	2	3	4
P(x)	$\frac{15}{61}$	$\frac{10}{61}$	$\frac{30}{61}$	$\frac{6}{61}$

10

Find the variance of the discrete random variable X with the probability mass function $P_x(X)=\begin{cases}\frac{1}{3};x=0\\ \frac{2}{3};x=2\end{cases}$

(Nov/Dec 2015 , Nov/Dec 2015) BTL3

The probability distribution of X given by

<table>
<tr><td></td><td>
<table>
<tr><td>X</td><td>0</td><td>2</td></tr>
<tr><td>P(x)</td><td>$\frac{1}{3}$</td><td>$\frac{2}{3}$</td></tr>
</table>
$$E[X]=\sum x\,P(x)=(0)\left(\frac{1}{3}\right)+(2)\left(\frac{2}{3}\right)=0+\frac{4}{3}=\frac{4}{3}$$
$$E[X^2]=\sum x^2P(x)=(0)^2\left(\frac{1}{3}\right)+(2)^2\left(\frac{2}{3}\right)=\frac{8}{3}$$
$$VarX=E[X^2]-(E[X])^2=\frac{8}{3}-\left(\frac{4}{3}\right)^2=\frac{8}{3}-\frac{16}{9}$$
</td></tr>
<tr><td>11</td><td>
Test whether the function defined as follows a density function ? $f(x)=\begin{cases}0 & x<2\\ \frac{1}{18}(3+2x) & 2\le x\le 4\\ 0 & x>4\end{cases}$ BTL4
$$\int_2^4 f(x)\,dx=\int_2^4\frac{1}{18}(3+2x)dx=\frac{1}{18}\left[3(x)_2^4+2\left(\frac{x^2}{2}\right)_2^4\right]$$
$$=\frac{1}{18}[3(4-2)+(16-4)]=\frac{1}{18}(18)=1$$
Hence the given function is a density function.
</td></tr>
<tr><td>12</td><td>
Show that the function $f(x)=\begin{cases}e^{-x} & x\ge 0\\ 0 & x<0\end{cases}$ is a probability density function of a random variable X. BTL5
$$\int f(x)\,dx=\int_0^\infty e^{-x}\,dx=\left[-e^{-x}\right]_0^\infty=-[0-1]=1$$
Hence the given function is a density function.
</td></tr>
<tr><td>13</td><td>
Assume that X is a continuous random variable with the probability density function $f(x)=\begin{cases}\frac{3}{4}(2x-x^2) & 0<x<2\\ 0 & otherwise\end{cases}$. Find P(X>1). BTL3
$$P[X>1]=\int_1^2\frac{3}{4}(2x-x^2)dx=\frac{3}{4}\left[2\left(\frac{x^2}{2}\right)_1^2-\left(\frac{x^3}{3}\right)_1^2\right]$$
$$=\frac{3}{4}\left[(4-1)-\left(\frac{8}{3}-\frac{1}{3}\right)\right]=\frac{1}{2}$$
</td></tr>
<tr><td>14</td><td>
A random variable X is known to have a distributive function $F(x)=u(x)\left[1-e^{-x^2/b}\right], b>0$ is a constant.
</td></tr>
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	Determine density function. BTL 3 $f(x)=F_x(x)=\frac{d}{dx}\left[u(x)\left(1-e^{-x^2/b}\right)\right]$ $=u(x)\left(e^{-x^2/b}\left(-\frac{2x}{b}\right)\right)+u'(x)\left(1-e^{-x\ /b}\right)$ $=\frac{2}{b}x\,u(x)\,e^{-x^2/b}\ +u'(x)\left(1-e^{-x^2/b}\right)$
15	**If** $f(x)=\frac{x^2}{3},-1<x<2$ **is the PDF of the random variable X then find P[0<X<1]. (Apr/May 2018)** BTL3 $\int f(x)\,dx=\int_0^1\frac{x^2}{3}dx=\frac{1}{3}\left[\frac{x^3}{3}\right]_0^1=\frac{1}{9}[1-0]=\frac{1}{9}$
16	**A continuous random variable X has probability density function** $f(x)=\begin{cases}3x^2 & 0\le x\le 1\\ 0 & otherwise\end{cases}$ **Find 'k' such that P[X>k]=0.5** . BTL4 P[X>k] = 0.5 $\Rightarrow\int_k^1 f(x)\,dx=0.5$ $\Rightarrow\int_k^1 3x^2dx=0.5$ $\Rightarrow 3\left[\frac{x^3}{3}\right]_k^1=0.5\ \Rightarrow 1-k^3=0.5$ $\Rightarrow k^3=1-0.5=0.5\Rightarrow k=(0.5)^{\frac{1}{3}}=0.7937$
17	**The cumulative distribution function of the random variable X is given by** $F_x(X)=\begin{cases}0 & ;x<0\\ x+\frac{1}{2} & ;0\le x\le\frac{1}{2}\\ 1 & ;\ x>\frac{1}{2}\end{cases}$ **.** **Find** $P[X>\frac{1}{4}]$. BTL3 $P\left[X>\frac{1}{4}\right]=1-P\left[X\le\frac{1}{4}\right]=1-F\left[\frac{1}{4}\right]=1-\left[\frac{1}{4}+\frac{1}{2}\right]=\frac{1}{4}$
18	**Find the moment generating function of Binomial distribution. (May/June 2013)** BTL3 The P.M.F of Binomial distribution is $P[X=x]=nC_x p^x q^{n-x}$, $x=0,1,2,\ldots,n$

	$M_x(t)=\sum_{x=0}^{n} e^{tx} p(x) = \sum_{x=0}^{n} e^{tx} nC_x p^x q^{n-x}$ $= \sum_{x=0}^{n} nC_x q^{n-x} \left(pe^t\right)^x$ $= nC_0 q^{n-0}\left(pe^t\right)^0 + nC_1 q^{n-1}\left(pe^t\right)^1 + nC_2\, nC_0 q^{n-0}\left(pe^t\right)^0 q^{n-2}\left(pe^t\right)^2 + \ldots + nC_n q^{n-n}\left(pe^t\right)^n$ $= q^n + nC_1 q^{n-1}\left(pe^t\right) + nC_2 q^{n-2}\left(pe^t\right)^2 + \ldots + \left(pe^t\right)^n = \left(q+pe^t\right)^n$
19	**The mean & variance of Binomial distribution are 5 and 4. Determine the distribution.(Apr/May 2015)** BTL4 Given: Mean = np = 5, variance = npq =4 $= 5q = 4 \Rightarrow q = \frac{4}{5}$ $p = 1-q = 1-\frac{4}{5} = \frac{1}{5}$ $np = n\left(\frac{1}{5}\right) = 5 \Rightarrow n = 25$ The P.M.F of the binomial distribution is $P[X=x] = nC_x p^x q^{n-x} \quad x = 0,1,2, \ldots n$ $P[X=x] = 25C_x \left(\frac{1}{5}\right)^x \left(\frac{4}{5}\right)^{n-x}, \; x = 0,1,2,\ldots,25$
20	**Balls are tossed at random into 50 boxes. Find the expected number of tosses required to get the first ball in the fourth box. (Apr/May 2017)** BTL3 Let probability of success be $p = \frac{1}{50}$ According to Geometric distribution, Expected number of tosses to get the first ball in the fourth box = $E[x] = \frac{1}{p} = 50$
21.	**A random variable is uniformly distributed between 3 and 15. Find the variance of X. (Nov/Dec 2015)** BTL3 $Var\ X = \frac{(b-a)^2}{12}$ $= \frac{(15-3)^2}{12} = \frac{144}{12} = 12$
22.	**Messages arrive at a switchboard in a poisson manner at an average rate of six per hour. Find the**

	probability for exactly 2 messages arrive within one hour. (Apr/May 2018) BTL3 $Mean = \lambda = 6\ per\ hour$ $P[X = x] = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-6}6^x}{x!}$ $P[X = 2] = \frac{e^{-6}6^2}{2!} = 0.0446$
23.	**Find the moment generating function of Poisson distribution. (Nov/Dec 2014, Apr/May 2015)** BTL2 The P.M.F of Poisson distribution is $P[X = x] = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0,1,2,\ldots \qquad \lambda > 0$ $M_x(t) = E[e^{tx}] = \sum e^{tx} p(x)$ $= \sum_{x=0}^{\infty} e^{tx} \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda}\sum_{x=0}^{\infty}\frac{(\lambda e^t)^x}{x!}$ $= e^{-\lambda}\left[1 + \frac{(\lambda e^t)^1}{1!} + \frac{(\lambda e^t)^2}{2!} + \ldots\right]$ $= e^{-\lambda} e^{\lambda e^t}$
24.	**Let X be a random variable with M.G.F $M_x(t) = \frac{(2e^t + 1)^4}{81}$. Find its mean and variance. (May/June 2016)** BTL3 $M_x(t) = \frac{(1 + 2e^t)^4}{81} = \left(\frac{1 + 2e^t}{3}\right)^4 = \left(\frac{1}{3} + \frac{2e^t}{3}\right)^4$ Comparing the M.G.F of Binomial distribution , $M_x(t) = (q + pe^t)^n$, we have $p = \frac{2}{3}, q = \frac{1}{3}, n = 4$ Hence $Mean = np = 4\left(\frac{2}{3}\right) = \frac{8}{3}$ $Variance = npq = 4\left(\frac{2}{3}\right)\left(\frac{1}{3}\right) = \frac{8}{9}$
25.	If **X and Y are independent random variables with variance 2 and 3. Find the variance of 3X+4Y. (May/June 2014)** BTL3 Given : *Var(x)= 2 and Var(y) = 3* $Var(aX+bY) = a^2Var(X) + b^2Var(Y)$ $Var(3X+4Y) = 9(2)+16(3)=66$
26.	**If** $f(x) = \begin{cases} cxe^{-x} & x > 0 \\ 0 & elsewhere \end{cases}$ **is the p.d.f of a random variable X. Find 'c'. (NovDec-2019)**BTL5

$$\int_0^\infty cxe^{-x}\,dx = 1$$

W.K.T $c\left[x\left(\frac{e^{-x}}{-1}\right)-(1)\left(e^{-x}\right)\right]_0^\infty = 1$

$$c[(0)-(0-1)] = 1$$

$$c = 1$$

27. **Find the second moment about the origin of the Geometric distribution with parameter p.(Apr/May-2019)BTL-3**

Soln.

Wkt Geometric distribution with parameter p is $P[X = x] = pq^{n-1} \quad n = 0,1,2, ...$

Therefore the second moment about the origin is $M''_X(0) = \frac{1+q}{p^2}$

PART * B

1 **A random variable X has the following probability distribution**

X=x	-2	-1	0	1	2	3
P(X=x)	0.1	K	0.2	2k	0.3	3k

Find (i) The value of 'k'

(ii) Evaluate P(X>2) and P(-2<X<2)

(iii)Find the cumulative distribution of X

(iv) Evaluate the mean of X (8M)(May/June 2010, Nov/Dec 2011, Nov/Dec 2017) BTL5**.**

Answer:Page: 1.80-Dr.A. Singaravelu

- Total Probability $\sum P(x) = 1$
- C.D. F $F(x) = P(X \le x) = \sum_{t \le x} p(t)$
- Mean $E(x) = \sum xP(x)$
- $E(x^2) = \sum x^2 P(x)$
- $VarX = E(X^2) - [E(x)]^2$

- Using $\sum P(x) = 1$, we have $k = \frac{1}{15}$. (1M)
- P(X<2)=0.5, $P(-2 < X < 2) = \frac{2}{5}$. (2M)

- C.D. F , F(-2)=0.1, F(-1)= 0.17, F(0)= 0.37, F(1)=0.5, F(2)= 0.8, F(3)=1. (3M)
- Mean E(x) = $\frac{16}{15}$. (2M)

2

A random variable X has the following probability function

X	0	1	2	3	4	5	6	7
P(x)	0	K	2k	2k	3k	K^2	$2k^2$	$7k^2+k$

Find (i) the value of 'k'

(ii) Evaluate $P[1.5 < X < 4.5 / X > 2]$

(iii) **The smallest value of** λ **for which** $P[X \le \lambda] > \frac{1}{2}$ **(8M) (Nov/Dec 2012,May/June 2012, May/June 2014, A/M 2015)** BTL5

Answer: Page: 1.74-Dr.A.Singaravelu

- Total Probability $\sum P(x) = 1$
- C.D. F $F(x) = P(X \le x) = \sum_{t \le x} p(t)$
- Mean $E(x) = \sum xP(x)$
- $E(x^2) = \sum x^2 P(x)$
- $VarX = E(X^2) - [E(x)]^2$
- Value of $k = \frac{1}{10}$. (2M)
- $P[1.5 < X < 4.5 / X > 2] = \frac{P[1.5 < X < 4.5 \cap X > 2]}{P(X > 2)} = \frac{5}{7}$. (3M)
- The minimum value of $\lambda = 4$. (3M)

3

If the probability mass function of a random variable X is given by $P(X = r) = kr^3$ ***r=1,2,3,4*** **Find the value of 'k',** $P\left(\frac{1}{2} < X < \frac{5}{2} / X > 1\right)$**, mean and variance of X. (8M)(Apr/May 2015)** BTL5

Answer: Page: 1.24- Dr.G. Balaji

- Total Probability $\sum P(x) = 1$
- C.D. F $F(x) = P(X \le x) = \sum_{t \le x} p(t)$
- Mean $E(x) = \sum xP(x)$
- $E(x^2) = \sum x^2 P(x)$
- $VarX = E(X^2) - [E(x)]^2$

	• Value of $k=\frac{1}{100}$. (2M) • $P\left(\frac{1}{2}<X<\frac{5}{2}/X>1\right)=\frac{P\left(\frac{1}{2}<X<\frac{5}{2}\cap X>1\right)}{P(X>1)}=\frac{8}{99}$. (3M) • Mean $E(X)=3.54$, Var(X)= 0.4684. (3M)
4	**If the moments of a random variable 'X' are defined by E(X^r) =0.6; r=1,2,3,… Show that P(X=0)=0.4, P(X=1)=0.6,** $P(X\geq 2)=0$ BTL5 **Answer: Page: 1.70-Dr.G. Balaji** • $M_x(t)=E(e^{tx})=\sum_{x=0}^{\infty}e^{tx}p(x)$ • $M_x(t)=\sum_{x=0}^{\infty}\frac{t^r}{r!}\mu_r'$ • $M_x(t)=\sum_{x=0}^{\infty}\frac{t^r}{r!}\mu_r'=0.4+(0.6)e^t$ • But $M_x(t)=E(e^{tx})=\sum_{x=0}^{\infty}e^{tx}p(x)=p(0)+e^t p(1)+e^{2t}p(2)$. (3M) • Comparing P(X=0) = 0.4, P(X=1)=0.6. (3M) • $P(X\geq 2)=0$. (2M)
5	**A continuous random variable X that can assume any value between x=2 and x=5 has a density function f(x) = k(1+x). Find P[X<4]. (8M) (Nov/Dec 2012, Apr/May 2015)** BTL5 **Answer: Page: 1.88- Dr.A.Singaravelu** • Total probability $\int_{-\infty}^{\infty}f(x)dx=1\Rightarrow\int_{2}^{5}k(1+x)dx=1$. (2M) • The value of $k=\frac{2}{27}$. (3M) • $P[X<4]=\int_{2}^{4}f(x)dx=\frac{16}{27}$. (3M)

6

If the density function of a continuous random variable X is given by $f(x)=\begin{cases} ax & ,0\le x\le 1 \\ a & ,1\le x\le 2 \\ 3a-ax & ,2\le x\le 3 \\ 0 & ,otherwise \end{cases}$ **.Find the value of 'a, and find the c.d.f of X. (8M) (Apr/May 2015)**BTL5

Answer : Page: 1.118- Dr. A. Singaravelu

- $\int_{-\infty}^{\infty} f(x)dx=1 \Rightarrow \int_0^1 ax\,dx+\int_1^2 a\,dx+\int_2^3 (3a-ax)\,dx=1$ (1M)
- Value of a=0.5. (1M)
- For c.d.f , If x<0, F(x)=0. (1M)
- If $0\le x\le 1, F(x)=\frac{x^2}{4}$. (1M)
- $1\le x\le 2, F(x)=\frac{x}{2}-\frac{1}{4}$. (2M)
- $2\le x\le 3, F(x)=-\frac{x^2}{4}+\frac{3}{2}x-\frac{5}{4}$, For x>3, F(x)=1. (2M)

7

A continuous random variable 'X' has the density function f(x) given by . $f(x)=\frac{k}{1+x^2}, -\infty<x<\infty$ **Find the value of 'k' and the cumulative distribution of 'X'.(8M) (Nov/Dec 2014, Apr/May 2018)** BTL5

Answer: Page: 1.123- Dr. A. Singaravelu

- $\int_{-\infty}^{\infty} f(x)dx=1 \Rightarrow \int_0^1 \frac{k}{1+x^2}dx=1$. (2M)
- The value of $k=\frac{1}{\pi}$. (2M)
- The c.d.f is $F(x)=\int_{-\infty}^{x} f(x)dx=\int_{-\infty}^{x}\frac{1}{\pi}\left(\frac{1}{1+x^2}\right)dx=\frac{1}{\pi}\left[\tan^{-1}x+\frac{\pi}{2}\right]$. (4M)

8

Let 'X' be the random variable that denotes the outcome of the roll of a fair die. Compute the mean and variance of 'X'.(8M)(Apr/May 2018) BTL4

Answer : Page: 1.177- Dr. A. Singaravelu

- $P(X=i)=\frac{1}{6}, i=1,2,\ldots,6$. (1M)
- $M_x(t)=\sum_{i=1}^{6} e^{it}P(X=i)=\frac{1}{6}\left[e^t+e^{2t}+\ldots+e^{6t}\right]$. (2M)
- $E(x)=\left[M_x{}'(t)\right]_{t=0}=\frac{7}{2}$. (2M)
- $.E(x^2)=\left[M_x{}''(t)\right]_{t=0}=\frac{91}{6}$. (2M)
- $Var(X)=E(X^2)-\left[E(X)\right]^2=\frac{35}{12}$. (1M)

9	**For the triangular distribution** $f(x)=\begin{cases} x & ,0<x\le 1 \\ 2-x, & 1\le x\le 2 \\ 0 & ,otherwise \end{cases}$ **.Find the mean, variance, moment generating function. (8M) (Nov/Dec 2013)** BTL5 **Answer : Page: 1.180- Dr. A. Singaravelu** • $M_x(t)=E[e^{tx}]=\frac{[e^t-1]^2}{t^2}$. (3M) • Mean $E(X)=\int_{-\infty}^{\infty} x\, f(x)\,dx=1$. (2M) • $E(X^2)=\int_{-\infty}^{\infty} x^2\, f(x)\,dx=\frac{7}{6}$. (2M) • $Var(X)=E(X^2)-[E(X)]^2=\frac{1}{6}$ (1M)
10	**Find the M.G.F of the random variable X having the probability density function** $f(x)=\begin{cases} \frac{x}{4}e^{-x/2} & ,x>0 \\ 0 & ,elsewhere \end{cases}$ **(8M) (May/June2012, May/June 2014)** BTL5 **Answer: Page:1.74-Dr. G. Balaji** • $M_x(t)=E[e^{tx}]=\int_0^{\infty} e^{tx}\frac{x}{4}e^{-x/2}\,dx=\frac{1}{(1-2t)^2}$. (1M) • $M_x(t)=1+\frac{t}{1!}\mu_1'+\frac{t^2}{2!}\mu_2'+\frac{t^3}{3!}\mu_3'+\ldots$ (1M) • $M_x(t)=1+\frac{t}{1!}(4)+\frac{t^2}{2!}(24)+\frac{t^3}{3!}(192)+\ldots$ (2M) • $\mu_1'=coefficient\ of\ \frac{t}{1!}=4$. (1M) • $\mu_2'=coefficient\ of\ \frac{t^2}{2!}=24$. (1M) • $\mu_3'=coefficient\ of\ \frac{t^3}{3!}=192$. (1M) • $\mu_4'=coefficient\ of\ \frac{t^4}{4!}=1920$. (1M)
11	**Find the MGF of the Binomial distribution and hence find the mean and variance. (8M)(Apr/May 2011, May/June2019)**BTL2 **Answer : Page: 1.190- Dr. A. Singaravelu** • $P(x)=nC_x\,p^x q^{n-x}$, $x=0,1,2,\ldots,n$. (1M)

	• $M_x(t)=E[e^{tx}]=\left(q+pe^t\right)^n$. (2M) • Mean $E(X)=\left[M_x'(t)\right]_{t=0}=np$. (2M) • $E(X^2)=\left[M_x''(t)\right]_{t=0}=n^2p^2+npq$. (2M) • Var(X)= npq. (1M)
12	**Derive Poisson distribution form Binomial distribution. (8M)(Nov/Dec 2014, Apr/May 2019)**BTL2 **Answer : Page: 1.219 – Dr. A. Singaravelu** The Binomial distribution becomes Poisson distribution under the following conditions (2M) • The number of trials is very large • The probability of success is very small • $np=\lambda$ • $P(X=x)=\lim_{n\to\infty} nC_x p^x q^{n-x} = \lim_{n\to\infty}\frac{(1-1/n)(1-2/n)\ldots(1-(x-1)/n)}{x!}\lambda^x\frac{(1-\lambda/n)^n}{(1-\lambda/n)^x}$. (4M) • $P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!}$. (2M)
13	**It is known that the probability of an item produced by a certain machine will be defective is 0.05. If the produced items are sent to the market in packets of 20, find the number of packets containing atleast, exactly and atmost 2 defective items in a consignment of 1000packets using binomial and Poisson distribution.(8M) (Nov/Dec 2017)** BTL5 **Answer : Page: 1.116 – Dr. GBalaji** Probability of Binomial Distribution $P(X=x)=nC_x p^x q^{n-x}$ Probability of Poisson Distribution $P(X=x)=\frac{e^{-\lambda}\lambda^x}{x!}$ Binomial Distribution • Number of packets containing atleast 2 defective items $=NP(X\geq 2)=264$. (2M) • Number of packets containing exactly 2 defective items $=NP(X=2)=189$. (1M) • Number of packets containing atmost 2 defective items $=NP(X\leq 2)=925$. (1M) Poisson Distribution • Number of packets containing atleast 2 defective items $=NP(X\geq 2)=264$. (2M) • Number of packets containing exactly 2 defective items $=NP(X=2)=184$. (1M) • Number of packets containing atmost 2 defective items $=NP(X\leq 2)=920$. (1M)
14	**The number of monthly breakdown of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month (1)without a breakdown, (2)with only one breakdown and (3)with atleast one breakdown(8M) (Nov/Dec 2017)** BTL5

Answer : Page: 1.227- Dr. A. Singaravelu

Probability of Poisson Distribution $P(X=x)=\frac{e^{-\lambda}\lambda^{x}}{x!}$

- P(without a breakdown) = P(X=0) = 0.1653. (2M)
- P(with only one breakdown) = P(X=1)=0.2975. (2M)
- P(with atleast 1 breakdown)= $P(X\geq 1)=1-P(X<1)=0.8347$. (4M)

15

State and prove the Memoryless property of Geometric distribution.(8M)(Nov/Dec2015, May/June 2016) BTL1

Answer : Page: 1.254- Dr. A. Singaravelu

Probability of Geometric distribution P(X=x) = $q^{x-1}p$, x=1,2,…

- $P[X>m+n/X>m]=\frac{P[X>m+n\cap X>m]}{P[X>m]}$. (2M)
- P[X>k] = q^k (4M)
- $P[X>m+n/X>m]=\frac{P[X>m+n]}{P[X>m]}=q^{n}$. (2M)

16

If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test (a) on the fourth trial, (b) in fewer than 4 trials. (8M) (May/June2015) BTL5

Answer : Page: 1.137- Dr. G. Balaji

Probability of Geometric distribution P(X=x) = $q^{x-1}p$, x=1,2,…

- P(on the fourth trial) = P(X=4) = 0.0064. (4M)
- P(fewer than 4 trials) = P(X<4) = 0.992. (4M)

17

A coin is tossed until the first head occurs. Assuming that the tosses are independent and the probability of a head occurring is 'p', find the value of 'p' so that the probability that an odd number of tosses is required, is equal to 0.6. Can you find a value of 'p' so that the probability is 0.5 that an odd number of tosses is required? (8M)(Nov/Dec 2010, Nov/Dec 2016) BTL4

Answer : Page: 1.135- Dr. G. Balaji

Probability of Geometric distribution P(X=x) = $q^{x-1}p$, x=1,2,…

- P[X= odd number of tosses] $=\frac{1}{1+q}=0.6$ (3M)
- $q=\frac{2}{3}, p=1-q=\frac{1}{3}$. (1M)
- P[X= odd number of tosses] $=\frac{1}{1+q}=0.5$ (3M)
- q=1, p=0 . (1M)

18

Determine the moment generating function of Uniform distribution in (a,b) and hence find the mean and

variance. (8M) (Nov/Dec 2017, Apr/May 2018) BTL2

Answer : Page: 1.256- Dr. A. Singaravelu

The probability function of Uniform distribution is $f(x)=\begin{cases}\dfrac{1}{b-a}, a<x<b\\ 0 \quad ,otherwise\end{cases}$

- $M_x(t)=E[e^{tx}]=\int_a^b e^{tx} f(x)dx=\dfrac{\left(e^{bt}-e^{at}\right)}{t(b-a)}$. (3M)
- Mean $E(X)=\int_a^b x\, f(x)\, dx=\dfrac{b+a}{2}$. (2M)
- $E(X^2)=\int_a^b x^2\, f(x)\, dx=\dfrac{b^2+ab+a^2}{3}$. (2M)
- $Var(X)=\dfrac{(b-a)^2}{12}$. (1M)

19 **Suppose 'X' has an exponential distribution with mean=10, Determine the value of 'x' such that P(X<x)=0.95. (8M) (Nov/Dec 2015, Apr/May 2017)** BTL5

Answer : Page: 1.143- P. Sivaramakrishna Dass

The probability function of exponential distribution is $f(x)=\begin{cases}\lambda e^{-\lambda x}, x\geq 0\\ 0 \quad ,otherwise\end{cases}$

- $Mean=\dfrac{1}{\lambda}=10 \Rightarrow \lambda=\dfrac{1}{10}$. (2M)
- P(X<x)= 1-P(X>x) =0.95. (2M)
- $1-e^{-\frac{x}{10}}=0.95 \Rightarrow x=29.96$. (4M)

20 **The time in hours required to repair a machine is exponentially distributed with perimeter $\lambda=\dfrac{1}{2}$.**

(i) **What is the probability that the repair time exceeds 2h**

(ii) **What is the conditional probability that a repair takes atleast 10h given that its duration exceeds 9h? (8M) (May/June 2012, Nov/Dec 2016, Nov/Dec 2017)** BTL3

Answer : Page: 1.274- Dr. A. Singaravelu

The probability function of exponential distribution is $f(x)=\begin{cases}\lambda e^{-\lambda x}, x\geq 0\\ 0 \quad ,otherwise\end{cases}$

- P(the repair time exceeds 2h) $P(X>2)=\int_2^\infty \dfrac{1}{2}e^{-x/2}dx$ (2M)
- $P(X>2)=0.3679$. (2M)

	• $P(X \geq 10/X > 9) = P(X > 1) = \int_1^\infty \frac{1}{2} e^{-x/2} dx$. (2M) • $P(X \geq 10/X > 9) = 0.6065$. (2M)
21	**In a test 2000 electric bulbs, it was found that the life of a particular make, was normally distributed with an average life of 2040 hours and S.D. of 60 hours. Estimate the number of bulbs likely to burn for (i)more than 2150 hours, (ii)less than 1950 hours and (iii) more than 1920 hours but less than 2160 hours. (8M) (Nov/Dec 2017)** BTL5 **Answer: Page:1.293 -A. Singaravelu** • $z = \frac{X-\mu}{\sigma}$ • P(more than 2150 hrs) = P(X>2150) =P(z > 1.833) = 0.5-P(0 < z < 1.833) = 0.0336. (2M) • The number of bulbs expected to burn for more than 2150hrs = 2000 x 0.0336 = 67. (1M) • P(Less than 1950 hrs) = P(X< 1950) = P(z< -1.5) = 0.5 – P(0< z< 1.5) = 0.0668. (2M) • The number of bulbs expected to burn for less than 1950hrs = 2000 x 0.0668 = 134. (1M) • P(more than 1920 hrs but less than 2160 hrs) = P(1920 <X < 2160) = P(-2 < z < 2) = 0.9546.(1M) • The number of bulbs = 2000 x 0.9546 = 1909. (1M)
22	**In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and variance of the distribution. (8M) (Nov/Dec 2012, Nov/Dec 2015)** BTL5 **Answer: Page: 1.295- A. Singaravelu** • $z = \frac{X-\mu}{\sigma}$ • $45 - \mu = -0.49\sigma$. (2M) • P(Z > Z_1) = 0.8 or P(0 < Z < Z_2)= 0.42. (1M) • From tables , Z_2 = 1.40. (1M) • $64 - \mu = 1.40\sigma$. (2M) • Solving, $\sigma = 10, \mu = 50$. (2M)
23	**The contents of urns I, II, III are as follows:** **1 white, 2 red and 3 black balls** **2 white, 3 red and 1 black balls and** **3 white, 1 red and 2 black balls.** **One urn is chosen at random and 2 balls are drawn. They happen to be white and red. What is the probability that they came from urns I, II, III. (Nov/Dec 2019)** BTL5 **Answer: Page: 1.60-Dr. A. Singaravelu** Let $A_1, A_2, ..., A_n$ be 'n' mutually exclusive and exhaustive events with $P(A_i) \neq 0$ for I = 1,2,…n. Let 'B' be an event such that $B \subset \bigcup_{I=1}^{N} A_i$, $P(B) \neq 0$ then $P(A_i / B) = \frac{P(A_i).P(B/A_i)}{\sum_{i=1}^{n} P(A_i).P(B/A_i)}$

- $P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ (1M)
- $P(A/E_1) = \frac{1C_1 \times 2C_1}{6C_2} = \frac{2}{15}$, $P(A/E_2) = \frac{2C_1 \times 3C_1}{6C_2} = \frac{6}{15}$, $P(A/E_3) = \frac{3C_1 \times 1C_1}{6C_2} = \frac{3}{15}$ (2M)
- $P(E_2/A) = \frac{P(E_2).P(A/E_2)}{\sum_{i=1}^{3} P(E_i).P(A/E_i)} = \frac{6}{11}$ (2M)
- $P(E_3/A) = \frac{P(E_3).P(A/E_3)}{\sum_{i=1}^{3} P(E_i).P(A/E_i)} = \frac{3}{11}$ (2M)
- $P(E_1/A) = 1 - P(E_2/A) - P(E_3/A) = \frac{2}{11}$ (1M)

UNIT II – TWO - DIMENSIONAL RANDOM VARIABLES

Joint distributions – Marginal and conditional distributions – Covariance – Correlation and linear regression – Transformation of random variables – Central limit theorem (for independent and identically distributed random variables).

PART *A

Q.No.	Questions
1.	**State the basic properties of joint distribution of (X,Y) where X and Y are random variables. (May/June 2014)** BTL1 Properties of joint distribution of (X,Y) are (i) $F[-\infty, y] = 0 = F[x, -\infty]$ *and* $F[-\infty, -\infty] = 0, F[\infty, \infty] = 0$ (ii) $P[a < X < b, Y \le y] = F(b, y) - F(a, y)$ (iii) $P[X \le x, c < Y < d] = F(x, d) - F(x, c)$ (iv) $P[a < X < b, c < Y < d] = F(b, d) - F(a, d) - F(b, c) + F(a, c)$ (v) At points of continuity of *f(x,y)*, $\frac{\partial^2 F}{\partial x \partial y} = f(x, y)$
2	**The joint probability mass function of a two dimensional random variable (X,Y) is given by p(x,y) = f(2x + y); x = 1,2 and y = 1,2 where 'k' is a constant. Find the value of 'k'.(Nov/Dec 2015)** BTL5 The joint pmf of (X,Y) is

x \ y	1	2

	1	3k	4k
	2	5k	6k

We have $\sum\sum p(x,y)=1$

Therefore, *3k + 4k + 5k + 6k = 1*

18 k=1 $k=\frac{1}{18}$.

3

The joint probability density function of the random variables (X,Y) is given by $f(x,y)=kxye^{-(x^2+y^2)},\ x>0,y>0$. Find the value of 'k'. (Apr/May 2015) BTL5

$$\iint f(x,y)\,dkx\,dy=1$$

$$\int_0^\infty\int_0^\infty kxye^{-(x^2+y^2)}\,dx\,dy=1$$

$$k\int_0^\infty ye^{-y^2}dy\int_0^\infty xe^{-x^2}dx=1$$

Put $x^2=t$

$2xdx=dt$

$xdx=\frac{dt}{2}$

$$k\int_0^\infty ye^{-y^2}dy\int_0^\infty e^{-t}\frac{dt}{2}=1$$

$$\frac{k}{2}\int_0^\infty ye^{-y^2}\left[-e^{-t}\right]_0^\infty dy=1$$

$$\frac{k}{2}\int_0^\infty ye^{-y^2}[0+1]dy=1$$

$$\frac{k}{2}\int_0^\infty e^{-t}\frac{dt}{2}=1$$

We have $\frac{k}{4}\left[-e^{-t}\right]_0^\infty=1$

$$\frac{k}{4}[0+1]=1\Rightarrow k=4$$

4

If the function f(x,y) = c(1-x)(1-y) , 0 < x < 1 , 0 < y < 1 is to be a density function, find the value of 'c'.(8M) (Nov/Dec 2017) BTL5

$$\iint f(x,y)\,dx\,dy = 1$$

$$\int_0^1\int_0^1 c(1-x)(1-y)\,dx\,dy = 1$$

$$c\int_0^1 (1-y)dy \int_0^1 (1-x)dx = 1$$

$$c\left[y-\frac{y^2}{2}\right]_0^1\left[x-\frac{x^2}{2}\right]_0^1 = 1$$

$$c\left[1-\frac{1}{2}\right]\left[1-\frac{1}{2}\right] = 1$$

$$c\left[\frac{1}{2}\right]\left[\frac{1}{2}\right] = 1$$

$$c\left[\frac{1}{4}\right] = 1 \Rightarrow c = 4$$

5

The joint pdf of (X,Y) is $f_{xy}(x,y) = xy^2 + \frac{x^2}{8}, 0 \le x \le 2, 0 \le y \le 1$**. Find P(X<Y). (May/June 2013, Apr/May 2019)**BTL5

$$P(X<Y) = \int_0^1\int_0^y\left(xy^2+\frac{x^2}{8}\right)dx\,dy$$

$$= \int_0^1\left[y^2\left(\frac{x^2}{2}\right)_0^y + \frac{1}{8}\left(\frac{x^3}{3}\right)_0^y\right]dy$$

$$= \int_0^1\left[\frac{y^2}{2}\left(y^2\right) + \frac{1}{24}\left(y^3\right)\right]dy = \int_0^1\left[\frac{y^4}{2}+\frac{y^3}{24}\right]dy$$

$$= \frac{1}{2}\left(\frac{y^5}{5}\right)_0^1 + \frac{1}{24}\left(\frac{y^4}{4}\right)_0^1 = \frac{1}{10}(1-0) + \frac{1}{96}(1-0) = \frac{53}{480}$$

6

If the joint pdf of (X,Y) is $f(x,y) = \begin{cases} \frac{1}{4} & ,0 < x,y < 2 \\ 0 & ,otherwise \end{cases}$ **. Find** $P[X+Y \le 1]$BTL5

$$P[X+Y \le 1] = \int_0^1\int_0^{1-y}\left(\frac{1}{4}\right)dx\,dy = \frac{1}{4}\int_0^1 (x)_0^{1-y}\,dy$$

$$= \frac{1}{4}\int_0^1 (1-y)dy \quad = \frac{1}{4}\left[y-\frac{y^2}{2}\right]_0^1$$

$$= \frac{1}{4}\left[1-\frac{1}{2}\right] = \frac{1}{8}$$

7	**Find the marginal density function of X and Y if** $f(x,y)=\begin{cases}\frac{6}{5}\left(x+y^2\right) & ,0\le x,y\le 1\\ 0 & ,otherwise\end{cases}$ **(Nov/Dec 2012)** BTL5 Marginal density function of X is $f_x(x)=\int f(x,y)\,dy=\int_0^1\frac{6}{5}\left(x+y^2\right)dy=\frac{6}{5}\left[xy+\frac{y^3}{3}\right]_0^1=\frac{6}{5}\left[x+\frac{1}{3}\right]\ 0\le x\le 1$ Marginal density function of Y is $f_y(y)=\int f(x,y)\,dx=\int_0^1\frac{6}{5}\left(x+y^2\right)dy=\frac{6}{5}\left[\frac{x^2}{2}+y^2x\right]_0^1=\frac{6}{5}\left[\frac{1}{2}+y^2\right]\ 0\le y\le 1$
8	**The joint probability density function of the random variable X and Y is** $f(x,y)=\begin{cases}25e^{-5y} & ,\ 0<x<0.2,\ y>0\\ 0 & ,otherwise\end{cases}$ **.Find the marginal PDF of X and Y. (Nov/Dec 2016)** BTL5 Marginal density function of X is $f_x(x)=\int f(x,y)\,dy=\int_0^\infty 25e^{-5y}dy=25\left[\frac{e^{-5y}}{-5}\right]_0^\infty=-5[0-1]=5\ \ 0\le x\le 0.2$ Marginal density function of Y is $f_y(y)=\int f(x,y)\,dx=\int_0^{0.2}25e^{-5y}dx=25e^{-5y}[x]_0^{0.2}=2e^{-5y}[0.2-0]=5e^{-5y}\ \ y>0$
9	**If X and Y are independent random variables having the joint density function** $f(x,y)=\frac{1}{8}(6-x-y),0<x<2,\ 2<y<4$ **. Find P[X+Y<3]**. BTL5

	$P[X+Y<3]=\frac{1}{8}\int_2^3\int_0^{3-y}(6-x-y)\,dx\,dy$ $=\frac{1}{8}\int_2^3\left[(6-y)(x)-\frac{x^2}{2}\right]_0^{3-y}dy=\frac{1}{8}\int_2^3\left[(6-y)(3-y)-\frac{(3-y)^2}{2}\right]dy$ $=\frac{1}{8}\int_2^3\left[18-9y+y^2-\frac{1}{2}(3-y)^2\right]dy$ $=\left[18y-9\frac{y^2}{2}+\frac{y^3}{3}-\frac{1}{2}\frac{(3-y)^3}{-3}\right]_2^3$ $=\left[18(3)-\frac{9}{2}(9)+\frac{27}{3}+\frac{1}{6}(0)\right]-\left[18(2)-\frac{9}{2}(4)+\frac{8}{3}+\frac{1}{6}(1)\right]$ $=\left[18-\frac{45}{2}+\frac{19}{3}-\frac{1}{6}\right]=\frac{5}{24}$
10	**Let X and Y be random variables with joint density function** $f(x,y)=\begin{cases}4xy & ,0\le x\le 1,\ 0\le y\le 1\\ 0 & ,otherwise\end{cases}$ **Find E[XY].** BTL5 $E[XY]=\iint xy\,f(x,y)\,dx\,dy=\int_0^1\int_0^1 xy(4xy)\,dx\,dy$ $=4\int_0^1 x^2dx\int_0^1 y^2dy$ $=4\left[\frac{x^3}{3}\right]_0^1\left[\frac{y^3}{3}\right]_0^1=\frac{4}{9}(1)(1)=\frac{4}{9}$
11	**Let X and Y be a two-dimensional random variable. Define covariance of (X,Y). If X and Y are independent, what will be the covariance of (X,Y)? (May/June 2016)** BTL2 Covariance of (X,Y) is defined as *Cov(X,Y) = E[XY] – E[x]E[Y]* If X and Y are independent, then *Cov(X,Y) =0.*
12	**Two random variables X and Y have the joint pdf** $f(x,y)=\begin{cases}\frac{xy}{96} & ;0<x<4,1<y<5\\ 0 & ;otherwise\end{cases}$**. Find** ***Cov(X,Y)*. (May/June 2016)** BTL5 *Cov(X,Y) = E[XY] – E[x]E[Y]*

$$E[X]=\iint x\,f(x,y)\,dx\,dy=\int_1^5\int_0^4 x\left(\frac{xy}{96}\right)dx\,dy=\frac{1}{96}\int_1^5 y\,dy\int_0^4 x^2\,dx$$

$$=\frac{1}{96}\left[\frac{y^2}{2}\right]_1^5\left[\frac{x^3}{3}\right]_0^4=\frac{1}{576}[25-1][64]=\frac{8}{3}$$

$$E[Y]=\iint y\,f(x,y)\,dx\,dy=\int_1^5\int_0^4 y\left(\frac{xy}{96}\right)dx\,dy=\frac{1}{96}\int_1^5 y^2\,dy\int_0^4 x\,dx$$

$$==\frac{1}{96}\left[\frac{y^3}{3}\right]_1^5\left[\frac{x^2}{2}\right]_0^4=\frac{1}{576}[125-1][16]=\frac{31}{9}$$

$$E[XY]=\iint xy\,f(x,y)\,dx\,dy=\int_1^5\int_0^4 xy\left(\frac{xy}{96}\right)dx\,dy=\frac{1}{96}\int_1^5 y^2\,dy\int_0^4 x^2\,dx$$

$$=\frac{1}{96}\left[\frac{y^3}{3}\right]_1^5\left[\frac{x^3}{3}\right]_0^4=\frac{1}{864}[125-1][64]=\frac{248}{27}$$

$$\therefore\ Cov(X,Y)=\left[\frac{248}{27}\right]-\left[\frac{8}{3}\right]\left[\frac{31}{9}\right]=0$$

13 **Let X and Y be any two random variables a,b be constants. Prove that Cov(aX,bY)=abCov(X,Y).** BTL5

Cov(X,Y) = E[XY] –E[X]E[Y]

Cov(aX,bY)= E[aX bY] – E[aX] E[bY]

=ab E[XY] –ab E[X]E[Y]

=ab[*E[XY] –E[X]E[Y]]*

= ab *Cov(X,Y)*

14 **If Y = - 2X + 3, Find Cov(X,Y).** BTL3

Cov(X,Y) = E[XY] –E[X]E[Y]

=E[X(-2X+3)]-E[X]E[-2X+3]

= E[$-2X^2$+3X] – E[X][-2E[X]+3]

= -2E[X^2]+3E[X]+2$(E[X])^2$-3E[x]

= -2(E[X^2]-$(E[X])^2$) = -2Var X

15 **If X_1 has mean 4 and variance 9 while X_2 has mean -2 and variance 5 and the two are independent , find Var($2X_1+X_2-5$).** BTL3

E[X_1]=4 , E[X_2] = -2
Var[X_1] = 9, Var[X_2] = 5
Var($2X_1+X_2-5$) = 4 VarX_1 + VarX_2
= 4(9) + 5 = 41.

16 **If X and Y are independent random variables then show that E[Y/X] = E[Y] , E[X/Y] = E[X].**

	(Nov/Dec 2016) BTL5 $$E[Y/X]=\int y.\frac{f(x,y)}{f(x)}dy$$ Since X and Y are independent, $$E[Y/X]=\int y.\frac{f(x).f(y)}{f(x)}dy=\int y.f(y)dy=E[Y]$$ $$E[X/Y]=\int x.\frac{f(x,y)}{f(y)}dx$$ Since X and Y are independent, $$E[X/Y]=\int x.\frac{f(x).f(y)}{f(y)}dx=\int x.f(x)dx=E[X]$$
17	**Find the acute angle between the two lines of regression. (Apr/May 2015, Apr/May 2019)** BTL3 The equations of the regression are $$y-\bar{y}=r\frac{\sigma_y}{\sigma_x}(x-\bar{x}) \quad ----------(1)$$ $$x-\bar{x}=r\frac{\sigma_x}{\sigma_y}(y-\bar{y}) \quad ----------(2)$$ Slope of line (1) is $m_1=r\frac{\sigma_y}{\sigma_x}$ Slope of line (2) is $m_2=\frac{1}{r}\frac{\sigma_y}{\sigma_x}$ If θ is the acute angle between the two lines, then $$\tan\theta=\frac{\lvert m_1-m_2\rvert}{1+m_1m_2}$$ $$=\frac{\left\lvert r\frac{\sigma_y}{\sigma_x}-\frac{\sigma_y}{r\sigma_x}\right\rvert}{1+r\frac{\sigma_y}{\sigma_x}.\frac{\sigma_y}{r\sigma_x}}=\frac{\left\lvert\frac{(r^2-1)}{r}\frac{\sigma_y}{\sigma_x}\right\rvert}{1+\frac{\sigma_y^2}{\sigma_x^2}}$$ $$=\frac{\left\lvert\frac{-(1-r^2)}{r}\frac{\sigma_y}{\sigma_x}\right\rvert}{\frac{\sigma_x^2+\sigma_y^2}{\sigma_x^2}}=\frac{(1-r^2)\sigma_x\sigma_y}{\lvert r\rvert(\sigma_x^2+\sigma_y^2)}$$
18	**The regression equations are 3x + 2y = 26 and 6x + y = 31. Find the correlation coefficient between X and Y.** BTL5 Let 3x + 2y = 26 be the regression equation of Y on X. Therefore, $2y=-3x+26 \Rightarrow y=-\frac{3}{2}x+\frac{26}{2}$. The regression coefficient $b_{yx}=-\frac{3}{2}$

	Let 6x + y = 31 be the regression equation of X on Y. Therefore, $6x = -y + 31 \Rightarrow x = -\frac{1}{6}y + \frac{31}{6}$ The regression coefficient $b_{xy} = -\frac{1}{6}$ Hence, correlation coefficient r_{xy} is given by $r_{xy} = \pm\sqrt{b_{yx} \times b_{xy}} = \pm\sqrt{\left(\frac{-3}{2}\right)\left(\frac{-1}{6}\right)} = \pm\sqrt{\frac{1}{4}} = \pm 0.5$ =-0.5, since both the regression coefficients are negative.
19	**The two regression equations of two random variables X and Y are 4x – 5y +33 = 0 and 20x – 9y =107. Find the mean values of X and Y. (Nov/Dec 2015)** BTL5 Replace x and y as $\bar{x}$ and $\bar{y}$, we have $4\bar{x} - 5\bar{y} = -33 \quad ------(1)$ $20\bar{x} - 9\bar{y} = 107 \quad ------(2)$ Solving the equations (1) and (2), we have $\bar{x} = 13$ and $\bar{y} = 17$.
20	**Can y=5+2.8x and x=3-0.5y be the estimated regression equations of y on x and x on y respectively, explain your answer. (Nov/Dec 2016)** BTL4 Since the signs of regression co-effieients are not the same, the given equation is not estimated regression equation of y on x and x on y.
21	**If X has an exponential distribution with parameter 1. Find the pdf of $y = \sqrt{x}$.** BTL3 $y = \sqrt{x} \Rightarrow x = y^2$ Since $dx = 2y\,dy \Rightarrow \frac{dx}{dy} = 2y$ Since X has an exponential distribution with parameter 1, the pdf of X is given by, $f_x(x) = e^{-x}, x > 0 \qquad [f(x) = \lambda e^{-\lambda x}, \lambda = 1]$ $\therefore f_y(y) = f_x(x)\left\|\frac{dx}{dy}\right\|$ $= e^{-x}2y = 2ye^{-y^2} \quad y > 0$
22	**State Central limit theorem.** BTL1 If $X_1, X_2, \ldots, X_n, \ldots$ be a sequence of independent identically distributed random variables with $E(X_i) = \mu$ and $Var(X_i) = \sigma^2$, i=1,2,… and if $S_n = X_1 + X_2 + \ldots + X_n$, then under certain general conditions, S_n follows a normal distribution with mean $n\mu$ and variance $n\sigma^2$ as $n \to \infty$
23	**If X and Y have joint pdf of $f(x,y) = \begin{cases} x+y, & 0 < x, y < 1 \\ 0 & ,elsewhere \end{cases}$. Check whether X and Y are independent.** BTL4 The marginal function of X is $f(x) = \int_0^1 (x+y)dy = \left[xy + \frac{y^2}{2}\right]_0^1 = x + \frac{1}{2}$, 0<x<1 The marginal function of Y is

	$f(y)=\int_0^1 (x+y)dx=\left[\frac{x^2}{2}+y\right]_0^1=y+\frac{1}{2}$, 0<y<1 Now, $f(x).f(y)=\left(x+\frac{1}{2}\right)\left(y+\frac{1}{2}\right)=xy+\frac{1}{2}(x+y)+\frac{1}{4}\neq x+y\neq f(x,y)$ Hence X and Y are not independent.
24	**Assume that the random variables X and Y have the probability density function f(x,y). What is E[E[X/Y]]? (Apr/May 2017)** BTL5 $E[[X/Y]]=\int_{-\infty}^{\infty} E[X/Y]f(y)dy$ $=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\,f(x/y)dx\,f(y)dy$ $=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\,f(x/y)\;f(y)dx\,dy$ $=\int_{-\infty}^{\infty} x\int_{-\infty}^{\infty} f(x,y)\,dy\,dx$ $=\int_{-\infty}^{\infty} x\,f(x)dx=E(X)$
25	**Define the joint density function of two random variables X and Y.** BTL1 If (X,Y) is a two dimensional continuous random variables such that , a function f which assigns each (X,Y) a real number f(x,y) for all real x,y then f(x,y) is called the joint pdf of (X,Y), provided f(x,y) satisfies the following conditions (i) $f(x,y)\geq 0,\ for\ all\,(x,y)\in R$ (ii) $\iint_R f(x,y)dx\,dy=1$
	Part*B
1	**The joint pmf of (X,Y) is given by P(x,y) = k(2x + 3y), x = 0,1,2 ; y = 1,2,3. Find all the marginal and conditional probability distributions. Also, find the probability distribution of (X+Y). (10M) (Nov/Dec 2014, Nov/Dec 2019)** BTL5 **Answer: Pg. 2.8 – Dr. A. Singaravelu** • $k=\frac{1}{72}$. (1M) • Marginal distribution of X: $P(X=0)=\frac{18}{72},P(X=1)=\frac{24}{72},P(X=2)=\frac{30}{72}$ (1M) • Marginal distribution of Y: $P(Y=1)=\frac{15}{72},P(Y=2)=\frac{24}{72},P(Y=3)=\frac{33}{72}$ (1M) • Conditional distribution of X given Y: $P[X=x_i/Y=y_1]=\frac{1}{5},\frac{1}{3},\frac{7}{15}$ (1M) • $P[X=x_i/Y=y_2]=\frac{1}{4},\frac{1}{3},\frac{5}{12}$. (1M) • $P[X=x_i/Y=y_3]=\frac{9}{33},\frac{1}{3},\frac{13}{33}$. (1M)

	• Conditional distribution of Y given X: $P[Y=y_i/X=x_0]=\frac{1}{6},\frac{1}{3},\frac{1}{2}$. (1M) • $P[Y=y_i/X=x_1]=\frac{5}{24},\frac{1}{3},\frac{11}{24}$. (1M) • $P[Y=y_i/X=x_2]=\frac{7}{30},\frac{1}{3},\frac{13}{30}$. (1M) • Total probability distribution of X+Y is 1. (1M)
2	**The two dimensional random variable (X,Y) has the joint pmf** $f(x,y)=\frac{x+2y}{27}, x=0,1,2; y=0,1,2$ **Find the conditional distribution of Y for X=x. (8M) (Nov/Dec 2017)** BTL5 **Answer : Pg. 2.13 – Dr. A. Singaravelu** • Marginal distribution of X: $P(X=0)=\frac{6}{27}, P(X=1)=\frac{9}{27}, P(X=2)=\frac{12}{27}$ (1M) • Marginal distribution of Y: $P(Y=0)=\frac{3}{27}, P(Y=1)=\frac{9}{27}, P(Y=2)=\frac{15}{27}$ (1M) • Conditional distribution of Y given X: $P[Y=y_i/X=x_0]=0,\frac{1}{3},\frac{2}{3}$. (2M) • $P[Y=y_i/X=x_1]=\frac{1}{9},\frac{1}{3},\frac{5}{9}$. (2M) • $P[Y=y_i/X=x_2]=\frac{1}{6},\frac{1}{3},\frac{1}{2}$. (2M)
3	**Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denote the number of red balls drawn, find the joint probability distribution of (X,Y). (8M)(Apr/May 2015, May/June 2016)** BTL5 **Answer: Page: 2.20- Dr. G. Balaji** • Let X denote number of white balls drawn and Y denote the number of red balls drawn. • $P(X=0,Y=0)=\frac{1}{21}, P(X=0,Y=1)=\frac{3}{14}, P(X=0,Y=2)=\frac{1}{7}, P(X=0,Y=3)=\frac{1}{84}$ (3M) • $P(X=1,Y=0)=\frac{1}{7}, P(X=1,Y=1)=\frac{2}{7}, P(X=1,Y=2)=\frac{1}{14}$ (3M) • $P(X=2,Y=0)=\frac{1}{21}, P(X=2,Y=1)=\frac{1}{28}$ (2M)
4	**The joint pdf of the random variable (X,Y) is given by** $f(x,y)=Kxye^{-(x^2+y^2)}, x>0, y>0$**. Find the value of 'K' and also prove that X and Y are independent. (8M) (Apr/May 2015)** BTL5 **Answer : Pg. 2.25 – Dr.A. Singaravelu** • Marginal density function of X : $f(x)=\int_{-\infty}^{\infty} f(x,y)dy$ • Marginal density function ofY: $f(y)=\int_{-\infty}^{\infty} f(x,y)dx$ • X and Y are independent if f(x,y) = f(x). f(y)

- $\int_0^\infty\int_0^\infty Kxye^{=\left(x^2+y^2\right)}dx\,dy=1 \Rightarrow K=4$. (2M)
- Marginal density function of X : $f(x)=\int_0^\infty Kxye^{=\left(x^2+y^2\right)}dy=2xe^{-x^2}$. (2M)
- Marginal density function of Y : $f(y)=\int_0^\infty Kxye^{=\left(x^2+y^2\right)}dx=2ye^{-y^2}$. (2M)
- f(x). f(y) = $2xe^{-x^2}.2ye^{-y^2} = 4xye^{=\left(x^2+y^2\right)}=f(x,y)$. (2M)

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Given $f_{XY}(x,y)=Cx(x-y), 0<x<2, -x<y<x$ **and 0 elsewhere. (a)Evaluate C; (b)Find** $f_x(x)$ **; (c)** $f_{y/x}\left(\frac{y}{x}\right)$ **(d)**Find f_y(y). **(8M) (May, June 2013May/June 2016)**BTL5

Answer : Pg. 2.40 – Dr. A. Singaravelu

- $\int_{-\infty}^\infty\int_{-\infty}^\infty f(x,y)dx\,dy=1$
- Marginal density function of X : $f(x)=\int_{-\infty}^\infty f(x,y)dy$
- Marginal density function ofY: $f(y)=\int_{-\infty}^\infty f(x,y)dx$
- $\int_0^2\int_{-x}^x Cx(x-y)dy\,dx=1 \Rightarrow C=\frac{1}{8}$. (1M)
- $f_x(x)=\int_{-x}^x Cx(x-y)dy=\frac{x^3}{4}, 0<x<2$. (2M)
- $f\left(\frac{y}{x}\right)=\frac{f(x,y)}{f(x)}=\frac{x-y}{2x^2}, -x<y<x$. (2M)
- $f_y(y)=\begin{cases}\int_{-y}^2\frac{1}{8}x(x-y)dx, if -2\le y\le 0 = \frac{1}{3}-\frac{y}{4}+\frac{5}{28}y^3\\ \int_y^2\frac{1}{8}x(x-y)dx, if\ 0\le y\le 2 = \frac{1}{3}-\frac{y}{4}+\frac{1}{28}y^3\end{cases}$ (3M)

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The joint pdf of (X,Y) is given by $f(x,y)=e^{-(x+y)}, 0\le x,y\le\infty$**. Are X and Y independent. (8M)(Nov/Dec 2015, Apr/May 2018)** BTL4

Answer : Page:2.28 – Dr. A. Singaravelu

- Marginal density function of X : $f(x)=\int_{-\infty}^\infty f(x,y)dy$
- Marginal density function ofY: $f(y)=\int_{-\infty}^\infty f(x,y)dx$
- X and Y are independent if f(x,y) = f(x). f(y)

	• $f(x)=\int_0^\infty e^{-(x+y)}dy=e^{-x}$. (3M) • $f(y)=\int_0^\infty e^{-(x+y)}dx=e^{-y}$. (3M) • $f(x).f(y)=e^{-x}e^{-y}=e^{-(x+y)}=f(x,y)$. (2M)
7	**The joint p.d.f of a two dimensional random variable (X,Y) is given by $f(x,y)=xy^2+\frac{x^2}{8}, 0\le x\le 2, 0\le y\le 1$. Compute (i) $P\left(X>1/Y<\frac{1}{2}\right)$, (ii) $P\left(Y<\frac{1}{2}/X>1\right)$, (iii) P(X<Y), (iv) $P(X+Y\le 1)$ (8M) (Apr/May 2017)** BTL5 **Answer : Pg. 2.43 – Dr.A. Singaravelu** • $P\left(X>1/Y<\frac{1}{2}\right)=\frac{P\left(X>1,Y<\frac{1}{2}\right)}{P\left(Y<\frac{1}{2}\right)}=\frac{\frac{5}{24}}{\frac{1}{4}}=\frac{5}{6}$ (2M) • $P\left(Y<\frac{1}{2}/X>1\right)=\frac{P\left(X>1,Y<\frac{1}{2}\right)}{P(X>1)}=\frac{\frac{5}{24}}{\frac{19}{24}}=\frac{5}{19}$ (2M) • $P(X<Y)=\int_0^1\int_0^y\left(xy^2+\frac{x^2}{8}\right)dx\,dy=\frac{53}{480}$ (2M) • $P(X+Y\le 1)=\int_0^1\int_0^{1-y}\left(xy^2+\frac{x^2}{8}\right)dx\,dy=\frac{13}{480}$ (2M)
8	**Let X and Y have j.d.f f(x,y) = k , 0<x<y<2, Find the marginal pdf. Find the conditional density functions.(8M) (Nov/Dec 2016, Nov/Dec 2017)** BTL5 **Answer : Pg. 2.33 – Dr. A. Singaravelu** • $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dx\,dy=1$ • Marginal density function of X : $f(x)=\int_{-\infty}^{\infty} f(x,y)dy$ • Marginal density function ofY: $f(y)=\int_{-\infty}^{\infty} f(x,y)dx$ • The conditional density function of X given Y: $f(X/Y)=\frac{f(x,y)}{f(y)}$ • The conditional density function of Y given X: $f(Y/X)=\frac{f(x,y)}{f(x)}$

	• $\int_0^2\int_0^y k\,dx\,dy = 1 \Rightarrow k = \frac{1}{2}$. (2M) • $f(x) = \int_x^2 \frac{1}{2}dy = \frac{1}{2}(2-x), 0 < x < 1$ (2M) • $f(y) = \int_0^y \frac{1}{2}dx = \frac{y}{2}, 0 < y < 2$ (2M) • $f(X/Y) = \frac{1}{y}, 0 < x < y$ (1M) • $f(Y/X) = \frac{1}{2-x}, x < y < 2$ (1M)
9	**If the joint distribution function of X and Y is given by $F(x,y) = (1-e^{-x})(1-e^{-y}), x > 0, y > 0$. Find the marginal density function of X and Y. Check if X and Y are independent. Also find P(1<X<3 / 1<Y<2). (8M) (Apr/May 2015, May/June 2016)** BTL5 **Answer : Pg. 2.50 – Dr. A. Singaravelu** • $f(x,y) = \frac{\partial^2 F(x,y)}{\partial x \partial y} = e^{-(x+y)}$ • $f(x) = \int_0^\infty e^{-(x+y)}dy = e^{-x}$. (2M) • $f(y) = \int_0^\infty e^{-(x+y)}dx = e^{-y}$. (2M) • $f(x).f(y) = e^{-x}\,e^{-y} = e^{-(x+y)} = f(x,y)$. (2M) • $P(1 < X < 3, 1 < Y < 2) = \left(\frac{1-e^2}{e^3}\right)\left(\frac{1-e}{e^2}\right)$. (2M)
10	**Find the co-efficient of correlation between X and Y from the data given below.(8M) (May 2016)** BTL5 X: 65, 66, 67, 67, 68, 69, 70, 72 Y: 67, 68, 65, 68, 72, 72, 69, 71 **Answer : Page: 2.71- Dr. A. Singaravelu** • $\overline{X} = \frac{\sum X}{n} = \frac{544}{8} = 68$ (1M) • $\overline{Y} = \frac{\sum Y}{n} = \frac{552}{8} = 69$ (1M) • $\sigma_x = \sqrt{\frac{1}{n}\sum X^2 - \overline{X}^2} = 2.121$ (2M) • $\sigma_y = \sqrt{\frac{1}{n}\sum Y^2 - Y^2} = 2.345$ (2M) • $r(X,Y) = \frac{Cov(X,Y)}{\sigma_x.\sigma_y} = 0.6031$ (2M)
11	**Let X and Y be discrete random variables with pdf $f(x,y) = \frac{x+y}{21}, x = 1,2,3; y = 1,2$. Find**

	$\rho(X,Y)$ **(8M) (Nov/Dec-2019)**BTL5 **Answer : Pg. 2.78- Dr. A. Singaravelu** • $E(X)=\sum x\, f(x)=\frac{46}{21}$ (1M) • $E(Y)=\sum y\, f(y)=\frac{33}{21}$ (1M) • $E(X^2)=\sum x^2\, f(x)=\frac{114}{21}$ (1M) • $E(Y^2)=\sum y^2\, f(y)=\frac{57}{21}$ (1M) • $Var\ X=\sigma_x^2=E(X^2)-[E(X)]^2=\frac{278}{441}$ (1M) • $Var\ Y=\sigma_y^2=E(Y^2)-[E(Y)]^2=\frac{108}{441}$ (1M) • $E(XY)=\sum xy\, f(x,y)=\frac{72}{21}$ (1M) • $r(X,Y)=\frac{Cov(X,Y)}{\sigma_x.\sigma_y}=\frac{-6}{173.20}=-0.035$ (1M)
12	**If the joint pdf of (X,Y) is given by** $f(x,y)=x+y, 0\le x,y\le 1$. **Find** ρ_{xy}. **(8 M) (May/June 2014)** BTL3 **Answer : Page : 2.99 – Dr. A. Singaravelu** • $f(x)=\int_0^1 (x+y)dy=x+\frac{1}{2}, 0<x<1$ (1M) • $f(y)=\int_0^1 (x+y)dx=y+\frac{1}{2}, 0<y<1$ (1M) • $E(X)=\int x\, f(x)dx=\int_0^1 x\left(x+\frac{1}{2}\right)dx=\frac{7}{12}$ (1M) • $E(Y)=\int y\, f(y)dy=\int_0^1 y\left(y+\frac{1}{2}\right)dy=\frac{7}{12}$ (1M) • $E(X^2)=\int x^2\, f(x)dx=\frac{5}{12}$, $E(Y^2)=\int y^2\, f(y)dy=\frac{5}{12}$ (1M) • $Var\ X=\sigma_x^2=E(X^2)-[E(X)]^2=\frac{11}{144}$, $Var\ Y=\sigma_y^2=E(Y^2)-[E(Y)]^2=\frac{11}{144}$ (1M) • Cov(X,Y) = E(XY) – E(X). E(Y) = $\frac{-1}{144}$ (1M) • $r(X,Y)=\frac{Cov(X,Y)}{\sigma_x.\sigma_y}=\frac{-1}{11}$ (1M)
13	**Two independent random variables X and Y are defined by,** $f(x)=\begin{cases}4ax, 0\le x\le 1\\ 0\quad, otherwise\end{cases}$ $f(y)=\begin{cases}4by, 0\le y\le 1\\ 0\quad, otherwise\end{cases}$ **. Show that U=X + Y and V=X – Y are uncorrelated. (8 M)(May/June**

	2013) BTL4 **Answer : Page: 2.105 – Dr. A. Singaravelu** • $\int_0^1 f(x)\,dx = 1 \Rightarrow a = \frac{1}{2}$; $\int_0^1 f(y)\,dy = 1 \Rightarrow b = \frac{1}{2}$ (1M) • $E(U) = E(X) + E(Y) = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}$. (2M) • $E(V) = E(X) - E(Y) = \frac{2}{3} - \frac{2}{3} = 0$. (2M) • $E(UV) = E(X^2) - E(Y^2) = \frac{1}{2} - \frac{1}{2} = 0$. (2M) • Cov(U,V) = E(UV) – E(U).E(V) =0. (1M)
14	**If X and Y are two random variables having joint pdf** $f(x,y) = \frac{1}{8}(6 - x - y)$, **0<x<2, 2<y<4. Find (i)** r_{xy} **(ii)P(X<1 /Y<3) (8 M)** BTL5 **Answer : Page : 2.109 – Dr. A. Singaravelu** • $f(x) = \int_2^4 \frac{1}{8}(6 - x - y)\,dy = \frac{6-2x}{4}$ (1M) • $f(y) = \int_0^2 \frac{1}{8}(6 - x - y)\,dy = \frac{10-2y}{8}$ (1M) • $E(X) = \int x\, f(x)dx = \frac{5}{6}$ (1M) • $E(Y) = \int y\, f(y)dy = \frac{17}{6}$ (1M) • $E(X^2) = \int x^2\, f(x)dx = 1$ (1M) • $E(Y^2) = \int y^2\, f(y)dy = \frac{25}{3}$ (1M) • $E(XY) = \int\int x\, f(x)dx = \frac{7}{3}$ (1M) • $\sigma_x^2 = \frac{11}{36},\ \sigma_y^2 = \frac{11}{36}$ (1M) • $r_{xy} = \frac{Cov(X,Y)}{\sigma_x.\sigma_y} = -\frac{1}{11}$ (1M)
15	**The two lines of regression are** ***8x – 10y +66 = 0; 40x -18y -214 = 0.*** **The variance of 'x' is 9. Find th4e mean values of 'x' and 'y'. Also find the correlation coefficient between 'x' and 'y'.(8 M) (Apr/May 2015, May/June 2016)** BTL4 **Answer: Page : 2.129 – Dr.A. Singaravelu** • $\bar{x} = 13$, $\bar{y} = 17$ • From first equation $x = \frac{10}{8}y - \frac{66}{8} \Rightarrow b_{xy} = \frac{10}{8}$. (2M) • From the second equation $y = \frac{40}{18}x - \frac{214}{18} \Rightarrow b_{yx} = \frac{40}{18}$. (1M) • Correlation coefficient $r = 1.66$ which is not less than 1. (1M)

	• Now, From first equation $y=\frac{8}{10}x+\frac{66}{10} \Rightarrow b_{yx}=\frac{8}{10}$. (1M) • From the second equation $x=\frac{18}{40}y-+\frac{214}{40} \Rightarrow b_{yx}=\frac{18}{40}$. (1M) • Correlation coefficient $r=\pm 0.6$. (2M)
16	**If the pdf of a two dimensional random variable (X,Y) is given by $f(x,y)=x+y,;0\le(x,y)\le 1$. Find the pdf of U=XY. (8 M) (Apr/May 2015, Nov/Dec 2019)** BTL4 **Answer : Page : 2.156 – Dr.A.Singaravelu** • Take $u=xy$ and $v=y$. • $J=\frac{\partial(x,y)}{\partial(u,v)}=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix}=\frac{1}{v}$. (2M) • $f(u,v)=\lvert J\rvert\, f(x,y)=1+\frac{u}{v^2}$. (3M) • $f(u)=\int_u^1\left(1+\frac{u}{v^2}\right)dv=2-2u$. (3M)
17	**Let (X,Y) be a two-dimensional non-negative continuous random variable having the joint density $f(x,y)=\begin{cases}4xye^{-(x^2+y^2)} & ,\ x,y\ge 0\\ 0 & ,elsewhere\end{cases}$. Find the density function of $U=\sqrt{X^2+Y^2}$. (8 M) (May/June 2016, Apr/May 2018)** BTL5 **Answer : Page : 2.179 – Dr.A. Singaravelu** • Take $u^2=x^2+y^2,\ v=x$ • $J=\frac{\partial(x,y)}{\partial(u,v)}=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix}=\frac{u}{\sqrt{u^2-v^2}}$. (2M) • $f(u,v)=\lvert J\rvert\, f(x,y)=4uve^{-u^2}$. (3M) • $f(u)=\int_0^u\left(4uve^{-u^2}\right)dv=2u^3e^{-u^2}$. (3M)
18	**If X and Y are independent random variables with pdf $e^{-x},x\ge 0\,;e^{-y},y\ge 0$ respectively. Find the density function of $U=\frac{X}{X+Y}$ and *V=X + Y*. Are X and Y independent? (8 M) (Nov/Dec 2013, Apr/May 2017, Nov/Dec 2017)** BTL5 **Answer : Page : 2.176- Dr. A. Singaravelu** • Take $U=\frac{X}{X+Y}$ and *V=X + Y*. • $J=\frac{\partial(x,y)}{\partial(u,v)}=\begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v}\\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v}\end{vmatrix}=v$. (2M)

	• $f(u,v)=\lvert J\rvert f(x,y)=ve^{-v}$. (1M) • $f(u)=\int_0^\infty (ve^{-v})dv=1$ (2M) • $f(v)=\int_0^\infty (ve^{-v})du=ve^{-v}$. (2M) • $f(u).f(v)=1.\,ve^{-v}=ve^{-v}=f(u,v)$. (1M)
19	**If X_1, X_2, …, X_n are Poisson variables with parameter $\lambda=2$, use the central limit theorem to estimate P(120 < S_n < 160) where $S_n = X_1 + X_2 + \ldots + X_n$ and n=75. (8M) (Apr/May-2019)**BTL5 **Answer:Page: 2.187-Dr.A. Singaravelu** • $n\mu=150; n\sigma=\sqrt{150}$. (1M) • $z=\frac{S_n-n\mu}{\sigma\sqrt{n}}$; If $S_n=120, z=\frac{-30}{\sqrt{150}}$. (2M) • If $S_n=160, z=\frac{10}{\sqrt{150}}$. (2M) • $P(120<S_n<160)=P(-2.45\le S_n\le 0.85)=P(-2.45\le S_n\le 0)+P(0\le S_n\le 0.85)=0.7866$. (3M)

	UNIT III – Random Proccesses
	Classification – Stationary process – Markov process - Markov chain - Poisson process – Random telegraph process.
	PART *A
Q.No	**Questions**
1.	**Define a random process and give an example. (May/June 2016)** BTL1 A random process is a collection of random variables {X(s,t)} that are functions of a real variable, namely time 't' where $s\in S$ (Sample space) and $t\in T$ (Parameter set or index set). Example: $X(t)=A\cos(\omega t+\theta)$ where θ is uniformly distributed in $(0,2\pi)$, where 'A' and 'ω' are constants.
2	**State the two types of stochastic processes.** BTL1 The four types of stochastic processes are Discrete random sequence, Continuous random sequence, Discrete random process and Continuous random process.
3	**Define Stationary process with an example.(May/June 2016)** BTL1 If certain probability distribution or averages do not depend on 't', then the random process {X(t)} is called stationary process. Example: A Bernoulli process is a stationary process as the joint probability distribution is independent of time.
4	**Define first Stationary process. (Nov/Dec 2015)** BTL1 A random process {X(t)} is said to be a first order stationary process if $E[X(t)]=\mu$ is a constant.
5	**Define strict sense and wide sense stationary process.(Nov/Dec 2015, Apr/May 2017, Nov/Dec 2017)** BTL1 A random process is called a strict sense stationary process or strongly stationary process if all its finite dimensional distributions are invariant under translation of time parameter. A random process is called wide sense stationary or covariance stationary process if its mean is a constant and auto correlation depends only on the time difference.
6	**In the fair coin experiment we define {X(t)} as follows** $X(t)=\begin{cases}\sin \pi t & ,if\ head\ shows\\ 2t & ,if\ tail\ shows\end{cases}$ **.Find E[X(t)] and find**

	F(x,t) for t = 0.25. (Nov/Dec 2016) BTL3 $P[X(t)=\sin \pi t]=\frac{1}{2}$, $P[X(t)=2t]=\frac{1}{2}$ $E[X(t)]=\sum X(t)P[X(t)]=\sin \pi t\left(\frac{1}{2}\right)+2t\left(\frac{1}{2}\right)=\frac{1}{2}\sin \pi t+t$ $When\ t=0.25$, $P[X(0.25)=\sin \pi(0.25)]=P\left[X(0.25)=\frac{1}{\sqrt{2}}\right]=\frac{1}{2}$ $P[X(t)=2(0.25)]=P\left[X(t)=\frac{1}{2}\right]=\frac{1}{2}$ Hence F(x,t) for t= 0.25 is given by $F(x,t)=\begin{cases}0 & ,x<0\\ \frac{1}{2} & ,\frac{1}{2}\le x<\frac{1}{\sqrt{2}}\\ 1 & ,x\ge\frac{1}{\sqrt{2}}\end{cases}$
7	**Prove that a first order stationary random process has a constant mean. (Apr/May 2011)** BTL3 f[X(t)] = f[X(t+h)] as the process is stationary. $E[X(t)]=\int X(t)\ f[X(t+h)]d(t+h)$ $t+h=u \Rightarrow d(t+h)=du$ $Put \qquad =\int X(u)\ f[X(u)]du$ $=E[X(u)]$ Therefore, E[X(t+h)] = E[X(t)] Therefore, E[X(t)] is independent of 't'. Therefore , E[X(t)] is a constant.
8	**What is a Markov process. Give an example.(Nov/Dec 2014, Apr/May 2015, May/June 2016,Apr/May 2019, Nov 2019)** BTL1 Markov process is one in which the future value is independent of the past values, given the present value. (i.e.,)A random process X(t) is said to be a Markov process if for every $t_0< t_1 < t_2 <\dots t_n$, $P\{X(t_n)\le x_n / X(t_{n-1})=x_{n-1}, X(t_{n-2})=x_{n-2}, \dots, X(t_0)=x_0\}\Rightarrow P\{X(t_n)\le x_n / X(t_{n-1})=x_{n-1}\}$. Example: Poisson process is a Markov process. Therefore, number of arrivals in (0,t) is a Poisson process and hence a Markov process.
9	**Define Markov chain. When it is called homogeneous? Also define one-step transition probability. (Apr/May 2010)** BTL1 • If $\forall n, P[X_n=a_n / X_{n-1}=a_{n-1}, X_{n-2}=a_{n-2},\dots,X_0=a_0]=P[X_n=a_n / X_{n-1}=a_{n-1}]$ then the process $\{X_n\}$ n = 0,1,2,… is called a Markov chain. • In a Markov chain if the one-step transition probability $P[X_n=a_n / X_{n-1}=a_{n-1}]=P_{ij}(n-1,n)$ independent of the step 'n'. (i.e.,) $P_{ij}(n-1,n)=P_{ij}(m-1,m)$ for all m,n and I,j. Then the Markov chain is said to be homogeneous. • The conditional probability $P[X_n=a_j / X_{n-1}=a_j]$ is called the one step transition probability from state a_i to state a_j at the nth step.
10	**Define Poisson process.(Nov/Dec 2017)** BTL1

	If X(t) represents the number of occurrences of a certain event in (0,t), then the discrete process {X(t)} is called the Poisson process provided the postulates are satisfied: $P[1\ occurrence\ in\ (t,t+\Delta t)] = \lambda\Delta t + 0(\Delta t)$ $P[0\ occurrence\ in\ (t,t+\Delta t)] = 1-\lambda\Delta t + 0(\Delta t)$ $P[2\ occurrence\ in\ (t,t+\Delta t)] = 0(\Delta t)$ X(t) is independent of the number of occurrences of the event in any interval prior and after the interval (0,t) The provability that the event occurs a specified number of times in (t_0, t_0+t) depends only on 't', but not on 't_0'.
11	**State any two properties of Poisson process. (Nov/Dec 2015, Apr/May 2018)** BTL1 • The Poisson process is a Markov process • Sum of two different Poisson process is a Poisson process • Difference of two different Poisson process is not a Poisson process
12	**If the customers arrive at a bank according to a Poisson process with mean rate 2 per minute, find the probability that during a 1-minute interval no customers arrive. (Apr/May 2017)** BTL3 Mean arrival rate = $\lambda = 2$ The probability of Poisson process is $P[X(t)=n] = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$ $P[X(t)=0] = \frac{e^{-2}(2)^0}{0!} = e^{-2} = 0.1353.$
13	**Prove that the sum of two independent Poisson process is a Poisson process.(Nov/Dec 2012, Apr/May 2015, Apr/May 2017)** BTL5 $Let\ \ X(t) = [X_1(t) + X_2(t)]$ $E[X(t)] = E[X_1(t)+X_2(t)] = E[X_1(t)] + E[X_2(t)]$ $= \lambda_1 t + \lambda_2 t = (\lambda_1+\lambda_2)t$ $E[X^2(t)] = E[X_1(t)+X_2(t)]^2 = E[X_1^2(t) + 2X_1(t)X_2(t) + X_2^2(t)]$ $= E[X_1^2(t)] + 2E[X_1(t)]E[X_2(t)] + E[X_2^2(t)]$ $= \lambda_1^2t^2 + \lambda_1 t + 2(\lambda_1 t)(\lambda_2 t) + \lambda_2^2 t^2 + \lambda_2 t$ $= (\lambda_1+\lambda_2)^2 t^2 + (\lambda_1+\lambda_2)t$ Therefore $X(t) = [X_1(t)+X_2(t)]$ is a Poisson process.
14	**Prove that the sum of two independent Poisson process is a Poisson process.** BTL5 $Let\ \ X(t) = [X_1(t) - X_2(t)]$ $E[X(t)] = E[X_1(t) + -X_2(t)] = E[X_1(t)] - E[X_2(t)]$ $= \lambda_1 t - \lambda_2 t = (\lambda_1-\lambda_2)t$ $E[X^2(t)] = E[X_1(t)-X_2(t)]^2 = E[X_1^2(t) - 2X_1(t)X_2(t) + X_2^2(t)]$ $= E[X_1^2(t)] - 2E[X_1(t)]E[X_2(t)] + E[X_2^2(t)]$ $= \lambda_1^2t^2 + \lambda_1 t - 2(\lambda_1 t)(\lambda_2 t) + \lambda_2^2 t^2 + \lambda_2 t$ $= (\lambda_1-\lambda_2)^2 t^2 + (\lambda_1+\lambda_2)t$ $\neq (\lambda_1-\lambda_2)^2 t^2 + (\lambda_1-\lambda_2)t$

	Therefore $X(t)=[X_1(t)-X_2(t)]$ is not a Poisson process.
15	**Patients arrive randomly and independently at a doctor's consulting room form 8 A.M at an average rate of 1 for every 5 minutes. The waiting room can hold 12 persons. What is the probability that the room will be full when the doctor arrives at 9 A.M?. (Nov/Dec 2016)** BTL3 Given $\lambda = \frac{1}{5}\ per\ \min = \frac{1}{5}\times 60 = 12\ per\ hour$ The probability law of Poisson process is $P[X(t)=n]=\frac{e^{-\lambda t}(\lambda t)^n}{n!}$ $P[X(1)=12]=\frac{e^{-12}(12)^{12}}{12!}=0.1144$
16	**Define Semi- Random telegram signal process. (Apr/May 2015)** BTL1 If N(t) represents the number of occurrences of a specified event in (0,t) and X(t) = $(-1)^{N(t)}$, then {X(t)} is called a semi-random telegraph signal process.
17	**Define Random telegraph process.** BTL1 A random telegraph process is a discrete random process X(t) satisfying the following conditions: • X(t) assumes only one of the two possible values 1 or -1 at any time 't', randomly • X(0) = 1 or -1 with equal probability ½. • The number of level transitions or flips, $N(\tau)$, from one value to another occurring in any interval of length τ is a Poisson process with rate λ so that the probability of exactly 'r' transitions is $P[N(\tau)=r]=\frac{e^{-\lambda\tau}(\lambda\tau)^r}{r!},\ r=0,1,2,\ldots$
18	**Write the properties of Random telegraph process.** BTL1 • P[X(t)=1] = $\frac{1}{2}$ = P[X(t)= - 1] for any t >0 • E[X(t)] = 0 and Var[X(t)] = 1 • X(t) is a WSS process
19	**Consider the random process** $X(t)=\cos(t+\phi)$ **where** ϕ is a random variable with density function $f(\phi)=\frac{1}{\pi},\ -\frac{\pi}{2}<\phi<\frac{\pi}{2}$. **Check whether or not the process is stationary.** BTL3

$$E[X(t)] = \int_{-\infty}^{\infty} X(t)\, f(\phi)\, d\phi$$

$$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(t+\phi)\, \frac{1}{\pi}\, d\phi$$

$$= \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(t+\phi)\, d\phi$$

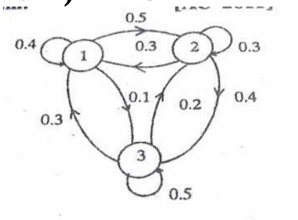
$$= \frac{1}{\pi} \left[\sin(t+\phi)\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{1}{\pi}\left[\sin(\frac{\pi}{2}+t) - \sin(-\frac{\pi}{2}+t)\right]$$

$$= \frac{1}{\pi}\left[\cos(t)+\cos(t)\right] = \frac{2}{\pi}\cos(t)$$

Therefore E[X(t)] is not a constant. Hence X(t) is not stationary.

20 **Find the transition probability matrix of the process represented by the transition diagram. (Apr/May 2011)** BTL3



$$\begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}$$

21 **If the tpm of the markov chain is $\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$, find the steady-state distribution of the chain.** BTL5

Given : P= $\begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

Let the steady- state probability distribution be $\pi = (\pi_1 \quad \pi_2)$ we have

	$\pi P=\pi$ ....(1) $\pi_1+\pi_2=1$..........(2) $(1)\Rightarrow(\pi_1 \quad \pi_2)\begin{bmatrix}0 & 1\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}=(\pi_1 \quad \pi_2)$ $\left[\pi_1(0)+\pi_2\left(\frac{1}{2}\right) \quad \pi_1(1)+\pi_2\left(\frac{1}{2}\right)\right]=(\pi_1 \quad \pi_2)$ $\Rightarrow\left[\pi_2\left(\frac{1}{2}\right) \quad \pi_1+\pi_2\left(\frac{1}{2}\right)\right]=(\pi_1 \quad \pi_2)$ $\Rightarrow\frac{1}{2}\pi_2=\pi_1$...........(3) $\pi_1+\pi_2\left(\frac{1}{2}\right)=\pi_2$.........(4) $Now\ (2)\Rightarrow\pi_1+\pi_2=1$, *substitute* (3) *in* (2) $\Rightarrow\frac{1}{2}\pi_2+\pi_2=1\Rightarrow\frac{3}{2}\pi_2=1\Rightarrow\pi_2=\frac{2}{3}$ *Sub* π_2 *in* (3), $\frac{1}{2}.\frac{2}{3}=\pi_1\Rightarrow\pi_1=\frac{1}{3}$ The steady state distribution of the chain is $\pi=\left(\frac{1}{3} \quad \frac{2}{3}\right)$
22	**Let** $A=\begin{pmatrix}0 & 1\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}$ **be a stochastic matrix. Check if it is regular. (Nov/Dec 2016)** BTL4 $A^2=\begin{pmatrix}0 & 1\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}0 & 1\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}=\begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{4} & \frac{3}{4}\end{pmatrix}$ Since all the entries of A^2 are positive , 'A' is regular.
23	**What is the autocorrelation function of the Poisson process. Is Poisson process stationary? (**Apr/May 2019) BTL2 Let X(t) be a Poisson process then $P[X(t)=n]=\frac{e^{-\lambda t}(\lambda t)^n}{n!}$ n=0,1,2,… Autocorrelation function $R_{xx}(t_1,t_2)=\lambda^2 t_1 t_2+\lambda\min\{t_1,t_2\}$ Since $R_{xx}(t_1,t_2)$ is not a function of time difference t_1-t_2, Poisson process is not stationary.
24	**When is a Random process said to be evolutionary. Give an example. (Apr/May 2015)** (BTL1) A random process that is not stationary at any sense is called evolutionary process. Semi-random telegraph signal process is an example of evolutionary random process.
25	**Define irreducible Markov chain and state Chapman-Kolmogorov theorem.** BTL1 A Markov chain is said to be irreducible if every state can be reached from every other state, where $p_{ij}^{(n)}>0$ for some 'n' and for all 'i' and 'j'. If 'P' is the tpm of a homogeneous Markov chain, then the n-step tpm $P^{(n)}$ is equal to P^n. (i.e.,) $\left[P_{ij}^{(n)}\right]=\left[P_{ij}\right]^n$.

Part*B

1 **The process {X(t)} whose probability distribution under certain conditions is given by,**

$$P\{X(t)=n\}=\frac{(at)^{n-1}}{(1+at)^{n+1}},\ n=1,2...$$

$$=\frac{at}{1+at},\ n=0$$

Show that it is not stationary(evolutionary). (8M)(Nov/Dec 2014, Nov/Dec 2016, Apr/May 2018) BTL5

Answer: Page: 3.33 –Dr. A. Singaravelu

- $E[X(t)]=\sum_{n=0}^{\infty} n\, p_n = 0+(1)\frac{1}{(1+at)^2}+(2)\frac{at}{(1+at)^3}+....\ =1.$ (3M)
- $E[X^2(t)]=\sum_{n=0}^{\infty} n^2\, P_n=\sum_{n=0}^{\infty}([n(n+1)-n]P_n=1+2at.$ (3M)
- $Var[X(t)]=E[X^2(t)]-E[X(t)]=2at\neq constant.$ (2M)

2 **If the random process X(t) takes the value -1 with probability $\frac{1}{3}$ and takes the value 1 with probability $\frac{2}{3}$, find whether X(t) is a stationary process or not. (6M)(Apr/May 2019)** BTL4

Answer:Page: 3.12 – Dr. G. Balaji

X(t)=n	-1	1
P_n	1/3	2/3

- $E[X(t)]=\sum_{n=-1}^{1} n\, P_n=\frac{1}{3}$ (2M)
- $E[X^2(t)]=\sum_{n=-1}^{1} n^2\, P_n=1$ (2M)
- $Var[X(t)]=E[X^2(t)]-E[X(t)]=\frac{8}{9}$ = constant. (2M)

3 **Show that the process $X(t)=A\cos(\omega t+\theta)$ where A, ω are constants, θ is uniformly distributed in $(-\pi,\pi)$ is wide sense stationary. (8M) (May/June 2016, Nov/Dec 2016)** BTL5

Answer:Page: 3.15-Dr. A. Singaravelu

- $E[X(t)]=\int_{-\infty}^{\infty} X(t)\, f(\theta)\, d\theta=\int_{-\pi}^{\pi} A\cos(\omega t+\theta)\frac{1}{2\pi}\, d\theta=0=constant$ (2M)
- $R_{XX}(t,t+\tau)=E[X(t)X(t+\tau)]=E[A\cos(\omega t+\theta).A\cos(\omega(t+\tau)+\theta)]$ (1M)
- $E[A\cos(\omega t+\theta).A\cos(\omega(t+\tau)+\theta)]=\frac{A^2}{2}\{E(\cos\omega\tau)+E[\cos(2\omega t+2\theta+\omega\tau)]\}$ (2M)
- $E[\cos(2\omega t+2\theta+\omega\tau)]=0$ (2M)
- $R_{XX}(t,t+\tau)=\frac{A^2}{2}\cos\omega\tau$ =a function of τ. (1M)

4 **Show that the process $X(t)=A\cos(\omega t+\theta)$ where A, ω are constants, θ is uniformly distributed in $(0,2\pi)$ is WSS. (8M) (Nov/Dec 2017)** BTL5

Answer:Page: 3.24-Dr. G. Balaji

- $E[X(t)]=\int_{-\infty}^{\infty} X(t)\, f(\theta)\, d\theta=\int_{0}^{2\pi} A\cos(\omega t+\theta)\frac{1}{2\pi}\, d\theta=0=constant$ (2M)

- $R_{XX}(t,t+\tau)=E[X(t)X(t+\tau)]=E[A\cos(\omega t+\theta).A\cos(\omega(t+\tau)+\theta)]$ (1M)
- $E[A\cos(\omega t+\theta).A\cos(\omega(t+\tau)+\theta)]=\frac{A^2}{2}\{E(\cos\omega\tau)+E[\cos(2\omega t+2\theta+\omega\tau)]\}$ (2M)
- $E[\cos(2\omega t+2\theta+\omega\tau)]=0$ (2M)
- $R_{XX}(t,t+\tau)=\frac{A^2}{2}\cos\omega\tau$ =a function of τ . (1M)

5

Show that the process $X(t)=A\cos\lambda t+B\sin\lambda t$ is strict sense stationary of order 2. A and B are random variables if E[A] = E[B] = 0 ; E[A^2] = E[B^2]; E[AB] = 0.

(OR)

If $X(t)=A\cos\lambda t+B\sin\lambda t$, $t\geq 0$ is a random process where A and B are independent $N(0,\sigma^2)$ random variables. Examine the WSS process of X(t). (8M) (Apr/May 2015, Apr/May 2017) BTL5

Answer:Page: 3.13-Dr. A. Singaravelu

- $E\{X(t)\}=E\{A\cos\lambda t+B\sin\lambda t\}=0=cons\tan t$ (2M)
- $R_{XX}(t,t+\tau)=E[X(t)X(t+\tau)]=E\{[A\cos\lambda t+B\sin\lambda t][A\cos\lambda(t+\tau)+B\sin\lambda(t+\tau)]\}$ (2M)
- $R_{XX}(t,t+\tau)=K^2[\cos\lambda t\cos\lambda(t+\tau)+\sin\lambda t\sin\lambda(t+\tau)]=K^2\cos\lambda\tau$ (4M)

6

A random variable {X(t)} is defined by $X(t)=A\cos t+B\sin t, -\infty<t<\infty$ where A and B are independent random variables each of which has a value -2 with probability $\frac{1}{3}$ and a value 1 with probability $\frac{2}{3}$. Show that X(t) is wide sense stationary. (8M) (Nov/Dec 2015, Apr/May 2017, Apr/May 2018) BTL5

Answer:Page: 3.44-Dr. G. Balaji

- $E[A]=\sum A_i\, P(A_i)=0$ (1M)
- $E[B]=\sum B_i\, P(B_i)=0$ (1M)
- $E[A^2]=\sum A_i^{\,2}\, P(A_i)=2$ (1M)
- $E[B^2]=\sum B_i^{\,2}\, P(B_i)=2$ (1M)
- $E[X(t)]=E[Y\cos t+Z\sin t]=0=cons\tan t$ (2M)
- $R_{XX}(t,t+\tau)=E[X(t)X(t+\tau)]=E[(Y\cos t_1+Z\sin t_1)(Y\cos t_2+Z\sin t_2)]=2\cos\tau$ (2M)

7

The transition probability matrix of a Markov chain {X$_n$}, n=1,2,… having 3 states 1,2 and 3 is $P=\begin{bmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}$ and the initial distribution is $P^{(0)}=(0.7\quad 0.2\quad 0.1)$. Find (i) $P\{X_2=3\}$ and (ii) $P\{X_3=2, X_2=3, X_1=3, X_0=2\}$.

Answer:Page: 3.60-Dr. A. Singaravelu

- $P^{(1)}=P^{(0)}P=[0.7\quad 0.2\quad 0.1]\begin{bmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}=[0.22\quad 0.43\quad 0.35]$ (2M)
- $P^{(2)}=P^{(1)}P=[0.22\quad 0.43\quad 0.35]\begin{bmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}=[0.385\quad 0.336\quad 0.279]$ (2M)
- $P\{X_2=3\}=0.279$ (1M)
- $P\{X_3=2, X_2=3, X_1=3, X_0=2\}=P_{32}^1\, P_{33}^1\, P_{23}^1\, P[X_0=2]=0.0048$ (3M)

8	**A man either drives a car or catches a train to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drive to work if and only if a 6 appeared. Find (i) The probability that he drives to work in the long run and (ii) The probability that he takes a train on the third day. (8M) (May/June 2016, Nov/Dec 2017)** BTL4 **Answer:Page: 3.71-Dr. A. Singaravelu** • $P=\begin{bmatrix}0 & 1\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$ (2M) • $\pi=(\pi_1 \quad \pi_2)=\left(\frac{1}{3} \quad \frac{2}{3}\right)$ (3M) • $P^{(2)}=P^{(1)}P=\left(\frac{1}{12} \quad \frac{11}{12}\right)$ (1M) • $P^{(3)}=P^{(2)}P=\left(\frac{11}{24} \quad \frac{13}{22}\right)$ (2M)
9	**If $\{X_n; n=1,2,3\ldots\}$ be a Markov chain on the space S={1,2,3} with one-step $\begin{bmatrix}0 & 1 & 0\\ \frac{1}{2} & 0 & \frac{1}{2}\\ 1 & 0 & 0\end{bmatrix}$.Sketch the transition diagram. Is the chain irreducible? Explain. Is the chain ergodic? Explain. (8M) (May/June 2013, Nov/Dec 2019)** BTL4 **Answer:Page:3.141-Dr. G. Balaji** • $P^4=P^3P=P.P=P^2$ (1M) • $P^5=P^4P=P^2.P=P^3=P$ (1M) • 1st state $P_{00}^{(2)}>0, P_{00}^{(4)}>0, P_{00}^{(6)}>0 \ldots\Rightarrow d_i=GCD(2,4,6,\ldots)=2$ (1M) • 2nd state $P_{11}^{(2)}>0, P_{11}^{(4)}>0, P_{11}^{(6)}>0 \ldots\Rightarrow d_i=GCD(2,4,6,\ldots)=2$ (1M) • 3rd state $P_{22}^{(2)}>0, P_{22}^{(4)}>0, P_{22}^{(6)}>0 \ldots\Rightarrow d_i=GCD(2,4,6,\ldots)=2$ (1M) • The states are aperiodic with period 2. • We find $P_{ij}^{(n)}>0$. So the Markov chain is irreducible (2M) • The chain is finite and irreducible so it is non- null persistant. But not ergodic. (1M)
10	**Find the mean, variance and auto correlation of Poisson process. (8M) (May/June 2014, Apr/May 2015)** BTL2 **Answer: Page:3.93- Dr. A. Singaravelu** • The probability of Poisson distribution is $P\{X(t)=n\}=\frac{e^{-\lambda t}(\lambda t)^n}{n!}$, n=0,1,2,… (1M) • $E[X(t)]=\sum_{x=0}^{\infty} x\frac{e^{-\lambda t}(\lambda t)^n}{n!}=\lambda t$ (2M) • $E[X^2(t)]=\sum_{x=0}^{\infty} x^2\frac{e^{-\lambda t}(\lambda t)^n}{n!}=(\lambda t)^2+\lambda t$ (2M) • $Var[X(t)]=\lambda t$ (1M) • $R_{xx}(t_1,t_2)=E[X(t_1)X(t_2)]=\lambda^2 t_1 t_2+\lambda\min(t_1,t_2)$ (2M)

11	**(i) Prove that the interval between two successive occurrences of a Poisson process with parameter λ has an exponential distribution.** **(ii)Show that Poisson process is a Markov process. (8M) (Apr/May 2018)** BTL5 **Answer: Page:3.98- Dr. A. Singaravelu** (i) • P(T > t) = P(E_{i+1} did not occur in (t_i, t_{i+1}) = P(X(t)=0) = $e^{-\lambda t}$ **(**1M) • $F(t)=P(T\le t)=1-P(T>t)=1-e^{-\lambda t}$ (2M) • The pdf of T is given by $\lambda e^{-\lambda t}$ which is an exponential distribution. (1M) (ii) • $P[X(t_3)=n_3 / X(t_2)=n_2; X(t_1)=n_1] = \frac{e^{-\lambda(t_3-t_2)}\lambda^{n_3-n_2}(t_3-t_2)^{n_3-n_2}}{(n_3-n_2)!}$ (3M) • $P[X(t_3)=n_3 / X(t_2)=n_2; X(t_1)=n_1] = P[X(t_3)=n_3 / X(t_2)=n_2]$ which is Markov process. (1M)
12	**Suppose that customers arrive at a bank according to a Poisson process with mean rate of 3 per minute; find the probability that during a time interval of 2 min (i) exactly 4 customers arrive and (ii) more than 4 customers arrive. (iii) fewer than 4 customers arrive. (8M) (Nov/Dec 2015)** BTL5 **Answer: Page:3.100- Dr. A. Singaravelu** • The probability of Poisson distribution is $P\{X(t)=n\}=\frac{e^{-\lambda t}(\lambda t)^n}{n!}$, n=0,1,2,… (1M) • P[4 customers arrive in 2 min time interval] = P{X(2)=4} = 0.1339 (2M) • P[More than 4 customers arrive in 2 min interval] =P{X(2)>4} = $1-P[X(2)\le 4]$= 0.715(3M) • P[Fewer than 4 customers arrive in 2 min interval] = P{X(2)<4} = 0.1512. (2M)
13	**A fisherman catches a fish at a Poisson rate of 2 per hour from a large lake with lots of fish. If he starts fishing at 10.00 a.m. What is the probability that he catches one fish by 10.30 a.m and three fishes by noon? (8M) (Apr/May 2017)** BTL5 **Answer: Classwork** • The probability of Poisson distribution is $P\{X(t)=n\}=\frac{e^{-\lambda t}(\lambda t)^n}{n!}$, n=0,1,2,… (2M) • P[He catches one fish by 10.30 a.m] =P[X(0.5)=1] = $\frac{e^{-1}(1)^1}{1!}$ =0.3679 (3M) • P[He catches three fishes by noon] = P[X(2) = 3] = $\frac{e^{-4}(4)^3}{3!}=0.1954$ (2M)
14	**A hard disk fails in a computer system and it follows Poisson process with mean rate of 1 per week. Find the probability that 2 weeks have elapsed since the last failure. If there are 5 extra hard disks and the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks. (8M) (Nov/Dec 2017)** BTL5 **Answer: Page:3.102- Dr. A. Singaravelu** • The probability of Poisson distribution is $P\{X(t)=n\}=\frac{e^{-\lambda t}(\lambda t)^n}{n!}$, n=0,1,2,… (2M) • P[No failure in 2 weeks since last failure] = P[X(2)=0] = e^{-2} = 0.135 (3M) • $P[X(10)\le 5]=P[X(10)=0]+[X(10)=1]+[X(10)=2]+[X(10)=3]+[X(10)=4]+[X(10)=5]=0.067$ (3M)
15	**If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is (i) more than 1 minute, (ii) between 1**

	min and 2 min and (iii) 4 min or less. (8M) (May/June 2012) BTL5 **Answer: Page: 3.100- Dr. A. Singaravelu** • Using inter arrival property of Poisson process, $f(t)=\lambda e^{-\lambda t}$ (1M) • $P(T>1)=\int_1^\infty 2e^{-2t}dt=0.135$ (2M) • $P(1<T<2)=\int_1^2 2e^{-2t}dt=0.117$ (2M) • $P(T\le 4)=\int_0^4 2e^{-2t}dt=1$ (3M)
16	**If {X₁(t)} and {X₂(t)} are two independent Poisson process with parameter λ_1 and λ_2 respectively, show that P[X₁(t) =x / X₁(t)+ X₂(t) = n] is Binomial where $P=\frac{\lambda_1}{\lambda_1+\lambda_2}$.(8M) (Apr/May 2018)** BTL5 **Anwer: Page: 3.84-Dr G. Balaji** • $P\left[X_1(t)=x/X_1(t)+X_2(t)=n\right]=\frac{P\left[\{X_1(t)=x\}\cap\{X_1(t)+X_2(t)=n\}\right]}{P\left(X_1(t)+X_2(t)=n\right)}$ (3M) • $P\left[X_1(t)=x/X_1(t)+X_2(t)=n\right]=\frac{\frac{e^{-\lambda_1 t}(\lambda_1 t)^x}{x!}\cdot\frac{e^{-\lambda_2 t}(\lambda_2 t)^{n-x}}{(n-x)!}}{\frac{e^{-(\lambda_1+\lambda_2)t}((\lambda_1+\lambda_2)t)^n}{n!}}$ (3M) • $P\left[X_1(t)=x/X_1(t)+X_2(t)=n\right]=nC_x P^x q^{n-x}$ where $P=\frac{\lambda_1}{\lambda_1+\lambda_2}$ and $q=\frac{\lambda_2}{\lambda_1+\lambda_2}$ (2M)
17	**Define semi-random telegraph signal process and random telegraph signal process and prove that the former is evolutionary and the latter is wide sense stationary(Covariance stationary process). (16M) (Nov/Dec 2013, Nov/Dec 2017, Apr/May 2015, Apr/May 2017)** BTL5 **Answer: 3.106- -Dr.A. Singaravelu** • A random telegraph process is a discrete random process X(t) satisfying the following conditions: X(t) assumes only one of the two possible values 1 or -1 at any time 't', randomly X(0) = 1 or -1 with equal probability ½. The number of level transitions or flips, $N(\tau)$, from one value to another occurring in any interval of length τ is a Poisson process with rate λ so that the probability of exactly 'r' transitions is $P[N(\tau)=r]=\frac{e^{-\lambda\tau}(\lambda\tau)^r}{r!}, r=0,1,2,\dots$ (2M) • If N(t) represents the number of occurrences of a specified event in (0,t) and $X(t)=(-1)^{N(t)}$, then {X(t)} is called a semi-random telegraph signal process. (2M) • P{X(t) = 1} = P{N(t) is even} = $e^{-\lambda t}\cosh\lambda t$ (1M) • P{X(t) = -1} = P{N(t) is odd} = $e^{-\lambda t}\,Sinh\,\lambda t$ (1M) • $E[X(t)]=e^{-2\lambda t}$ (1M) • $P\left[X(t_1)=1,X(t_2)=1\right]=P\left[X(t_1)=1/X(t_2)=1\right]\times P\left[X(t_2)=1\right]=e^{-\lambda\tau}\cosh\lambda\tau\,e^{-\lambda t_2}\cosh\lambda t_2$ (1M) • $P\left[X(t_1)=-1,X(t_2)=-1\right]=e^{-\lambda\tau}\cosh\lambda\tau\,e^{-\lambda t_2}\,Sinh\,\lambda t_2$ (1M) • $P\left[X(t_1)=1,X(t_2)=-1\right]=e^{-\lambda\tau}\sinh\lambda\tau\,e^{-\lambda t_2}\,Sinh\,\lambda t_2$ (1M)

- $P[X(t_1)=-1, X(t_2)=1] = e^{-\lambda\tau} \sinh \lambda\tau \, e^{-\lambda t_2} \cosh \lambda t_2$ (1M)
- $P[X(t_1)\times X(t_2)=1] = e^{-\lambda\tau} \cosh \lambda\tau$ (1M)
- $P[X(t_1)\times X(t_2)=-1] = e^{-\lambda\tau} \sinh \lambda\tau$
- $R(t_1,t_2)=E[X(t_1)X(t_2)]=e^{-2\lambda(t_2-t_1)}$ (1M)
- {X(t)} is evolutionary
- For Random telegraph signal process Y(t), $P(\alpha=1)=\frac{1}{2}$, $P(\alpha=-1)=\frac{1}{2}$ (1M)
- $E(\alpha)=0, E(\alpha^2)=1$ (1M)
- $R_{YY}(t_1,t_2)=E[Y(t_1)Y(t_2)]=E[\alpha^2 X(t_1)X(t_2)]=e^{-2\lambda(t_2-t_1)}$ which is WSS. (1M)

UNIT-IV CORRELATION AND SPECTRAL DENSITIES

Auto correlation functions – Cross correlation functions – Properties – Power spectral density - Cross spectral density – Properties.

Q.No	PART*A
1.	**List any two properties of an autocorrelation function. [N/D14] BTL 1** • $R(\tau)$ is an even function of τ. • If $R(\tau)$ is the autocorrelation function of a stationary process $\{X(t)\}$ with no periodic component, then $\lim_{\tau\to\infty} R(\tau)=\mu_x^2$, provided the limit exists.
2	**Prove that for a WSS process $\{X(t)\}$, $R_{XX}(\tau)=R_{XX}(-\tau)$. [A/M11,N/D11,N/D12,N/D15,M/J16,*N/D16*,A/M17,N/D17] BTL 5** $R_{XX}(\tau)=E[X(t)X(t-\tau)]$ $R_{XX}(-\tau)=E[X(t)X(t+\tau)]=E[X(t+\tau)X(t)]=R_{XX}(\tau)$ Therefore $R(\tau)$ is an even function of τ.
3	**Show that the autocorrelation function $R_{XX}(\tau)$ is maximum at $\tau=0$. [*N/D17*] BTL 5** $R_{XX}(\tau)$ is maximum at $\tau=0$ *i.e.* $\lvert R(\tau)\rvert \le R(0)$ Cauchy-Schwarz inequality is $(E[XY])^2 \le E[X^2]E[Y^2]$ Put $X=X(t)$ and $Y=X(t-\tau)$, then

	$\left(E\left[X(t)X(t-\tau)\right]\right)^2 \le E\left[X^2(t)\right]E\left[X^2(t-\tau)\right]$ *i.e.* $\left(R(\tau)\right)^2 \le \left(E\left[X^2(\tau)\right]\right)^2$ [Since $E[X(t)]$ and $Var[X(t)]$ are constants for a stationary process] $\left[R(\tau)\right]^2 \le \left[R(0)\right]^2$ Taking square root on both sides, $\lvert R(\tau)\rvert \le R(0)$. [Since $R(0)=E\left[X^2(t)\right]$ is positive].
4	**The autocorrelation function of a stationary process is** $R_{XX}(\tau)=16+\frac{9}{1+6\tau^2}$**. Find the mean and variance of the process. [A/M10, A/M11, M/J12] BTL5** Given $R_{XX}(\tau)=16+\frac{9}{1+6\tau^2}$ $\mu_x^2 = \lim_{\tau\to\infty} R(\tau) = \lim_{\tau\to\infty}\left(16+\frac{9}{1+6\tau^2}\right)=16+\lim_{\tau\to\infty}\left(\frac{9}{1+6\tau^2}\right)$ $=16+0=16$ Mean $=\mu_x = E\left[X(t)\right]=4$ $E\left[X^2(t)\right]=R_{XX}(0)=16+\frac{9}{1+6(0)}=16+9=25$ Variance $=E\left[X^2(t)\right]-\left(E\left[X(t)\right]\right)^2 = 25-(4)^2=25-16=9$.
5	**If the autocorrelation function of a stationary processes is** $R_{XX}(\tau)=25+\frac{4}{1+6\tau^2}$**. Find the mean and variance of the process.[N/D11,M/J14,N/D14,*N/D15*,*A/M18*] BTL5** Given $R_{XX}(\tau)=25+\frac{4}{1+6\tau^2}$ $\mu_x^2 = \lim_{\tau\to\infty} R(\tau) = \lim_{\tau\to\infty}\left(25+\frac{4}{1+6\tau^2}\right)=25+\lim_{\tau\to\infty}\left(\frac{4}{1+6\tau^2}\right) = 25+0=25$ Mean $=\mu_x = E\left[X(t)\right]=5$ $E\left[X^2(t)\right] = R_{XX}(0)=25+\frac{4}{1+6(0)}=25+4=29$ Variance $=E\left[X^2(t)\right]-\left(E\left[X(t)\right]\right)^2 = 29-(5)^2=29-25=4$.
6	**Find the variance of the stationary process** $\{X(t)\}$ **whose autocorrelation function is given by** $R_{XX}(\tau)=2+4e^{-2\lvert\tau\rvert}$**. [N/D10,N/D12,A/M17,*N/D19*] BTL5** Given $R_{XX}(\tau)=2+4e^{-2\lvert\tau\rvert}$ $\mu_x^2 = \lim_{\tau\to\infty} R(\tau) = \lim_{\tau\to\infty}\left(2+4e^{-2\lvert\tau\rvert}\right)=2+\lim_{\tau\to\infty}\left(4e^{-2\lvert\tau\rvert}\right)=2+0=2$ Mean $=\mu_x = E\left[X(t)\right]=\sqrt{2}$

	$E\left[X^2(t)\right] = R_{XX}(0) = 2 + 4e^{-2(0)} = 2 + 4 = 6$ Variance $= E\left[X^2(t)\right] - \left(E\left[X(t)\right]\right)^2$ $= 6 - \left(\sqrt{2}\right)^2 = 6 - 2 = 4$.
7	**Define cross correlation function and state any two of its properties. [N/D10, M/J13, M/J14,*A/M15* ,M/J19] BTL1** If the process $\{X(t)\}$ and $\{Y(t)\}$ are jointly wide sense stationary, then $E[X(t)Y(t-\tau)]$ is a function of τ, denoted by $R_{XY}(\tau)$. This function $R_{XY}(\tau)$ is called the cross correlation function of the process $\{X(t)\}$ and $\{Y(t)\}$. Properties of cross correlation function are: i. $R_{XY}(-\tau) = R_{YX}(\tau)$. ii. If the process $\{X(t)\}$ and $\{Y(t)\}$ are orthogonal, then $R_{XY}(\tau) = 0$. iii. If the process $\{X(t)\}$ and $\{Y(t)\}$ are independent, then $R_{XY}(\tau) = E[X(t)]E[Y(t-\tau)]$.
8	**Prove that $R_{XY}(\tau) = R_{YX}(-\tau)$. [*M/J16*] BTL5** By definition, we have $R_{YX}(\tau) = E[Y(t)X(t-\tau)]$ $R_{YX}(-\tau) = E[Y(t)X(t+\tau)]$ $R_{YX}(-\tau) = E[X(t+\tau)Y(t)]$ $R_{YX}(-\tau) = R_{XY}(\tau)$ [by definition] Therefore, $R_{XY}(\tau) = R_{YX}(-\tau)$.
9	**. Define power spectral density function of stationary random processes $X(t)$. [N/D13,A/M15] BTL1** If $\{X(t)\}$ is a stationary process with autocorrelation function $R(\tau)$, then the Fourier transform of $R(\tau)$ is called the power spectral density function of $\{X(t)\}$ and denoted as $S(\omega)$ or $S_{XX}(\omega)$. *i.e.* $S(\omega) = \int_{-\infty}^{\infty} R(\tau)e^{-i\omega\tau}\,d\tau$.
10	**. A random process $X(t)$ is defined by $X(t) = k\cos\omega t, t \geq 0$ where ω is a constant and k is uniformly distributed over $(0,2)$. Find the autocorrelation function of $X(t)$.[M/J13] BTL5** Given k is uniformly distributed over $(0,2)$, the density function is given by $f_K(k) = \frac{1}{2-0} = \frac{1}{2}$, $0 < k < 2$ The autocorrelation function $R_{XX}(\tau)$ is given by $R_{XX}(\tau) = E[X(t)X(t-\tau)] = \int_0^2 X(t)X(t-\tau)f(k)dk = \int_0^2 k\cos\omega t.k\cos\omega(t-\tau)\frac{1}{2}dk$

$$= \frac{\cos\omega t\cos\omega(t-\tau)}{2}\int_0^2 k^2\,dk = \frac{\cos\omega t\cos\omega(t-\tau)}{2}\left[\frac{k^3}{3}\right]_0^2 = \frac{8}{6}\cos\omega t\cos\omega(t-\tau)$$

$$R_{XX}(\tau) = \frac{4}{3}\cos\omega t\;\cos\omega(t-\tau)\,.$$

11. **If $R_{XX}(\tau) = \dfrac{25\tau^2+36}{6.25\tau^2+4}$. Find the mean and variance of X. [A/M15] BTL5**

Given $R_{XX}(\tau) = \dfrac{25\tau^2+36}{6.25\tau^2+4}$

$$\mu_x^{\,2} = \lim_{\tau\to\infty} R_{XX}(\tau) = \lim_{\tau\to\infty}\frac{25\tau^2+36}{6.25\tau^2+4} = \lim_{\tau\to\infty}\frac{\tau^2\left(25+\frac{36}{\tau^2}\right)}{\tau^2\left(6.25+\frac{4}{\tau^2}\right)}$$

$$= \lim_{\tau\to\infty}\frac{25+\frac{36}{\tau^2}}{6.25+\frac{4}{\tau^2}} = \frac{25+0}{6.25+0} = \frac{25}{6.25} = 4$$

Mean = $\mu_x = E[X(t)] = 2$

$$E[X^2(t)] = R_{XX}(0) = \frac{25(0)+36}{6.25(0)+4} = \frac{36}{4} = 9$$

Variance $= E[X^2(t)] - (E[X(t)])^2 = 9-(2)^2 = 9-4 = 5$.

12. **Write any two properties of the power spectral density of the WSS process. [*A/M18*] BTL1**

(i) The spectral density of a real random process is an even function.

(ii) The spectral density of a process $\{X(t)\}$, real or complex is a real function of ω and non negative.

13. **Prove that the spectral density of a real random process is an even function. [*N/D15*] BTL5**

By definition, we have

$$S_{XX}(\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)e^{-i\omega\tau}\,d\tau$$

$$S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)e^{i\omega\tau}\,d\tau$$

Put $\tau = -u$ when $\tau = -\infty,\ u = \infty$

$d\tau = -du$ when $\tau = \infty,\ u = -\infty$

$$S_{XX}(-\omega) = \int_{\infty}^{-\infty} R_{XX}(-u)e^{-i\omega u}(-du)$$

	$S_{XX}(-\omega) = -\int_{\infty}^{-\infty} R_{XX}(-u)e^{-i\omega u}\,du$ $S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(-\tau)e^{-i\omega\tau}\,d\tau$, treating u as a dummy variable $S_{XX}(-\omega) = \int_{-\infty}^{\infty} R_{XX}(\tau)e^{-i\omega\tau}\,d\tau$, since $R_{XX}(-\tau) = R_{XX}(\tau)$ $S_{XX}(-\omega) = S_{XX}(\omega)$. Hence the spectral density of a real random process is an even function.
14	. **State any two properties of cross-power density spectrums. [A/M17] BTL1** Properties of cross-power density spectrums are (1) $S_{YX}(\omega) = S_{XY}(-\omega)$ (2) If $\{X(t)\}$ and $\{Y(t)\}$ are independent, then $S_{XY}(\omega) = S_{YX}(\omega) = 0$
15	ate and prove any one of the properties of the cross spectral density function.[A/M15] Cross spectral density function is not an even function of ω, but it has a symmetry relationship. *i.e.* $S_{YX}(\omega) = S_{XY}(-\omega)$ <u>Proof</u>: $S_{XY}(\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau)e^{-i\omega\tau}\,d\tau$ $S_{XY}(-\omega) = \int_{-\infty}^{\infty} R_{XY}(\tau)e^{i\omega\tau}\,d\tau$ Putting $\tau = -u$ when $\tau = -\infty, u = \infty$ $d\tau = -du$ when $\tau = \infty, u = -\infty$ $S_{XY}(-\omega) = \int_{\infty}^{-\infty} R_{XY}(-u)e^{-i\omega u}(-du)$ $S_{XY}(-\omega) = \int_{-\infty}^{\infty} R_{YX}(u)e^{-i\omega u}\,du \quad \because R_{XY}(-\tau) = R_{YX}(\tau)$ $= \int_{-\infty}^{\infty} R_{YX}(\tau)e^{-i\omega\tau}\,d\tau = S_{YX}(\omega)$ *i.e.* $S_{YX}(\omega) = S_{YX}(-\omega)$.
16	**An autocorrelation function $R(\tau)$ of $\{X(t); \tau \in T\}$ is given by c$e^{-\alpha\|\tau\|}; c > 0; \alpha > 0$. Obtain the spectral density of $X(t)$. [N/D16] BTL5** Given $R_{XX}(\tau) = ce^{-\alpha\|\tau\|}$

	$S(\omega)=\int_{-\infty}^{\infty}R_{XX}(\tau)e^{-i\omega\tau}d\tau \quad =\int_{-\infty}^{\infty}c\,e^{-\alpha\|\tau\|}e^{-i\omega\tau}d\tau$ $=c\int_{-\infty}^{\infty}e^{-\alpha\|\tau\|}(\cos\omega\tau-i\sin\omega\tau)d\tau=c\int_{-\infty}^{\infty}e^{-\alpha\|\tau\|}(\cos\omega\tau)d\tau-ic\int_{-\infty}^{\infty}e^{-\alpha\|\tau\|}(\sin\omega\tau)d\tau$ $=2c\int_{0}^{\infty}e^{-\alpha\|\tau\|}\cos\omega\tau\,d\tau$ (Since the first integrand is even and the second integral is odd) $=2c\int_{0}^{\infty}e^{-\alpha\tau}\cos\omega\tau\,d\tau=2c\left[\frac{e^{-\alpha\tau}}{(-\alpha)^2+\omega^2}(-\alpha\cos\omega\tau+\omega\sin\omega\tau)\right]_0^{\infty}$ $=2c\left[0-\frac{1}{\alpha^2+\omega^2}(-\alpha+0)\right]\Rightarrow S(\omega)=\frac{2c\alpha}{\alpha^2+\omega^2}$
17	**An autocorrelation function $R(\tau)$ of $\{X(t);\tau\in T\}$ is given by $ce^{-\alpha\|\tau\|};c>0;\alpha>0$. Obtain the spectral density of $X(t)$. [N/D16] BTL5** Given $R_{XX}(\tau)=ce^{-\alpha\|\tau\|}$ $S(\omega)=\int_{-\infty}^{\infty}R_{XX}(\tau)e^{-i\omega\tau}d\tau \quad =\int_{-\infty}^{\infty}c\,e^{-\alpha\|\tau\|}e^{-i\omega\tau}d\tau$ $=c\int_{-\infty}^{\infty}e^{-\alpha\|\tau\|}(\cos\omega\tau-i\sin\omega\tau)d\tau=c\int_{-\infty}^{\infty}e^{-\alpha\|\tau\|}(\cos\omega\tau)d\tau-ic\int_{-\infty}^{\infty}e^{-\alpha\|\tau\|}(\sin\omega\tau)d\tau$ $=2c\int_{0}^{\infty}e^{-\alpha\|\tau\|}\cos\omega\tau\,d\tau$ (Since the first integrand is even and the second integral is odd) $=2c\int_{0}^{\infty}e^{-\alpha\tau}\cos\omega\tau\,d\tau=2c\left[\frac{e^{-\alpha\tau}}{(-\alpha)^2+\omega^2}(-\alpha\cos\omega\tau+\omega\sin\omega\tau)\right]_0^{\infty}$ $=2c\left[0-\frac{1}{\alpha^2+\omega^2}(-\alpha+0)\right]\Rightarrow S(\omega)=\frac{2c\alpha}{\alpha^2+\omega^2}$
18	**Find the power spectral density of the random process $\{X(t)\}$ whose autocorrelation is $R(\tau)=\begin{cases}-1;-3<\tau<3\\ 0\ ;otherwise\end{cases}$. [*N/D16*] BTL5** Given $R(\tau)=\begin{cases}-1;-3<\tau<3\\ 0\ ;otherwise\end{cases}$ $S(\omega)=\int_{-\infty}^{\infty}R_{XX}(\tau)e^{-i\omega\tau}d\tau=\int_{-3}^{3}(-1)e^{-i\omega\tau}\,d\tau=-\left[\frac{e^{-i\omega\tau}}{-i\omega}\right]_{-3}^{3}$ $=\frac{1}{i\omega}\left(e^{-i\omega3}-e^{i\omega3}\right)=-\frac{1}{i\omega}\left(e^{i3\omega}-e^{-i3\omega}\right)=\frac{i}{\omega}(2\sin h3\omega)=\frac{2i}{\omega}\sin h3\omega$.

19	**he autocorrelation function of the random telegraph signal process is given by $R(\tau)=a^2e^{-2\lambda\|\tau\|}$. Determine the power density spectrum of the random telegraph signal. BTL3** Given $R(\tau)=a^2e^{-2\lambda\|\tau\|}$ $S(\omega)=\int_{-\infty}^{\infty}R_{XX}(\tau)e^{-i\omega\tau}d\tau = \int_{-\infty}^{\infty}a^2e^{-2\lambda\|\tau\|}e^{-i\omega\tau}d\tau = a^2\int_{-\infty}^{\infty}e^{-2\lambda\|\tau\|}(\cos\omega\tau - i\sin\omega\tau)d\tau$ $= a^2\int_{-\infty}^{\infty}e^{-2\lambda\|\tau\|}(\cos\omega\tau)d\tau - ia^2\int_{-\infty}^{\infty}e^{-2\lambda\|\tau\|}(\sin\omega\tau)d\tau$ $= 2a^2\int_{0}^{\infty}e^{-2\lambda\|\tau\|}\cos\omega\tau\, d\tau$ (Since the first integrand is even and the second integral is odd) $= 2a^2\left[\frac{e^{-2\lambda\tau}}{(-2\lambda)^2+\omega^2}(-2\lambda\cos\omega\tau+\omega\sin\omega\tau)\right]_0^{\infty}$ $= 2a^2\left[0-\frac{1}{4\lambda^2+\omega^2}(-2\lambda+0)\right] \Rightarrow S(\omega)=\frac{4a^2\lambda}{4\lambda^2+\omega^2}$
20	**Find the power spectral density of a WSS process with autocorrelation function $R(\tau)=e^{-\alpha\tau^2}$. BTL4** Given $R(\tau)=e^{-\alpha\tau^2}$ $S(\omega) = \int_{-\infty}^{\infty}R(\tau)e^{-i\omega\tau}d\tau$ $= \int_{-\infty}^{\infty}e^{-\alpha\tau^2}e^{-i\omega\tau}d\tau = \int_{-\infty}^{\infty}e^{-\alpha\left(\tau^2+\frac{i\omega\tau}{\alpha}\right)}d\tau = \int_{-\infty}^{\infty}e^{-\alpha\left(\tau^2+\frac{i\omega\tau}{\alpha}+\left(\frac{i\omega}{2\alpha}\right)^2-\left(\frac{i\omega}{2\alpha}\right)^2\right)}d\tau$ $= \int_{-\infty}^{\infty}e^{-\alpha\left(\tau+\frac{i\omega}{2\alpha}\right)^2}e^{-\alpha\left(\frac{\omega^2}{4\alpha^2}\right)}d\tau = e^{-\frac{\omega^2}{4\alpha}}\int_{-\infty}^{\infty}e^{-\alpha\left(\tau+\frac{i\omega}{2\alpha}\right)^2}d\tau$ Put $\sqrt{\alpha}\left(\tau+\frac{i\omega}{2\alpha}\right)=x \Rightarrow \sqrt{\alpha}d\tau=dx \Rightarrow d\tau=\frac{dx}{\sqrt{\alpha}}$ When $\tau=-\infty, x=-\infty$ When $\tau=\infty, x=\infty$ $S(\omega) = e^{-\frac{\omega^2}{4\alpha}}\int_{-\infty}^{\infty}e^{-x^2}\frac{dx}{\sqrt{\alpha}} = \frac{e^{-\frac{\omega^2}{4\alpha}}}{\sqrt{\alpha}}\int_{-\infty}^{\infty}e^{-x^2}dx = \frac{e^{-\frac{\omega^2}{4\alpha}}}{\sqrt{\alpha}}\sqrt{\pi} = \sqrt{\frac{\pi}{\alpha}}e^{-\frac{\omega^2}{4\alpha}}$
21	**If the power spectral density of a WSS process is given by $S(\omega)=\begin{cases}\frac{b}{a}(a-\|\omega\|) & ,\|\omega\|<a\\ 0 & ,\|\omega\|>a\end{cases}$. Find $R(\tau)$. BTL4**

	$R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)e^{i\omega\tau}\,d\omega$ $= \frac{1}{2\pi}\left[\int_{-\infty}^{-a} S(\omega)e^{i\omega\tau}\,d\omega + \int_{-a}^{a} S(\omega)e^{i\omega\tau}\,d\omega + \int_{a}^{\infty} S(\omega)e^{i\omega\tau}\,d\omega\right]$ $= \frac{1}{2\pi}\left[\int_{-a}^{a}\frac{b}{a}(a-\lvert\omega\rvert)e^{i\tau\omega}\,d\omega\right] = \frac{b}{2\pi a}\int_{-a}^{a}(a-\lvert\omega\rvert)(\cos\tau\omega + i\sin\tau\omega)\,d\omega$ $= \frac{b}{2\pi a}\int_{-a}^{a}(a-\lvert\omega\rvert)(\cos\tau\omega)\,d\omega + i\frac{b}{2\pi a}\int_{-a}^{a}(a-\lvert\omega\rvert)(\sin\tau\omega)\,d\omega$ $= \frac{b}{2\pi a}2\int_{0}^{a}(a-\lvert\omega\rvert)(\cos\tau\omega)\,d\omega + i\frac{b}{2\pi a}(0) = \frac{b}{\pi a}\int_{0}^{a}(a-\omega)(\cos\tau\omega)\,d\omega$ $= \frac{b}{\pi a}\left[(a-\omega)\frac{\sin\tau\omega}{\tau} - \frac{\cos\tau\omega}{\tau^2}\right]_0^a = \frac{b}{\pi a}\left[\left(0-\frac{\cos a\tau}{\tau^2}\right)-\left(0-\frac{1}{\tau^2}\right)\right]$ $= \frac{b}{\pi a}\left(\frac{1}{\tau^2}-\frac{\cos a\tau}{\tau^2}\right)$ $R(\tau) = \frac{b}{\pi a\tau^2}(1-\cos a\tau).$
22	**The power spectral density function of a zero mean wide sense stationary process** $\{X(t)\}$ **is given by** $S(\omega)=\begin{cases}1 & ;\lvert\omega\rvert<\omega_0\\ 0 & ;Elsewhere\end{cases}$**. Find** $R(\tau)$**. BTL4** $R(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S(\omega)e^{i\omega\tau}\,d\omega$ $= \frac{1}{2\pi}\left[\int_{-\infty}^{-\omega_0} S(\omega)e^{i\omega\tau}\,d\omega + \int_{-\omega_0}^{\omega_0} S(\omega)e^{i\omega\tau}\,d\omega + \int_{\omega_0}^{\infty} S(\omega)e^{i\omega\tau}\,d\omega\right]$ $= \frac{1}{2\pi}\left[\int_{-\omega_0}^{\omega_0} 1.e^{i\omega\tau}\,d\omega\right] = \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0}(\cos\tau\omega + i\sin\tau\omega)\,d\omega$ $= \frac{1}{2\pi}\int_{-\omega_0}^{\omega_0}(\cos\tau\omega)\,d\omega + i\frac{1}{2\pi}\int_{-\omega_0}^{\omega_0}(\sin\tau\omega)\,d\omega$ $= \frac{1}{2\pi}2\int_{0}^{\omega_0}(\cos\tau\omega)\,d\omega + i\frac{1}{2\pi}(0)$ [$\because$ The 1st integrand is even and the 2nd is odd] $= \frac{1}{\pi}\left(\frac{\sin\tau\omega}{\tau}\right)_0^{\omega_0} = \frac{1}{\pi\tau}(\sin\tau\omega_0 - 0) \quad \Rightarrow R(\tau) = \frac{\sin\omega_0\tau}{\pi\tau}.$

23

Find the auto correlation function whose spectral density is $S(\omega)=\begin{cases}\pi\ ;|\omega|<1\\ 0\ ;\ otherwise\end{cases}$ **. [*A/M15, M/J16* ,M/J16] BTL4**

$$R(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}S(\omega)e^{i\omega\tau}d\omega$$

$$=\frac{1}{2\pi}\left[\int_{-\infty}^{-1}S(\omega)e^{i\omega\tau}d\omega+\int_{-1}^{1}S(\omega)e^{i\omega\tau}d\omega+\int_{1}^{\infty}S(\omega)e^{i\omega\tau}d\omega\right]$$

$$=\frac{1}{2\pi}\left[\int_{-1}^{1}\pi.e^{i\omega\tau}d\omega\right]=\frac{1}{2\pi}\int_{-1}^{1}\pi(\cos\tau\omega+i\sin\tau\omega)d\omega$$

$$=\frac{1}{2}\int_{-1}^{1}(\cos\tau\omega)d\omega+i\frac{1}{2}\int_{-1}^{1}(\sin\tau\omega)d\omega$$

$$=\frac{1}{2}2\int_{0}^{1}(\cos\tau\omega)d\omega+i\frac{1}{2\pi}(0)\,[\because \text{The } 1^{st} \text{ integrand is even and the } 2^{nd} \text{ is odd}]$$

$$=\left(\frac{\sin\tau\omega}{\tau}\right)_0^1=\frac{1}{\tau}(\sin\tau-0)\quad\Rightarrow\ R(\tau)=\frac{\sin\tau}{\tau}.$$

24

Determine the autocorrelation function of the random process with the spectral density given by $S_{XX}(\omega)=\begin{cases}S_0\ \ ;|\omega|<\omega_0\\ 0\ \ ;\ otherwise\end{cases}$ **. [A/M17,*A/M18*] BTL3**

$$R(\tau)=\frac{1}{2\pi}\int_{-\infty}^{\infty}S(\omega)e^{i\tau\omega}d\omega$$

$$=\frac{1}{2\pi}\left[\int_{-\infty}^{-\omega_0}S(\omega)e^{i\tau\omega}d\omega+\int_{-\omega_0}^{\omega_0}S(\omega)e^{i\tau\omega}d\omega+\int_{\omega_0}^{\infty}S(\omega)e^{i\tau\omega}d\omega\right]$$

$$=\frac{1}{2\pi}\left[\int_{-1}^{1}S_0\,e^{i\tau\omega}d\omega\right]=\frac{S_0}{2\pi}\int_{-\omega_0}^{\omega_0}(\cos\tau\omega+i\sin\tau\omega)d\omega=\frac{S_0}{2\pi}\int_{-\omega_0}^{\omega_0}(\cos\tau\omega)d\omega+i\frac{S_0}{2\pi}\int_{-\omega_0}^{\omega_0}(\sin\tau\omega)d\omega$$

$$=\frac{S_0}{2\pi}2\int_{0}^{\omega_0}(\cos\tau\omega)d\omega+i\frac{S_0}{2\pi}(0)\,[\because \text{The } 1^{st} \text{ integrand is even and the } 2^{nd} \text{ is odd}]$$

$$=\frac{S_0}{\pi}\left(\frac{\sin\tau\omega}{\tau}\right)_0^{\omega_0}=\frac{S_0}{\pi\tau}(\sin\tau\omega_0-0)\quad\Rightarrow\ R(\tau)=\frac{S_0}{\pi\tau}\sin\omega_0\tau\ .$$

25

State Wiene–Khinchine theorem.[N/D13,N/D15] OR Write the Wiener–Khinchine relation. [N/D14,N/D16,N/D17] BTL1

	If $X_T(\omega)$ is the Fourier transform of the truncated random process defined as $X_T(t)=\begin{cases} X(t) & for\ \|t\|\le T \\ 0 & for\ \|t\|>T \end{cases}$ where $\{X(t)\}$ is a real WSS process with power spectral density function $S(\omega)$, then $S(\omega)=\lim_{T\to\infty}\frac{1}{2T}E\left[\| X_T(\omega)\|^2\right]$.
	PART * B
1	**Consider two random processes $X(t)=3\cos(\omega t+\theta)$ and $Y(t)=2\cos(\omega t+\phi)$ where $\phi=\theta-\frac{\pi}{2}$ and θ is uniformly distributed random variable over $(0,2\pi)$. Verify whether $\|R_{XY}(\tau)\|\le\sqrt{R_{XX}(0)R_{YY}(0)}$. [*__A/M15__*,A/M17,*__A/M2019__*] BTL3** **Answer:Page: 4.26-Dr.A. Singaravelu** • RXX(0) = 9 / 2 (2M) • RYY(0) = 2 (2M) • RXY(τ) = 3 sin ωτ (2M) • |RXY(τ)| ≤ √RXX(0) . RYY(0) (2M)
2	**Find the power spectral density function whose autocorrelation function is given by $R_{XX}(\tau)=\frac{A^2}{2}\cos(\omega_0\tau)$. [M/J12] BTL4** **Answer:Page: 4.50-Dr.A. Singaravelu** • Sxx(W) $=\int_{-\infty}^{\infty}RXX(\tau)e^{-i\omega\tau}\,d\tau$ (2M) • Sxx(W) $=\frac{\pi A^2}{2}\left[\sigma(W+W_0)+\delta(W-W_0)\right]$ (6M)
3	**If $\{X(t)\}$ and $\{Y(t)\}$ are two random processes with autocorrelation function $R_{XX}(\tau)$ and $R_{YY}(\tau)$ respectively, then prove that $\|R_{XY}(\tau)\|\le\sqrt{R_{XX}(0)R_{YY}(0)}$. Establish any two properties of autocorrelation function $R_{XX}(\tau)$. [N/D10,N/D12,*__M/J16__*,*__N/D16__*] BTL5** **Answer:Page: 4.23-Dr.A. Singaravelu** • $RXX(0)=E[x^2(t)], RYY(0)=E[y^2(t+\tau)]$ (2M) • $\|RXY(\tau)\|\le\sqrt{RXX(0)\,RYY(0)}$ (4M) • $RXX(\tau)=RXX, \|RXX(\tau)\|\le RXX(0)$ (2M)
4	**If $X(t)=5\sin(\omega t+\phi)$ and $Y(t)=2\cos(\omega t+\theta)$ where ω is a constant, $\theta+\phi=\frac{\pi}{2}$ and ϕ is a random variable uniformly distributed in $(0,2\pi)$, find $R_{XX}(\tau)$, $R_{YY}(\tau)$, $R_{XY}(\tau)$ and $R_{YX}(\tau)$. Verify two properties of autocorrelation function and cross correlation function. [*__N/D16__*] BTL5**

	Answer:Page: 4.26-Dr.A. Singaravelu • $RXX(\tau) = \frac{25}{2}\cos w\,\tau$ (2M) • $RYY(\tau) = 2\cos w\,\tau$ (3M) • $RXY(\tau) = 5\sin w\,\tau$ (3M) • $\lvert RXY(\tau)\rvert \le \sqrt{RXX(0)\,RYY(0)}$ (4M) • $\lvert RXY(\tau)\rvert \le \frac{1}{2}\,[RXX(0) + RYY(0)]$ (4M)
5	**Two random processes $X(t)$ and $Y(t)$ are defined as follows: $X(t) = A\cos(\omega t+\theta)$ and $Y(t) = B\sin(\omega t+\theta)$ where A, B and ω are constants; θ is a uniform random variable over $(0,2\pi)$. Find the cross correlation function of $X(t)$ and $Y(t)$. [M/J13,*N/D15*] BTL5** **Answer:Page: 4.24-Dr.A. Singaravelu** • $RXY(t,t+\tau) = E\,[X(t).Y(t+\tau)]$ (2M) • $RXY(t,t+\tau) = \frac{A^2}{2}\sin w\,\tau$ (6M)
6	**Define spectral density of a stationary random process $X(t)$. Prove that for a real random process $X(t)$ the power spectral density is an even function. [M/J13,N/D17] BTL5** **Answer:Page: 4.33-Dr.A. Singaravelu** • $Sxx(w) = \int_{-\infty}^{\infty} Rxx(\tau)\,e^{-iw\tau}\,d\tau$ (2M) • $Sxx(-w) = Sxx(w)$ (2M)
7	**State and prove Wiener Khintchine theorem and hence find the power spectral density of a WSS process $X(t)$ which has an autocorrelation $R_{XX}(\tau)=A_0\left[1-\frac{\lvert\tau\rvert}{T}\right], -T\le\tau\le T$. [Nov/Dec2019] BTL5** **Answer:Page: 4.43-Dr.A. Singaravelu** • $Sxx(w) = \lim_{T\to\infty}\left(\frac{1}{2T}\,E\{\lvert X_T(W)\rvert^2\}\right)$ (2M) • $Sxx(w) = \frac{2}{Tw^2}\,[1-\cos WT]$ (6M)
8	**State Wiener-Khinchine relation and define cross power spectral density and its properties. [M/J16] BTL5** **Answer:Page: 4.36-Dr.A. Singaravelu** • $Sxx(W) = \lim_{T\to\infty}[\frac{1}{2T}\,E\{\lvert x_T(W)\rvert^2\}]$ (3M) • $Sxy(W) = \int_{-\infty}^{\infty} Rxy(\tau)e^{-1w\tau}d\tau$ (2M) • $Sxy(w) = Syx(-w)$ (1M) • $Sxy(w) = Syx(w) - 2\pi\,E(x)E(y)\,\sigma(w)$ (1M) If {x(t)} and {y(t)} are orthogonal then $Sxy(w) = 0$ and $Syx(w) = 0$ (1M)
9	**Find the power spectral density of a random signal with auto correlation function $e^{-\lambda\lvert\tau\rvert}$.**

	[A/M15,Apr/May19] BTL5 **Answer:Page: 4.42-Dr.A. Singaravelu** • $Sxx(w) = \int_{-\infty}^{\infty} Rxx(\tau)e^{-iw\tau}d\tau$ (2M) • $Sxx(w) = \frac{2\lambda}{\lambda^2 + w^2}$ (6M)
10	**Given that a process $X(t)$ has an autocorrelation function $R_{XX}(\tau) = Ae^{-\alpha\|\tau\|}\cos\omega_0\tau$ where $A>0, \alpha>0$ and ω_0 are real constants, find the power spectral density of $X(t)$. [N/D16,*A/M18*] BTL5** **Answer:Page: 4.49-Dr.A. Singaravelu** • $Sxx(w) = \int_{-\infty}^{\infty} Rxx(\tau)e^{-iw\tau}\, d\tau$ (2M) • $Sxx(w) = \frac{2A\alpha}{\alpha^2 + w^2}$ (6M)
11	**Autocorrelation function of an ergodic process $\{X(t) = X\}$ is $R_{XX}(\tau) = \begin{cases} 1-\|\tau\| & , \|\tau\| \le 1 \\ 0 & , otherwise \end{cases}$.** **Obtain the spectral density of X . [N/D10,N/D12, *M/J16*,*N/D17*] BTL5** **Answer:Page: 4.44-Dr.A. Singaravelu** • $Sxx(w) = \int_{-\infty}^{\infty} Rxx(\tau)e^{-iw\tau}\, d\tau$ (2M) • $Sxx(w) = [\frac{\int in\,(\frac{W}{2})}{\frac{W}{2}}]^2$ S(6M)
12	**The autocorrelation function of the random telegraph signal process is given by $R(\tau) = a^2 e^{-2\|\tau\|}$. Determine the power density spectrum of the random telegraph signal.[N/D13] (OR) The autocorrelation function of the random telegraph signal process is given by $R(\tau) = a^2 e^{-2\gamma\|\tau\|}$. Determine the power density spectrum of the random telegraph signal. [*N/D15* , M/J16] BTL5** **Answer:Page: 4.42-Dr.A. Singaravelu** • $Sxx(w) = \int_{-\infty}^{\infty} Rxx(\tau)e^{-iw\tau}\, d\tau$ (2M) • $Sxx(w) = \frac{4a^2r}{4r^2w^2}$ (6M)
13	**Find the spectral density of a WSS random process $\{X(t)\}$ whose autocorrelation function is $e^{-\frac{\alpha^2\tau^2}{2}}$. [N/D15,*Nov/Dec19*] BTL5** **Answer:Page: 4.46-Dr.A. Singaravelu** • $\delta xx(w) = \int_{-\infty}^{\infty} Rxx(\tau)e^{-iw\tau}\, d\tau$ (2M) • $Sxx(w) = \frac{4a^2r}{4r^2w^2}$ (6M) Type equation here.

14	**The autocorrelation function of the random process $X(t)$ is given by $R(\tau)=\begin{cases}1-\frac{\lvert\tau\rvert}{T} & ,\lvert\tau\rvert\le T\\ 0 & ,\lvert\tau\rvert>T\end{cases}$.** **Find the power spectrum of the process $X(t)$. [A/M10,*A/M15*,M/J16,N/D16] BTL5** **Answer:Page: 4.43-Dr.A. Singaravelu** • $Sxx(w) = \int_{-\infty}^{\infty} Rxx(\tau)e^{-iw\tau}\, d\tau$ (2M) • $Sxx(w) = \frac{2}{TW^2}[1-\cos WT]$ (6M)
15	**If the power spectral density of a WSS process is given by $S(\omega)=\begin{cases}\frac{b}{a}(a-\lvert\omega\rvert) & ,\lvert\omega\rvert\le a\\ 0 & ,\lvert\omega\rvert>a\end{cases}$** **Find the autocorrelation function of the process. [N/D13,N/D14,*N/D16*,*N/D17*] BTL5** **Answer:Page: 4.60-Dr.A. Singaravelu** • $Rxx(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Sxx(w)e^{iw\tau}\, dw$ (2M) • $Rxx(\tau) = \frac{b}{a\pi\tau^2}\, 2\, sin^2\left(\frac{a\tau}{2}\right)$ (6M)
16	**The autocorrelation function of the Poisson increment process is given by $R(\tau)=\begin{cases}\lambda^2 & for\ \lvert\tau\rvert>\varepsilon\\ \lambda^2+\frac{\lambda}{\varepsilon}\left(1-\frac{\lvert\tau\rvert}{\varepsilon}\right) & for\ \lvert\tau\rvert\le\varepsilon\end{cases}$. Find the power spectral density of the process.** **[N/D11] BTL5** **Answer:Page: 4.51-Dr.A. Singaravelu** • $Sxx(w) = \int_{-\infty}^{\infty} Rxx(\tau)e^{-iw\tau}\, d\tau$ (2M) • $Sxx(w) = 2\pi\lambda^2\sigma(w) + 4\lambda\frac{sin^2(\frac{ew}{2})}{e^2w^2}$ (6M)
17	**If $X(t)$ and $Y(t)$ are uncorrelated random processes, then find the power spectral density of $Z(t)$ if $Z(t)=X(t)+Y(t)$. Also find the cross spectral density $S_{XZ}(\omega)$ and $S_{YZ}(\omega)$.[N/D16] BTL5** **Answer:Page: 4.81-Dr.A. Singaravelu** • $Szz(w) = Sxx(w) + Syy(w) + Sxy(w) + Syx(w)$ (4M) • $Sxz(w) = Sxx(w) + Sxy(w)$ (2M) • $Syz(w) = Syy(w) + Syx(w)$ (2M)
18	**The power spectral density of a zero mean WSS process $\{X(t)\}$ is given by**

	$S(\omega)=\begin{cases}1 \;;\; \lvert\omega\rvert<\omega_0\\ 0 \;;\; elsewhere\end{cases}$. **Find** $R(\tau)$ **and show also that** $X(t)$ **and** $X\left(t+\frac{\pi}{\omega_0}\right)$ **are uncorrelated. [A/M11] BTL5** **Answer:Page: 4.63-Dr.A. Singaravelu** • $Rxx(\tau) = \frac{1}{\pi\tau}\sin(w_0\tau)$ (4M) • $c\left[x(t).x\left(t+\frac{\pi}{w_0}\right)\right] = 0$ (4M)
19	**Find the autocorrelation function of the process** $\{X(t)\}$ **for which the power spectral density is given by** $S_{XX}(\omega)=1+\omega^2 \;\; for \; \lvert\omega\rvert<1$ **and** $S_{XX}(\omega)=0 \;\; for \; \lvert\omega\rvert>1$. **[A/M10,N/D16,*A/M17*] BTL5** **Answer:Page: 4.68-Dr.A. Singaravelu** • $Rxx(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Sxx(w)e^{iw\tau}\,dw$ (2M) • $Rxx(\tau) = \frac{2}{\pi\tau^2}[\tau^2\sin\tau + \tau\cos\tau - \sin\tau]$ (6M)
20	**If the power spectral density of a continuous process is** $S_{XX}(\omega)=\frac{\omega^2+9}{\omega^4+5\omega^2+4}$**, find the mean square value of the process. [N/D11,*A/M15, M/J16*] BTL5** **Answer:Page: 4.67-Dr.A. Singaravelu** • $Rxx(\tau) = F^{-1}\left[\frac{w^2+9}{w^4+5w^2+4}\right]$ (2M) • $Rxx(\tau) = \frac{8}{6}e^{-\lvert\tau\rvert} - \frac{5}{12}e^{-2\lvert\tau\rvert}$ (3M) • $Rxx(0) = \frac{11}{12}$ (3M)
21	**The power spectrum of a Wide sense stationary process** $\{X(t)\}$ **is given by** $S(\omega)=\frac{1}{(1+\omega^2)^2}$**. Find its autocorrelation function** $R(\tau)$**. [A/M15,N/D15,*N/D15*,A/M17] BTL5** **Answer:Page: 4.63-Dr.A. Singaravelu** • $Rxx(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Sxx(w)e^{iw\tau}\,dw$ (2M) • $Rxx(\tau) = \frac{1}{4}(1+\tau)e^{-\tau}$ (6M)
22	**The cross power spectrum of real random process** $X(t)$ **and** $Y(t)$ **is given by** $S_{XY}(\omega)=\begin{cases}a+jb\omega \;;\; \lvert\omega\rvert<1\\ 0 \;;\; elsewhere\end{cases}$. **Find the cross correlation function. [N/D10,A/M11, N/D11,N/D15, *M/J16* , M/J16,N/D16,*A/M17*,*A/M18*] BTL4** **Answer:Page: 4.77-Dr.A. Singaravelu**

	• $Rxy(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Sxy(w)e^{iw\tau}\,dw$ (2M) • $Rxx(\tau) = \frac{1}{\pi\tau^2}\,[(a\tau - b)\sin\tau + \ b\,\tau\cos\tau]$ (6M)
23	**If the cross power spectral density of $X(t)$ and $Y(t)$ is given by $S_{XY}(\omega) = a + \frac{ib\omega}{\alpha}$, $-\alpha<\omega<\alpha$, $\alpha>0$ where a and b are constants, find the cross correlation function. [M/J13,*<u>N/D17</u>*] BTL4** **Answer:Page: 1.80-Dr.A. Singaravelu** • $Rxy(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} Sxy(w)e^{iw\tau}\,dw$ (2M) • $Rxy(\tau) = \frac{1}{\pi\tau^2}\left[\left(\tau a - \frac{b}{w}\right)\sin w\tau + \ \tau b\cos\omega\tau\right]$ (6M)
24	**If $\{X(t)\}$ is a WSS process with autocorrelation function $R_{XX}(\tau)$ and if $Y(t) = X(t+a) - X(t-a)$. Show that $R_{YY}(\tau) = 2R_{XX}(\tau) - R_{XX}(\tau+2a) - R_{XX}(\tau-2a)$. BTL5** **Answer:Page: 4.47-Dr.A. Singaravelu** • $Rxx(\tau) = E[x(t).x(t+\ \tau)]$ (1M) • $Ryy(\tau) = E[y(t).y(t+\ \tau)]$ (1M) • $Ryy(\tau) = 2\,Rxx(\tau) - \ Rxx(\tau + 2a) - \ Rxx(\tau - 2a)$ (6M)
	UNIT V-LINEAR SYSTEM WITH RANDOM INPUTS
	Linear time invariant system- System transfer function – Linear system with random inputs –Auto correlation and cross correlation functions of input and output.
	PART*A
1	**Define a system. When is it called a linear system? [M/J14,A/M15,M/J16,N/D17] BTL1** A system is a functional relationship between the input $x(t)$ and the output $y(t)$. The functional relationship is written as $y(t) = f[x(t)]$. If $f[a_1 X_1(t) \pm a_2 X_2(t)] = a_1 f[X_1(t)] \pm a_2 f[X_2(t)]$, then f is called a linear system.
2	**Define linear time- invariant system. [A/M10, M/J13,N/D16] BTL1** If $f[a_1 X_1(t) \pm a_2 X_2(t)] = a_1 f[X_1(t)] \pm a_2 f[X_2(t)]$, then f is called a linear system. If $Y(t+h) = f[x(t+h)]$ where $Y(t) = f[X(t)]$, f is called a time – invariant system or $X(t)$ and $Y(t)$ are said to form a time invariant system.
3	**Define casual system. [*<u>N/D15</u>*] BTL1** If the value of the output $Y(t)$ at $t = t_1$ depends only on the past values of the input $X(t)$, $t \le t_1$ (ie) $Y(t_1) = f[X(t); t \le t_1]$, then the system is called a casual system.
4	**When a system is said to be stable? BTL5** A linear time invariant system, $y(t) = f[x(t)]$ is said to be stable if its response to any bounded

	input is bounded.
5	**Prove that $Y(t) = 2X(t)$ is linear. [*A/M15*] BTL5** Let $Y_1(t) = 2X_1(t)$ and $Y_2(t) = 2X_2(t)$ If the input $X(t) = a_1 X_1(t) + a_2 X_2(t)$, then $Y(t) = 2\,(a_1 X_1(t) + a_2 X_2(t)) = 2a_1 X_1(t) + 2a_2 X_2(t) = a_1(2X_1(t)) + a_2(2X_2(t))$ $Y(t) = a_1 Y_1(t) + a_2 Y_2(t)$. Hence $Y(t) = 2X(t)$ is linear.
6	**Check whether the system $Y(t) = X^3(t)$ is linear or not. [N/D15,A/M17,*A/M17*] BTL5** Let $Y_1(t) = X_1^3(t)$ and $Y_2(t) = X_2^3(t)$ If the input $X(t) = a_1 X_1(t) + a_2 X_2(t)$, then $Y(t) = (a_1 X_1(t) + a_2 X_2(t))^3 = a_1^3 X_1^3(t) + 3a_1^2 a_2 X_1^2(t) X_2(t) + 3a_1 a_2^2 X_1(t) X_2^2(t) + a_2^3 X_2^3(t)$ $Y(t) \neq a_1 Y_1(t) + a_2 Y_2(t)$. Hence $Y(t) = X^3(t)$ is not linear.
7	**State the properties of linear system. [N/D11] BTL1** The properties of linear system are (i) If a system is such that its input $X(t)$ and its output $Y(t)$ are related by a convolution integral, then the system is a linear time invariant system. (ii) If the input to a time-invariant, stable linear system is a WSS process, the output will also be a WSS process. (iii) The power spectral densities of the input and output processes in the system are connected by the relation $S_{YY}(\omega) = \|H(\omega)\|^2 S_{XX}(\omega)$, where $H(\omega)$ is the Fourier transform of unit impulse response function $h(t)$.
8	**Define system weighting function. BTL1** If the output $Y(t)$ of a system is expressed as the convolution of the input $X(t)$ and a function $h(t)$ (ie) $Y(t) = \int_{-\infty}^{\infty} h(u) X(t-u)\, du$, then $h(t)$ is called the system weighting function.
9	**Prove that the mean of the output process is the convolution of the mean of the input process and the impulse response.[*A/M18*] BTL5** The output $Y(t)$ is expressed as a convolution of the input $X(t)$ with a system weighting function $h(t)$. i.e. the input-output relationship will be of the form $Y(t) = X(t) * h(t)$. Hence, the mean of the output process is $E[Y(t)] = E[X(t)] * h(t)$ (i.e) the convolution of the mean of the input process and the impulse response.
10	**State the relation between input and output of a linear time invariant system. [*A/M15*] BTL1** The output $Y(t)$ is expressed as a convolution of the input $X(t)$ with a system weighting function $h(t)$. i.e. the input-output relationship will be of the form $Y(t) = \int_{-\infty}^{\infty} h(u) X(t-u)\, du$.

11	**What is unit impulse response of a system? Why is it called so? [M/J12, N/D17] BTL5** If a system $Y(t)=\int_{-\infty}^{\infty} h(u)X(t-u)\,du$, then the system weighting function $h(t)$ is also called unit impulse response of the system. It is called so because the response (output) $Y(t)$ will be $h(t)$, when the input $X(t)$= the unit impulse function $\delta(t)$.
12	**Prove that if the input of a system is the unit impulse function then the output is the system wieghting function. [N/D17] BTL5** If the input of a linear system is a Gaussian random process, then the output will also be a Gaussian random process.
13	**If the input $X(t)$ of the system $Y(t)=\int_{-\infty}^{\infty} h(u)X(t-u)\,du$ is the unit impulse function, prove that $Y(t)=h(t)$. BTL5** Given $Y(t)=\int_{-\infty}^{\infty} h(u)X(t-u)\,du$ Put $X(t)=\delta(t)$ Therefore , $X(t-u)=\delta(t-u)$ $Y(t)=\int_{-\infty}^{\infty} h(u)\delta(t-u)\,du$ $Y(t)=\int_{-\infty}^{\infty} h(t-u)\delta(u)\,du$ (By the property of convolution) $Y(t)=h(t-o)=h(t)$.
14	**If a system is defined as $Y(t)=\frac{1}{T}\int_{0}^{\infty} X(t-u)e^{-\frac{u}{T}}\,du$, find its unit impulse function. BTL4** Given $Y(t)=\frac{1}{T}\int_{0}^{\infty} X(t-u)e^{-\frac{u}{T}}\,du$ $Y(t)=\int_{0}^{\infty} \frac{1}{T}e^{-\frac{u}{T}}X(t-u)\,du$ Unit impulse function is given by $h(t)=\begin{cases}\frac{1}{T}e^{-\frac{t}{T}} & , t\geq 0\\ 0 & , elsewhere\end{cases}$.
15	**If $\{X(t)\}$ and $\{Y(t)\}$ in the system $Y(t)=\int_{-\infty}^{\infty} h(u)X(t-u)\,du$ are WSS processes, how are their autocorrelation functions related? [N/D11] BTL4** The autocorrelation functions are related as $R_{YY}(\tau)=R_{XY}(\tau)*h(\tau)$

	(or) $R_{XY}(\tau) = R_{XX}(\tau) * h(-\tau)$ where $*$ denotes convolution.
16	**If the input and output of the system** $Y(t) = \int_{-\infty}^{\infty} h(u) X(t-u)\, du$ **are WSS processes, how are their power spectral densities related? BTL5** The power spectral densities are related as $S_{YY}(\omega) = S_{XX}(\omega)\lvert H(\omega)\rvert^2$ where $H(\omega)$ is the Fourier transform of $h(t)$.
17	**Define the power transfer function or system function of the system. [*N/D15*] BTL5** The power transfer function or system function of the system is the Fourier transform of the unit impulse response function of the system.
18	**If the system has the impulse response** $h(t) = \begin{cases} \frac{1}{2c} & for\ \lvert t\rvert \le c \\ 0 & , for\ \lvert t\rvert > c \end{cases}$ **. Write down the relation between the spectrums of input** $X(t)$ **and output** $Y(t)$ **. [*May2019*] BTL4** $H(\omega) = F[h(t)]$ $= \int_{-\infty}^{\infty} h(t) e^{-i\omega t}\, dt$ $= \int_{-c}^{c} \frac{1}{2c} e^{-i\omega t}\, dt = \frac{1}{2c}\int_{-c}^{c} (\cos\omega t - i \sin\omega t) dt$ $= \frac{1}{2c}\int_{-c}^{c} \cos\omega t\, dt - i\, \frac{1}{2c}\int_{-c}^{c} \sin\omega t\, dt$ $= \frac{1}{2c}\, 2\int_{0}^{c} \cos\omega t\, dt - i\, \frac{1}{2c}(0)$ [since the first integrand is an even function and second integrand is an odd function] $= \frac{1}{c}\left[\frac{\sin\omega t}{\omega}\right]_0^c = \frac{1}{\omega c}[\sin c\omega - 0] = \frac{\sin c\omega}{c\omega}$ $S_{YY}(\omega) = \lvert H(\omega)\rvert^2 S_{XX}(\omega) \quad \Rightarrow \quad S_{YY}(\omega) = \frac{\sin^2 c\omega}{c^2\omega^2} S_{XX}(\omega).$
19	**Find the system transfer function, if a linear time invariant system has an impulse function** $H(t) = \begin{cases} \frac{1}{2c} & ; \lvert t\rvert \le c \\ 0 & ; \lvert t\rvert \ge c \end{cases}$ **. [A/M11, N/D12] BTL5**

	$System\ transfer\ \ function = H(\omega) = F[H(t)] = \int_{-\infty}^{\infty} H(t) e^{-i\omega t}\, dt = \int_{-c}^{c} \frac{1}{2c}\, e^{-i\omega t}\, dt$ $= \frac{1}{2c} \int_{-c}^{c} (\cos\omega t - i \sin\omega t) dt = \frac{1}{2c} \int_{-c}^{c} \cos\omega t\, dt - i\, \frac{1}{2c} \int_{-c}^{c} \sin\omega t\, dt$ $= \frac{1}{2c}\, 2 \int_{0}^{c} \cos\omega t\, dt - i\, \frac{1}{2c}\, (0)$ [since the 1st integrand is even and 2nd integrand is odd] $= \frac{1}{c}\left[\frac{\sin\omega t}{\omega}\right]_0^c = \frac{1}{\omega c}[\sin c\omega - 0] = \frac{\sin c\omega}{c\omega}$
20	**If the input to a linear time invariant system is white noise $\{N(t)\}$, what is power spectral density function of the output? BTL5** If the input to a linear time invariant system is white noise $\{N(t)\}$, then the power spectral density of the output $S_{YY}(\omega)$ is given by $S_{YY}(\omega) = S_{XX}(\omega)\lvert H(\omega)\rvert^2 \Rightarrow S_{YY}(\omega) = \frac{N_0}{2}\lvert H(\omega)\rvert^2$ where $\{Y(t)\}$ is the output process and $H(\omega)$ is the power transfer function.
21	**A wide sense stationary noise process $N(t)$ has an autocorrelation function $R_{NN}(\tau) = P e^{-3\lvert\tau\rvert}, -\infty < \tau < \infty$ with P as a constant. Find its power density spectrum. BTL4** $S_{NN}(\omega) = \int_{-\infty}^{\infty} R_{NN}(\tau) e^{-i\omega\tau}\, d\tau = \int_{-\infty}^{\infty} P e^{-3\lvert\tau\rvert} e^{-i\omega\tau}\, d\tau$ $= \int_{-\infty}^{\infty} P e^{-3\lvert\tau\rvert} (\cos\omega\tau - i \sin\omega\tau) d\tau$ $= P\left[\int_{-\infty}^{\infty} e^{-3\lvert\tau\rvert} \cos\omega\tau\, d\tau - i \int_{-\infty}^{\infty} e^{-3\lvert\tau\rvert} \sin\omega\tau\, d\tau\right]$ $= P\, 2\int_{0}^{\infty} e^{-3\lvert\tau\rvert} \cos\omega\tau\, d\tau - P\, i\, (0)$ [since the first integrand is an even function and second integrand is an odd function] $= 2\, P \int_{0}^{\infty} e^{-3\tau} \cos\omega\tau\, d\tau = 2\, P \left[\frac{e^{-3\tau}}{(-3)^2 + \omega^2} (-3\cos\omega\tau + \omega \sin\omega\tau)\right]_0^\infty$ $= 2P\left[0 - \frac{1}{9+\omega^2}(-3+0)\right] = \frac{6P}{9+\omega^2}$.

22	**If** $X(t)$ **is a WSS process and if** $Y(t)=\int_{-\infty}^{\infty} h(u)X(t-u)du$ **then prove that** $R_{XY}(\tau)=R_{XX}(\tau)*h(-\tau)$**. [<u>A/M17</u>] BTL5** Given $Y(t)=\int_{-\infty}^{\infty} h(u)X(t-u)du$ $X(t+\tau)Y(t)=\int_{-\infty}^{\infty} X(t+\tau)X(t-u)h(u)du$ $E[X(t+\tau)Y(t)]=\int_{-\infty}^{\infty} E[X(t+\tau)X(t-u)du]$ $R_{XY}(\tau)=\int_{-\infty}^{\infty} R_{XX}(\tau+u)h(u)du$ Put $u=-\beta \Rightarrow du=-d\beta$ When $u=-\infty,\ \beta=\infty$ When $u=\infty,\ \beta=-\infty$ $R_{XY}(\tau)=\int_{\infty}^{-\infty} R_{XX}(\tau-\beta)h(-\beta)(-d\beta)=-\int_{\infty}^{-\infty} R_{XX}(\tau-\beta)h(-\beta)d\beta$ $=\int_{-\infty}^{\infty} R_{XX}(\tau-\beta)h(-\beta)d\beta=\int_{-\infty}^{\infty} R_{XX}(\tau-u)h(-u)du=R_{XX}(\tau)*h(-\tau)$
23	**Define Time invariant system. (Apr/May 2019)**BTL1 Let $y(t)=f[x(t)]$. If $y(t+h)=f[x(t+h)]$then 'f' is called a time variant system or x(t) and y(t) are said to form a time invariant system.
24	**Define memoryless system.** BTL1 If the value of the output y(t) at t = t_0 depends only on the past values of the input x(t), $t \le t_0$ i.e., if $\mathbf{y(t0)=f[x(t): t \le t_0]}$ then such a system is called a casual system. .
25	**Define stable system.** BTL1 A linear time invariant system is said to be stable if its response to any bounded input is bounded.
	PART*B
1	**Show that if** $\{X(t)\}$ **is a WSS process, then the output** $\{Y(t)\}$ **is a WSS process. Also find** $R_{XY}(\tau)$**. [N/D10,N/D11,N/D12,M/J13,M/J14,A/M15,<u>*A/M15*</u>,N/D16,<u>*N/D16*</u>,A/M17] BTL5** **Answer:Page: 5.6,5,7-Dr.A. Singaravelu** • $y(t)=\int_{-\infty}^{\infty} h(u)x(t-u)du$ (2M) • $Ryy(t,t+\tau)=g(\tau)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(u_1)h(u_2)du_1du_2$ (6M)

2	**If a system is connected by a convolution integral $Y(t)=\int_{-\infty}^{\infty} h(u)X(t-u)du$ where $X(t)$ is the input and $Y(t)$ is the output then prove that the system is a linear time invariant system. [*A/M17*] BTL5** **Answer:Page: 5.5-Dr.A. Singaravelu** • $y(t) = a_1y_1(t) + a_2y_2(t)$ (4M) • $y(t) = y(t+h)$ (4M)
3	**For a linear system with random input $x(t)$, the impulse response $h(t)$ and output $y(t)$, obtain the cross correlation function $R_{XY}(\tau)$ and the output autocorrelation function $R_{YY}(\tau)$ BTL4** **Answer:Page: 5.7-Dr.A. Singaravelu** • $y(t) = \int_{-\infty}^{\infty} h(u)x(t-u)du$ (2M) • $Rxy(\tau) = Rxx(\tau) * h(\tau)$ (3M) • $Ryy(\tau) = Rxx(\tau) * h(-\tau)$ (3M)
4	**For a linear system with random input $x(t)$, the impulse response $h(t)$ and output $y(t)$, obtain the power spectrum $S_{YY}(\omega)$ and cross power spectrum $S_{XY}(\omega)$. BTL4** **Answer:Page: 5.9-Dr.A. Singaravelu** • $y(t) = \int_{-\infty}^{\infty} h(u)x(t-u)du$ (2M) • $Sxy(w) = Sxx(w)\,H(w)$ (4M) • $Syy(w) = Sxx(w)\,\lvert H(w)\rvert^2$ (2M)
5	**Let $X(t)$ be a WSS process which is the input to a linear time invariant system with unit impulse $h(t)$ and output $Y(t)$, then prove that $S_{YY}(\omega)=\lvert H(\omega)\rvert^2\, S_{XX}(\omega)$. [N/D11 , M/J13,A/M15,*A/M15* , M/J16,*N/D16*,*A/M17*,*A/M18*] BTL5** **Answer:Page: 5.9-Dr.A. Singaravelu** • $Rxy(\tau) = Rxx(\tau) * h(\tau)$ (5M) • $S_{XY}(\omega) = S_{XX}(\omega)H^*(\omega)$ (6M) • $Syy(w) = Sxx(w)\,\lvert H(w)\rvert^2$ (5M)

6 **If $\{X(t)\}$ is a WSS process and if $Y(t)=\int_{-\infty}^{\infty} h(u)\times X(t-u)du$, prove that**

(i) $R_{XY}(\tau)=R_{XX}(\tau)*h(-\tau)$ **(ii)** $R_{YY}(\tau)=R_{XY}(\tau)*h(\tau)$ **where * denotes convolution**

(iii) $S_{XY}(\omega)=S_{XX}(\omega)H^*(\omega)$ **where** $H*(\omega)$ **is the complex conjugate of** $H(\omega)$

(iv) $S_{YY}(\omega)=S_{XX}(\omega)|H(\omega)|^2$. **[*N/D15*,*N/D17*,N/D17] BTL5**

Answer:Page: 5.6,5.9-Dr.A. Singaravelu

- $Rxy(\tau) = Rxx(\tau) * h(\tau)$ (5M)
- $Sxy(w) = Sxx(w)\,H(w)$ (6M)
- $Syy(w) = Sxx(w)\,|H(w)|^2$ (5M)

7 **A system has an impulse response function $h(t)=e^{-\beta t}u(t)$, find the power spectral density of the output $Y(t)$ corresponding to the input $X(t)$.**

[N/D10,12,M/J14,M/J16,*M/J16*,*N/D17*,*Apr/May2019*] BTL5

Answer:Page: 5.23-Dr.A. Singaravelu

- $H(w) = \int_{-\infty}^{\infty} h(t)e^{-iwt}\,dt$ (2M)
- $|H(w)|^2 = \frac{1}{B^2+W^2}$ (2M)
- $Syy(w) = \frac{1}{B^2+W^2}\,Sxx(w)$ (4M)

8 **. A linear time invariant system has an impulse response $h(t)=e^{-\beta t}u(t)$. Find the output auto correlation function $R_{YY}(\tau)$ corresponding to an input $X(t)$. [N/D15,*N/D16*] BTL4**

Answer:Page: 5.23-Dr.A. Singaravelu

- $|H(w)|^2 = \frac{1}{B^2+W^2}$ (3M)
- $Syy(w) = \frac{1}{B^2+W^2}\,Sxx(w)$ (3M)
- $Ryy(\tau) = F^{-1}\,[Syy(w)]$ (2M)

9 **. A circuit has an impulse response given by $h(t)=\begin{cases}\frac{1}{T}\end{cases}, 0\le t\le T$. Express $S_{YY}(\omega)$ in terms of $S_{XX}(\omega)$. [*A/M15*,N/D15, *M/J16*] BTL5**

Answer:Page: 5.21-Dr.A. Singaravelu

- $H(w) = \frac{1}{TW}\,[\sin w\tau - i(1-\cos w\tau)]$ (2M)
- $|H(w)|^2 = \frac{4}{T^2W^2}\,sin^2\left(\frac{WT}{2}\right)$ (1M)

	• $Syy(w) = \|H(w)\|^2\ Sxx(w)$ (1M) $= \left(\frac{\sin\frac{WT}{2}}{\frac{WT}{2}}\right)^2\ Sxx(w)$ (4M)
10	**. Given $R_{XX}(\tau) = Ae^{-\alpha\|\tau\|}$ and $h(t) = e^{-\beta t}u(t)$ where $u(t) = \begin{cases} 1\ ;\ t \geq 0 \\ 0\ ;\ otherwise \end{cases}$. Find the spectral density of the output $Y(t)$. [*<u>Apr/May2019</u>*] BTL4** **Answer:Page: 1.80-Dr.A. Singaravelu** • $\|H(w)\|^2 = \frac{1}{B^2 + W^2}$ (2M) • $Sxx(w) = \frac{2\alpha}{\alpha^2 + W^2}$ (3M) • $Syy(w) = \frac{1}{B^2 + W^2}\ \frac{2\alpha}{\alpha^2 + W^2}$ (3M)
11	**. A random process $X(t)$ is the input to a linear system whose impulse function is $h(t) = 2e^{-t}; t \geq 0$. The autocorrelation function of the process is $R_{XX}(\tau) = e^{-2\|\tau\|}$. Find the power spectral density of the output process $Y(t)$. [M/J13,*<u>Nov/Dec2019</u>*] BTL4** **Answer:Page: 5.26-Dr.A. Singaravelu** • $\|H(w)\|^2 = \frac{4}{W^2 + 1}$ (2M) • $Sxx(w) = \frac{4}{4 + W^2}$ (3M) • $Syy(w) = \frac{16}{(w^2+1)(w^2+4)}$ (3M)
12	**. A random process $X(t)$ with $R_{XX}(\tau) = e^{-2\|\tau\|}$ is the input to a linear system whose impulse response is $h(t) = 2e^{-t}, t > 0$. Find the cross correlation coefficient $R_{XY}(\tau)$ between the input process $X(t)$ and output process $Y(t)$. [A/M15,*<u>A/M18</u>*] BTL4** **Answer:Page: 5.15-Dr.A. Singaravelu** • $\|H(w)\|^2 = \frac{4}{W^2 + 1}$ (2M) • $Sxx(w) = \frac{4}{4 + W^2}$ (3M) • $Syy(w) = \frac{16}{(w^2+1)(w^2+4)}$ (3M)
13	**. $X(t)$ is the input voltage to a circuit (system) and $Y(t)$ is the output voltage. $\{X(t)\}$ is a**

	stationary random process with $\mu_x = 0$ and $R_{XX}(\tau) = e^{-\alpha\|\tau\|}$. Find μ_y , $S_{YY}(\omega)$ and $R_{YY}(\tau)$ if the power transfer function is $H(\omega) = \frac{R}{R + i L\omega}$.[N/D13, M/J14,A/M17,*N/D17*] BTL4 **Answer:Page: 5.16-Dr.A. Singaravelu** • $Sxx(w) = \frac{2\alpha}{\alpha^2 + W^2}$ (4M) • $E(y) = 0$ (2M) • $Syy(w) = \left(\frac{2\alpha}{\alpha^2 + w^2}\right) . \frac{R^2}{R^2 + W^2}$ (5M) • $Ryy(\tau) = \frac{\lambda}{2\alpha} e^{-\alpha\|\tau\|} + \frac{\mu}{2}\left(\frac{L}{R}\right) e^{-\frac{R}{L}\|\tau\|}$ (5M)
14	**. Consider a White Gaussian noise of zero mean and power spectral density $\frac{N_0}{2}$ applied to a low pass RC filter whose transfer function $H(f) = \frac{1}{1 + i 2\pi fRC}$. Find the autocorrelation function of the output random process. Also find the mean square value of the output process. [Nov/Dec2019] BTL4** **Answer:Page: 5.32-Dr.A. Singaravelu** • $Syy(w) = \frac{NO\ \beta^2}{2(\beta^2 + w^2)}$ (3M) • $Ryy(\tau) = \frac{NO\beta}{4} e^{-\beta\|\tau\|}$ (3M) • $E(y^2(t)] = Ryy(0) = \frac{NO\beta}{4}$ (2M)
15	**. Assume a random process $X(t)$ is given as input to a system with transfer function $H(\omega) = 1$ for $-\omega_0 < \omega < \omega_0$. If the autocorrelation function of the input process is $\frac{N_0}{2}\delta(\tau)$, find the autocorrelation function of the output process. [A/M10, *M/J16*] BTL4** **Answer:Page: 5.31-Dr.A. Singaravelu** • $Sxx(w) = \frac{NO}{2}$ (2M) • $Syy(w) = \frac{NO}{2}$ (3M) • $Ryy(\tau) = \frac{NO.\sin(wo\tau)}{2\pi\tau}$ (3M)
16	**. If $X(t)$ is the input voltage to a circuit and $Y(t)$ is the output voltage. $\{X(t)\}$ is a stationary**

	random process with $\mu_X = 0$ and $R_{XX}(\tau) = e^{-2\lvert\tau\rvert}$. Find the mean μ_Y and power spectrum $S_{YY}(\omega)$ of the output if the system transfer function is given by $H(\omega) = \frac{1}{\omega + 2i}$. [N/D10,Nov/Dec2019] BTL4 **Answer:Page: 5.16-Dr.A. Singaravelu** . • $E(y) = 0$ (1M) • $Sxx(w) = \frac{4}{w^2+4}$ (3M) • $Syy(w) = \frac{4}{(w^2+4)^2}$ (4M)
17	**A linear system is described by the impulse response $h(t) = \frac{1}{RC} e^{-\frac{t}{RC}} u(t)$. Assume an input process whose autocorrelation is $B\delta(\tau)$. Find the mean and autocorrelation function of the output process. [A/M11,N/D14,*A/M17*] BTL4** **Answer:Page: 5.33-Dr.A. Singaravelu** • $\lvert H(w)\rvert^2 = \frac{\beta^2}{\beta^2 + w^2}$ (2M) • $E[y(t)] = 0$ (1M) • $Syy(w) = \frac{\beta^2}{\beta^2 + w^2} . \beta$ (2M) • $Ryy(\tau) = \frac{\beta}{2RC} e^{-\frac{\lvert\tau\rvert}{RC}}, -\infty \le \tau \le \infty$ (3M)
18	**. If $\{N(t)\}$ is a band limited white noise centered at a carrier frequency ω_0 such that $S_{NN}(\omega) = \begin{cases} \frac{N_0}{2} & , for\ \lvert\omega - \omega_0\rvert < \omega_B \\ 0 & , elsewhere \end{cases}$. Find the autocorrelation of $\{N(t)\}$. [A/M11 , M/J12] BTL4** **Answer:Page: 5.36-Dr.A. Singaravelu** • $RNN(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \delta NN\ (w) . e^{iw\tau}\ dw$ (2M) • $RNN(\tau) = \frac{NOWB}{2\pi} \left(\frac{\sin wB\tau}{wB\tau}\right) \cos(wB\tau)$ (6M)